AP Physics 1

学生用书

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真实受力、牛顿第二定律与运动图

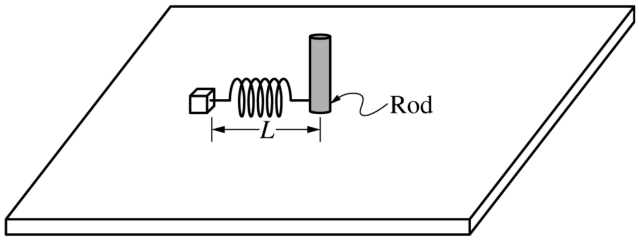
只画指定作用力：方向由施力者决定，长度表示大小

看到“draw the force exerted by the spring”，只画弹簧对物块的这一种力。箭头从物块的点出发，沿弹簧指向转杆；“向心”只是合力的方向，不是另一支力。

图中两次弹簧相同，原长是 L，额外伸长是 d。F_s 表示弹簧力大小，k 表示劲度（本题记为 k_0）。弹力由额外伸长决定；d 较大时，用更长的箭头表示较大的力。不要把弹簧的全长当作伸长。

$$F_s = kd.$$

例题 2023 Q3 (a)(i)



3. (12 points, suggested time 25 minutes)

A small block of mass m_0 is attached to the end of a spring of spring constant k_0 that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length L, as shown. All frictional forces are negligible.

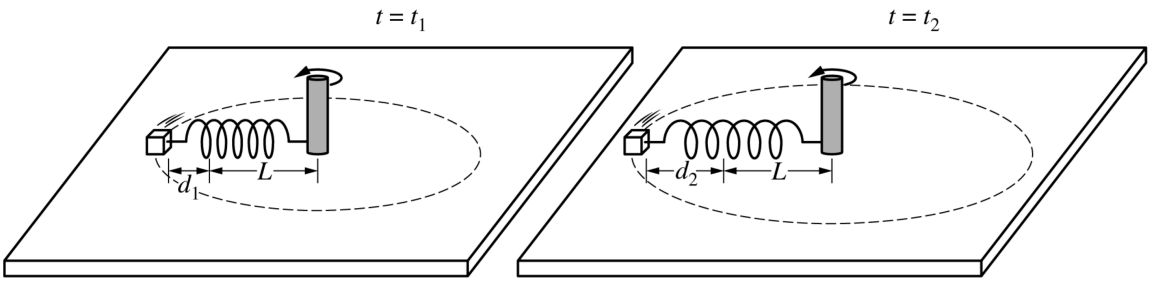
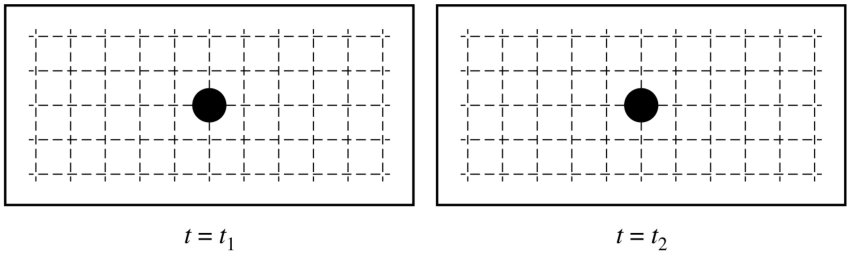


Figure 1

Figure 2

(a) At time $t = t_1$, the rod is spinning such that the block moves in a circular path with a constant tangential speed v_1 and the spring is stretched a distance d_1 from the spring's unstretched length, as shown in Figure 1. At time $t = t_2$, the rod is spinning such that the block moves in a circular path with a constant tangential speed v_2 and the spring is stretched a distance d_2 from the spring's unstretched length, where $d_2 > d_1$, as shown in Figure 2.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times $t = t_1$ and $t = t_2$. The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.
Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



解：

原图中物块在转杆左边，弹簧向右拉物块。在两个点上各画一支向右的弹簧力箭头，第二支更长。此问只要求弹簧施给物块的力，不画额外的“向心力”。



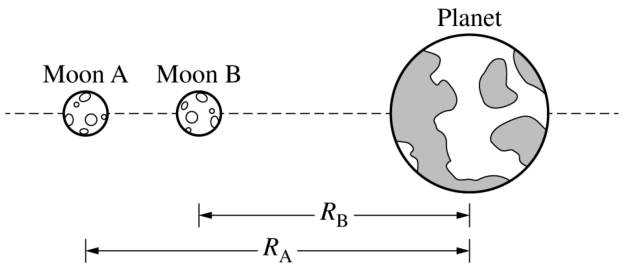
两次弹簧力都朝转杆，伸长较大时箭头较长。

画完整受力图：逐支标明实际施力者

先圈定研究的物体，再问“谁在拉它或推它”。每一种相互作用画一支箭头；别把合力或一个力的分量重复画进去。引力朝施力天体，重力竖直向下，绳力沿绳，支持力垂直接触面。

这里用点代表所研究的物体，箭头都从点出发。每支箭头标出具体施力者；两个方向相反的引力也要分别画出，不能只画它们合成之后的结果。

例题 2022 Q2 (a)



Note: Figure not drawn to scale.

2. (12 points, suggested time 25 minutes)

Two identical moons, Moon A and Moon B, orbit a planet. The mass m_0 of each moon is significant, but less than the mass m_p of the planet. At some point in their orbits, the planet and the two moons are aligned as shown in the figure.

(a) The following dots represent the two moons when they are at the locations shown in the previous figure. On each dot, draw and label the forces (not components) exerted on Moon A and on Moon B. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Moon A

Moon B

解：

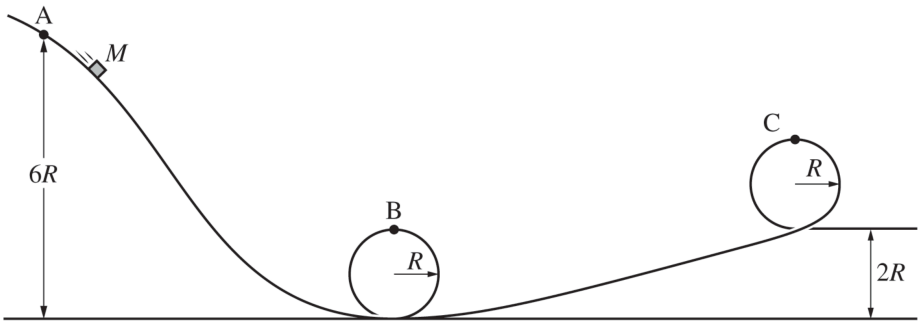
A 上画两支向右的力：行星对 A 的引力、B 对 A 的引力。B 上画行星对 B 向右的引力，以及 A 对 B 向左的引力。两个方向相反的力要分别标施力者。



箭头标方向与施力者；图中的长度不规定两种引力的实际大小。

在圆环顶端，圆心位于物块下方；轨道支持力朝接触面的法线。此问要求真实力，先分别找重力与轨道施力者。

练习 2024 Q1 (c)(i)



1. (7 points, suggested time 13 minutes)

A block of mass M is released from rest at Point A, a height $6R$ above the horizontal. After being released, the block slides down a track, as shown. When released from Point A, the block does not lose contact with the track at any point. Points B and C are located at the highest points of their respective circular loops, both of radius R. All frictional forces are negligible.

(c)

i. On the following dot that represents the block, **draw** and **label** the forces (not components) that are exerted on the block at the instant the block slides through Point C. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

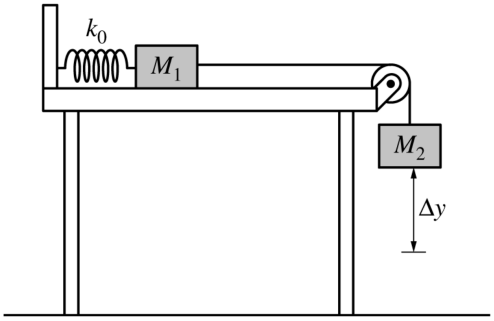


用直接受力解释先加速后减速

下落并不保证加速度向下。先比较物块直接受到的重力和张力，决定合力；再比较合力方向与当前速度方向：同向，速率增加；反向，速率减小。

弹簧越拉越长会使绳张力改变，但弹簧没有直接碰到悬挂的物块，所以不在它的受力图中额外加弹簧力。

例题 2022 Q1 (a)(i)



1. (7 points, suggested time 13 minutes)

Two blocks are connected by a string that passes over a pulley, as shown above. Block 1 is on a horizontal surface and is attached to a spring that is at its unstretched length. Frictional forces are negligible in the pulley's axle and between the block and the surface. Block 2 is released from rest and moves downward before momentarily coming to rest.

k_0 is the spring constant of the spring.
M_1 is the mass of block 1.
M_2 is the mass of block 2.
Δy is the distance block 2 moves before momentarily coming to rest.

(a)

i. Block 2 starts from rest and speeds up, then it slows down and momentarily comes to rest at a position below its initial position. In terms of only the forces directly exerted on block 2, explain why block 2 initially speeds up and explain why it slows down to a momentary stop.

解：

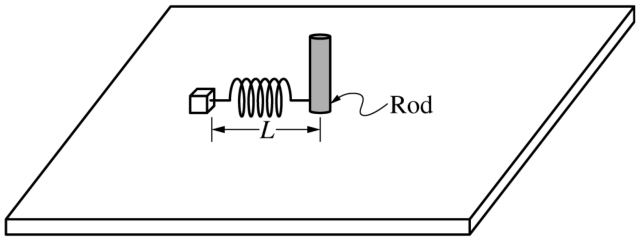
物块 2 直接受向下的重力 M_2g 与向上的张力 T。开始弹簧未伸长，张力较小，重力大于张力，合力向下，与下落速度同向，于是加速。弹簧越拉越长，张力增大；当张力超过重力时，合力向上而速度仍向下，于是减速直至首次停止。不要把弹簧力当作直接施给物块 2 的第三个力。

同一弹簧比较力：比较伸长，解释箭头长短

解释两支力箭头时，要把原题中的两个伸长量连到力的大小，而不是只重复“第二支较长”。同一弹簧劲度不变，弹力与额外伸长成正比。d_2 大于 d_1，才是判断大小的条件。

例题 2023 Q3 (a)(ii)

本问承接 2023 Q3 (a)(i)。



3. (12 points, suggested time 25 minutes)

A small block of mass m_0 is attached to the end of a spring of spring constant k_0 that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length L, as shown. All frictional forces are negligible.

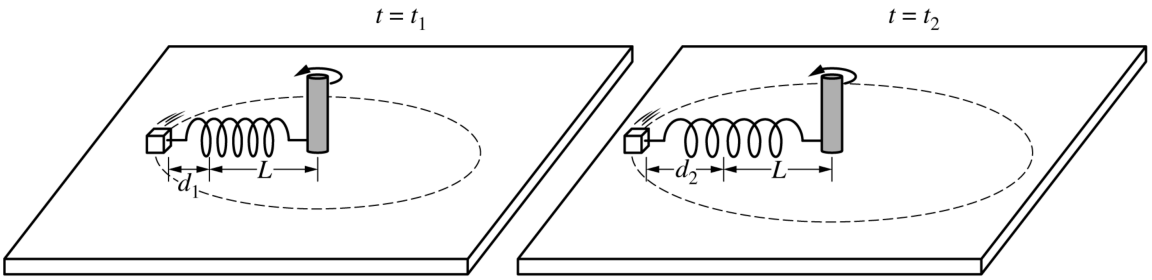


Figure 1

Figure 2

(a) At time $t = t_1$, the rod is spinning such that the block moves in a circular path with a constant tangential speed v_1 and the spring is stretched a distance d_1 from the spring's unstretched length, as shown in Figure 1. At time $t = t_2$, the rod is spinning such that the block moves in a circular path with a constant tangential speed v_2 and the spring is stretched a distance d_2 from the spring's unstretched length, where $d_2 > d_1$, as shown in Figure 2.

ii. Referencing d_1 and d_2, describe your reasoning for drawing the arrows the length that you did in part (a)(i).

解：

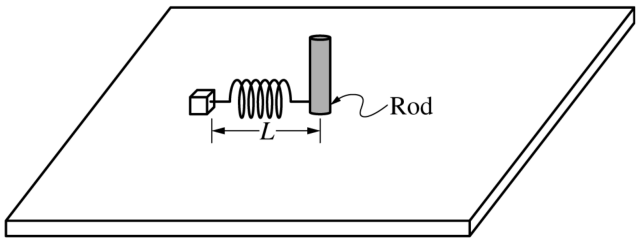
两次使用同一弹簧，劲度不变。第二次的伸长 d_2 大于 d_1，弹力因此更大；箭头长短比较的是伸长，而不是弹簧全长。

半径和合力同时变化：不用方程比较圆周速率

圆周运动即使速率不变，速度方向也一直转向圆心，需要向内的合力。相同速率下，圆周半径变大时，转向更缓，需要的径向加速度和合力反而较小。

题中伸长增加，会同时增大弹力和整个轨道半径。不能只看其中一个量：在更大的轨道上还需要更大的合力，速率必须发生相应变化。原题明确要求无方程论证，就用这两段因果说明。

例题 2023 Q3 (a)(iii)



3. (12 points, suggested time 25 minutes)

A small block of mass m_0 is attached to the end of a spring of spring constant k_0 that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length L, as shown. All frictional forces are negligible.

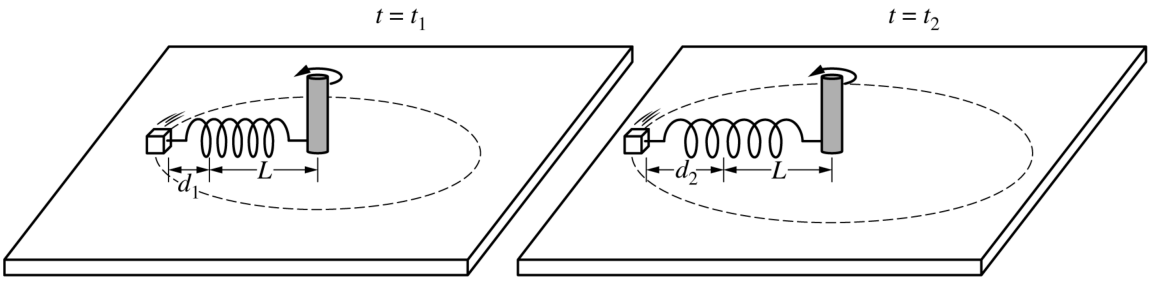


Figure 1 Figure 2

(a) At time $t = t_1$, the rod is spinning such that the block moves in a circular path with a constant tangential speed v_1 and the spring is stretched a distance d_1 from the spring's unstretched length, as shown in Figure 1. At time $t = t_2$, the rod is spinning such that the block moves in a circular path with a constant tangential speed v_2 and the spring is stretched a distance d_2 from the spring's unstretched length, where $d_2 > d_1$, as shown in Figure 2.

iii. Is the tangential speed v_1 of the block at time $t = t_1$ greater than, less than, or equal to the tangential speed v_2 of the block at time $t = t_2$?

_____ $v_1 > v_2$ _____ $v_1 < v_2$ _____ $v_1 = v_2$

Justify your answer without using equations.

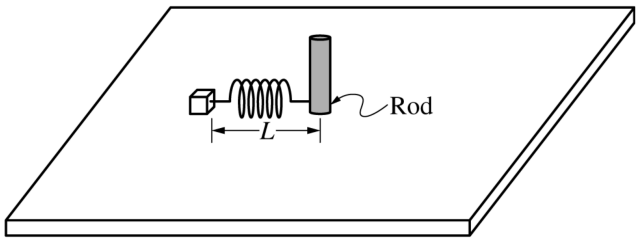
解：

选择第二次速率更大。弹簧提供全部水平合力。第二次轨道半径变大，而朝圆心的合力也变大；若速率不变，半径变大反而会使所需的向心合力减小。要在更大的轨道上获得更大的向心合力，第二次速率必须更大。这同时处理了力与半径的变化，没有偷偷把半径当作不变。

由实际力求合力：先消去互相平衡的方向

在水平桌面上，物块没有竖直加速度，向上的支持力与向下的重力抵消。合力只剩沿弹簧的力；把题给劲度和伸长写进胡克定律。L 是原长，它没有进入弹力。

例题 2023 Q3 (b)(i)



3. (12 points, suggested time 25 minutes)

A small block of mass m_0 is attached to the end of a spring of spring constant k_0 that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length L, as shown. All frictional forces are negligible.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance d from its unstretched length L.

i. Determine an expression for the magnitude of the net force F_{net} exerted on the block. Express your answer in terms of m_0, k_0, L, d, and fundamental constants, as appropriate.

解：

重力与支持力抵消，合力大小就是弹簧力：

$$F_{\text{net}} = k_0 d.$$

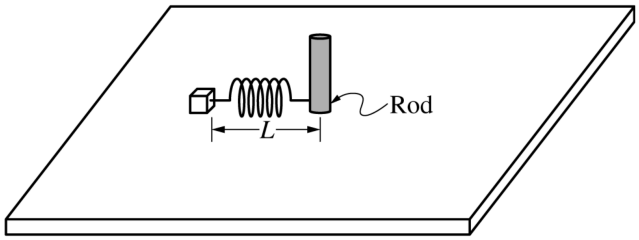
从径向第二定律推导速率：半径取全长

题目要求 derive 时，从牛顿第二定律起笔，不直接抄最后的速率式。转圈的径向加速度是速率平方除以半径；弹簧提供这个方向的合力。

弹簧力用伸长 d，轨道半径用 L+d。两者都测长度，却表示不同对象；把它们放对位置，再解出速率的正根。

$$F_{\text{net},r} = ma_r, \qquad a_r = \frac{v^2}{r}.$$

例题 2023 Q3 (b)(ii)



3. (12 points, suggested time 25 minutes)

A small block of mass m_0 is attached to the end of a spring of spring constant k_0 that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length L, as shown. All frictional forces are negligible.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance d from its unstretched length L.

ii. Derive an equation for the tangential speed v of the block. Express your answer in terms of m_0, k_0, L, d, and fundamental constants, as appropriate.

解：

从牛顿第二定律开始。圆周半径取当前全长 L+d，弹簧力只取伸长 d：

$$F_{\text{net},r} = m_0 a_r, \qquad k_0 d = \frac{m_0 v^2}{L+d}.$$

$$v^2 = \frac{k_0 d(L+d)}{m_0}, \qquad v = \sqrt{\frac{k_0 d(L+d)}{m_0}}.$$

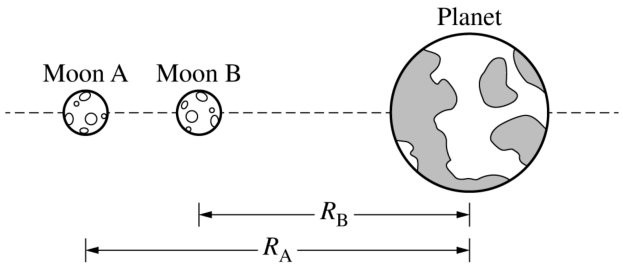
速率用非负根。题目中的 k_0、m_0、L 均为正，d 是伸长量。

同线引力比较：同向相加与反向抵消

月球 A 受两个同向引力，月球 B 受两个反向引力。比较合力大小，先确定同向相加、反向相减；“离行星更近”只决定其中一种引力，不决定总合力。

要说明“可能较大”，只需指出符合题设的一种情形。两项反向力接近时，合力可很小；月球间距很大时，月球之间的引力又可以很小。两种情形需要分别讨论。

例题 2022 Q2 (b)(i)



Note: Figure not drawn to scale.

2. (12 points, suggested time 25 minutes)

Two identical moons, Moon A and Moon B, orbit a planet. The mass m_0 of each moon is significant, but less than the mass m_p of the planet. At some point in their orbits, the planet and the two moons are aligned as shown in the figure.

(b) Consider the net gravitational force exerted on each moon due to the planet and the other moon.

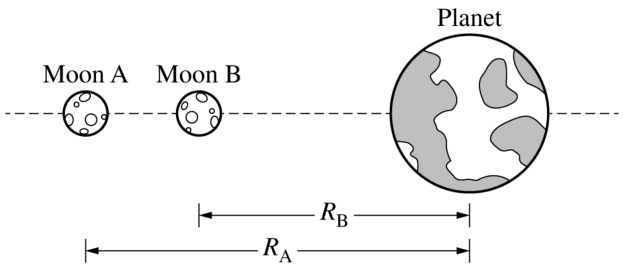
i. Justify why the magnitude of the net force exerted on Moon A could be larger than the magnitude of the net force exerted on Moon B.

解：

A 的两力同向相加，B 的两力反向抵消。如果 B 上两个引力大小接近，其合力可很小；此时 A 的合力可以更大。

上一问利用两项反向力相近。这一问换一种可能的距离关系：考虑两个月球分开很远，使月球之间的吸引较弱，再比较行星的作用。

例题 2022 Q2 (b)(ii)



Note: Figure not drawn to scale.

2. (12 points, suggested time 25 minutes)

Two identical moons, Moon A and Moon B, orbit a planet. The mass m_0 of each moon is significant, but less than the mass m_p of the planet. At some point in their orbits, the planet and the two moons are aligned as shown in the figure.

(b) Consider the net gravitational force exerted on each moon due to the planet and the other moon.

ii. Justify why the magnitude of the net force exerted on Moon B could be larger than the magnitude of the net force exerted on Moon A.

解：

B 离行星更近，行星对 B 的引力更大。如果 A 离得很远，两个月球间距也较大，月球间引力变弱；B 上朝行星的引力就能占优势，B 的合力可以更大。只凭这张位置关系图，不足以指定两种合力大小的唯一次序。

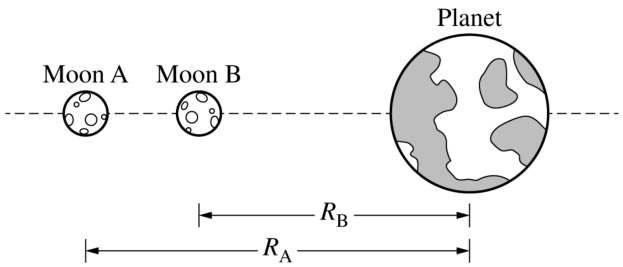
引力合力列式：距离取对应两物体的间距

先取一条正方向。每个引力大小由两物体质量和它们中心间距决定；同向正、反向负，然后相加。A 到行星的距离、B 到行星的距离以及两月球的间距是三个不同距离。

$$F_g = G\frac{m_1 m_2}{r^2}.$$

G 是引力常量，m_1、m_2 是这次相互吸引的两物体质量，r 是它们的间距。带符号的合力说明方向；比较合力大小时，应取该结果的绝对值。

例题 2022 Q2 (c)



Note: Figure not drawn to scale.

2. (12 points, suggested time 25 minutes)

Two identical moons, Moon A and Moon B, orbit a planet. The mass m_0 of each moon is significant, but less than the mass m_p of the planet. At some point in their orbits, the planet and the two moons are aligned as shown in the figure.

(c) Derive expressions for both of the following quantities. Express your answers in terms of m_0, m_p, R_A, R_B, and physical constants, as appropriate.

• The net force F_A exerted on Moon A

• The net force F_B exerted on Moon B

解：

取向右为正。月球与行星之间的距离分别为 R_A、R_B，而月球间距离为 R_A-R_B：

$$F_A = \frac{Gm_0m_p}{R_A^2} + \frac{Gm_0^2}{(R_A-R_B)^2},$$
$$F_B = \frac{Gm_0m_p}{R_B^2} - \frac{Gm_0^2}{(R_A-R_B)^2}.$$

这里 F_B 是有符号的水平合力；若结果为负，合力朝左。求合力大小时使用 $|F_B|$。

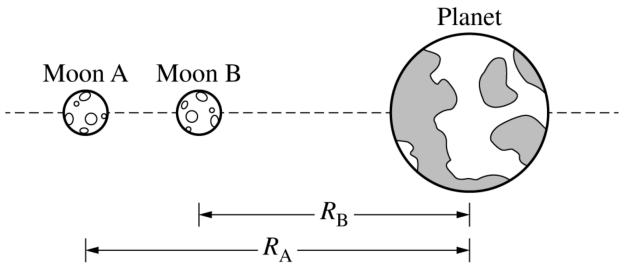
检验合力式能否支持某种大小关系

先找到待核验的原结论，再看式子中哪些项相加、哪些项抵消。不要把公式重新抄一遍当成解释：须说出什么符合题设的条件使结论可能成立。

例如一个合力含两正项、另一个含两项之差，后者的两项可接近而相消；换成相距很远时，含月球间距的项又会变小。

例题 2022 Q2 (d)(i)

本问承接 2022 Q2 (b)(i)、2022 Q2 (c)。



Note: Figure not drawn to scale.

2. (12 points, suggested time 25 minutes)

Two identical moons, Moon A and Moon B, orbit a planet. The mass m_0 of each moon is significant, but less than the mass m_p of the planet. At some point in their orbits, the planet and the two moons are aligned as shown in the figure.

(d)

i. Could the expressions in part (c) support your reasoning in part (b)(i) ?

___ Yes ___ No

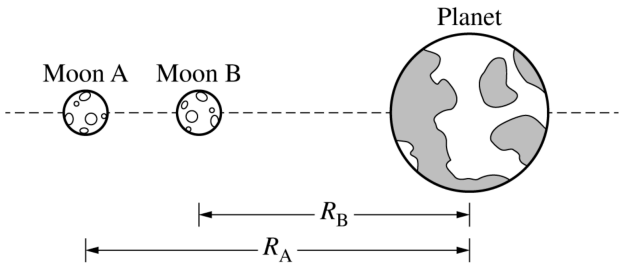
Explain your reasoning.

解：

可以支持。A 式是两个正项相加；B 式中两项相减，若大小接近，B 的合力大小很小，A 可以更大。

练习 2022 Q2 (d)(ii)

本问承接 2022 Q2 (b)(ii)、2022 Q2 (c)。



Note: Figure not drawn to scale.

2. (12 points, suggested time 25 minutes)

Two identical moons, Moon A and Moon B, orbit a planet. The mass m_0 of each moon is significant, but less than the mass m_p of the planet. At some point in their orbits, the planet and the two moons are aligned as shown in the figure.

(d)

ii. Could the expressions in part (c) support your reasoning in part (b)(ii) ?

___ Yes ___ No

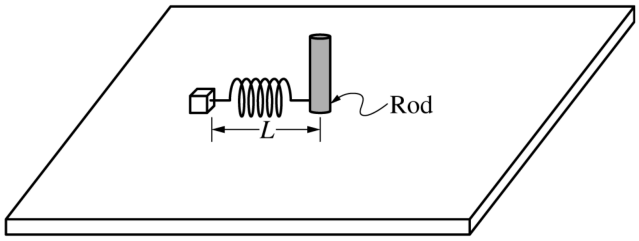
Explain your reasoning.

检验速率式：所有变量的变化是否同向

检验“伸长增大，速率是否增大”，先固定同一物块、同一弹簧和同一原长，逐个判断式子里含伸长的因子。两个正因子都变大时，乘积和正平方根都变大；这才把方程连回定性结论。

例题 2023 Q3 (c)

本问承接 2023 Q3 (a)(iii)、2023 Q3 (b)(ii)。



3. (12 points, suggested time 25 minutes)

A small block of mass m_0 is attached to the end of a spring of spring constant k_0 that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length L, as shown. All frictional forces are negligible.

(c) Does your equation for the tangential speed v of the block from part (b)(ii) agree with your reasoning from part (a) ?

____ Yes ____ No

Explain your reasoning.

解：

回答 Yes。伸长增加时，d 与 L+d 都增加，它们的乘积增加，开平方后的速率也增加，与(a)的定性比较一致。

恒加速度的图像：速度斜率与位置凹凸

先规定坐标正方向，再分清“正在往哪边走”和“加速度朝哪边”。速度—时间图的斜率是加速度；加速度恒定时，这幅图是直线。位置—时间图的切线斜率是速度，曲线凹凸由加速度的符号决定。

$$v = v_i + at, \qquad x = x_i + v_i t + \frac{1}{2}at^2.$$

v_i 和 x_i 是开始时的速度和位置，t 从该开始时刻计。水滴抛出后只有向下重力：水平分量恒定、竖直分量以负斜率变化。小车在斜坡坐标中上坡时，速度与向下的加速度可异号。

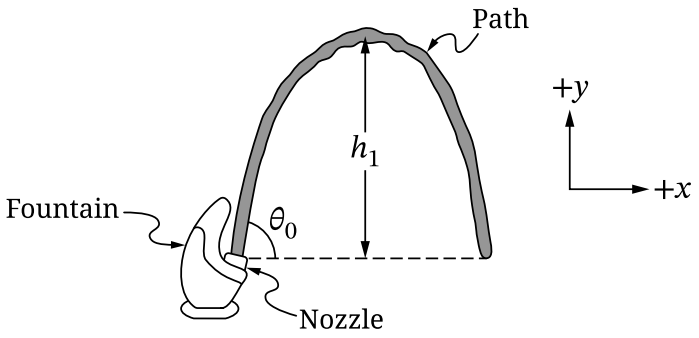
例题 2026 Q1 (a)(i)

Question 1: Version J

1. Water exits the nozzle of a fountain at an angle θ_0 above the horizontal.

At time $t = 0$, a droplet of water exits the nozzle and follows the path shown in Figure 1. The droplet reaches a maximum height h_1 above the nozzle.

Figure 1



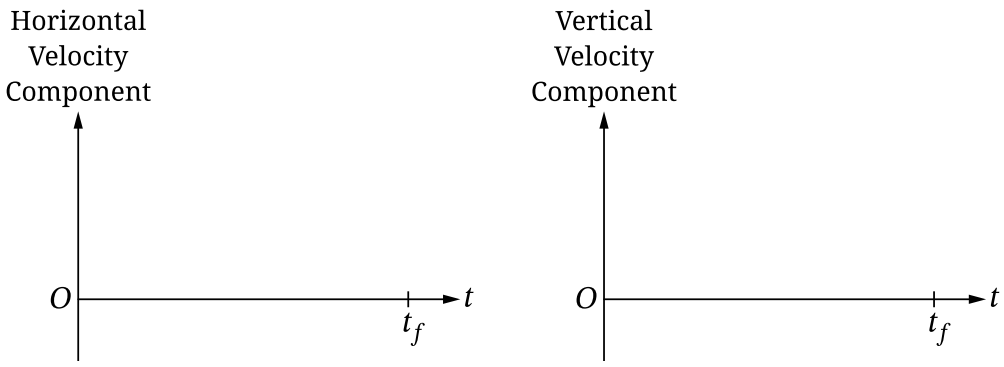
Note: Figure not drawn to scale.

At $t = t_f$, the droplet returns to the same height at which the droplet exited the nozzle.

Part A

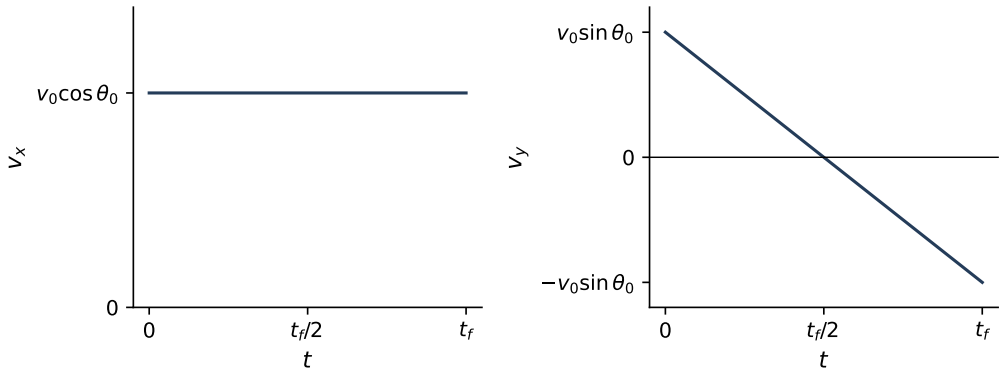
i. On the axes shown in Figure 2, **sketch** graphs of the horizontal and vertical components of the velocity of the water droplet as functions of t from $t = 0$ to $t = t_f$.

Figure 2



解：

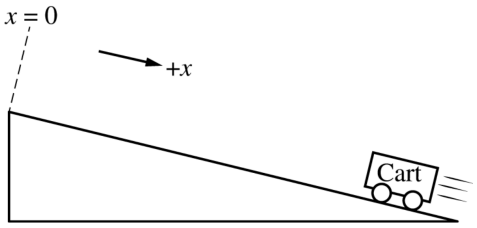
取向右、向上为正。水平速度为非零正常量，画水平直线；竖直速度从正值开始，以固定负斜率下降，在最高点过零，回到喷口高度时为与初始竖直速度等大、反向的负值。最高点在总飞行时间 t_f 的一半。



最高点的竖直速度为零，水平速度始终不为零。

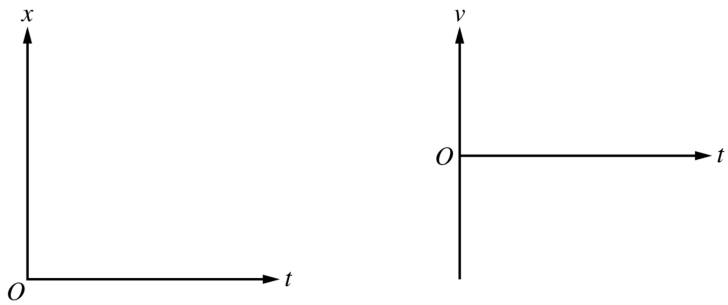
这次还要画位置图。沿原题坡向下为正，小车被推向坡顶时速度为负；读位置图时，用其切线斜率逐渐趋零来表示减速上坡。

例题 2023 Q2 (d)



The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The x-axis is still defined to be parallel to the ramp with the origin at the top.

(d) On the following graphs, sketch the position x and velocity v as functions of time t that correspond to the scenario shown while the cart moves up the ramp.

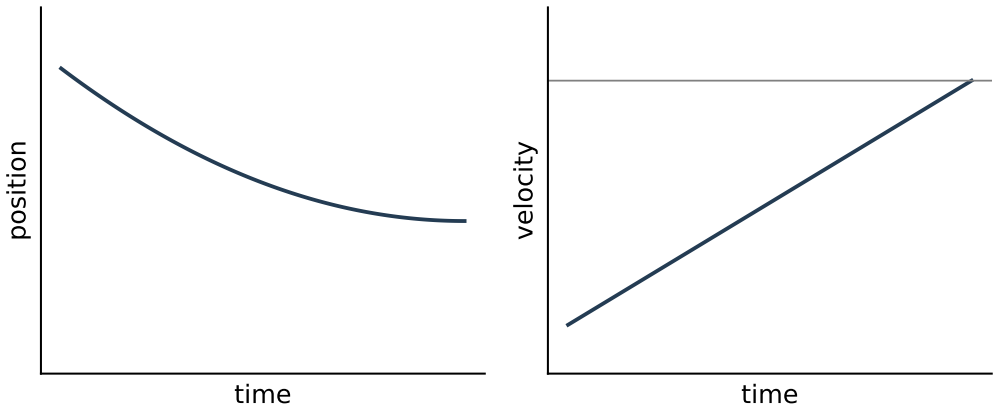


解：

仍沿坡向下为正。小车上坡时初速度为负，加速度为正；位置从正的起始位置下降，曲线向上凹，负切线斜率逐渐增到零。速度线从负值出发，以正斜率增加，首次停止处达到零。

$$x(t) = x_i + v_i t + \frac{1}{2}at^2, \qquad v(t) = v_i + at.$$

示意图只表达符号、斜率与凹凸，不给原题未提供的推车初速和时间赋数值。



上坡阶段速度为负，加速度为正；位置图向上凹。

利用最高点竖直速度为零反求喷口速率

水滴到最高点时，竖直速度为零，水平速度仍存在。最高高度只由初速度的竖直分量决定；先把初速投影到竖直方向，再用不含时间的恒加速度关系。

$$v_y^2 = v_{y,0}^2 + 2a_y\Delta y, \qquad v_{y,0} = v_0 \sin\theta_0.$$

向上为正时 a_γ=−g，Δy 是最高点相对喷口的高度。题给 θ_0 是喷射方向与水平的夹角，不能把总初速直接当竖直初速。

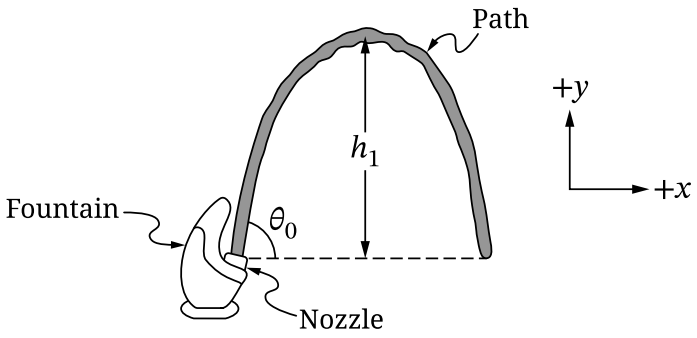
例题 2026 Q1 (a)(ii)

Question 1: Version J

1. Water exits the nozzle of a fountain at an angle θ_0 above the horizontal.

At time $t = 0$, a droplet of water exits the nozzle and follows the path shown in Figure 1. The droplet reaches a maximum height h_1 above the nozzle.

Figure 1



Note: Figure not drawn to scale.

At $t = t_f$, the droplet returns to the same height at which the droplet exited the nozzle.

Part A

ii. **Derive** an expression for the speed of the water exiting the nozzle. Express your final answer in terms of θ_0, h_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

先对竖直方向使用恒加速度关系。最高点竖直速度为零，竖直升高 h_1，初竖直速度为 $v_0 sin\theta_0$：

$$v_y^2 = v_{y,0}^2 + 2a_y\Delta y, \qquad 0 = (v_0 \sin\theta_0)^2 - 2gh_1,$$

$$v_0 = \frac{\sqrt{2gh_1}}{\sin\theta_0}.$$

喷射角向上，

$$\sin\theta_0 > 0$$

速率取正值。

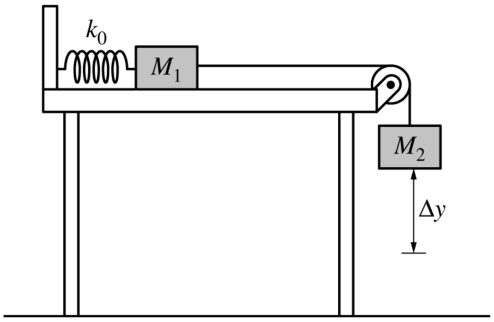
机械能、功与能量图

判断机械能是否守恒：先圈出系统

先写系统包含谁。地球在系统内时，重力作用用重力势能的转移描述；弹簧在系统内时，用弹簧势能。无摩擦等耗散，且外界不向系统净输入功时，这些能量与动能的总和保持不变。

物体在运动并不等于总能量增加。支持力若垂直运动，不做功；把系统内互相传递的能量误当成额外能量，会重复计算。

例题 2022 Q1 (b)



1. (7 points, suggested time 13 minutes)

Two blocks are connected by a string that passes over a pulley, as shown above. Block 1 is on a horizontal surface and is attached to a spring that is at its unstretched length. Frictional forces are negligible in the pulley's axle and between the block and the surface. Block 2 is released from rest and moves downward before momentarily coming to rest.

k_0 is the spring constant of the spring.
M_1 is the mass of block 1.
M_2 is the mass of block 2.
Δy is the distance block 2 moves before momentarily coming to rest.

(b) Indicate whether the total mechanical energy of the blocks-spring-Earth system changes as block 2 moves downward.

___ Changes ___ Does not change

Briefly explain your reasoning.

解：

选择 Does not change。物块—弹簧—地球系统无耗散；桌面支持力不沿运动方向做功，机械能只在动能、重力势能与弹簧势能之间转移。

静止端点之间的能量转移：重力势能变弹簧势能

选开始静止和首次停住的两个状态，动能在两端均为零。重力势能减少的大小由竖直下降量决定，弹簧势能增加由形变量的平方决定。两端关系能避开途中张力和加速度的变化。

式中的 K 是动能，U_g 是重力势能，U_s 是弹簧势能；下标 i 表示选定的开始状态，f 表示选定的结束状态。三种能量在每个状态分别相加，再比较两端总和。

$$K_i + U_{g,i} + U_{s,i} = K_f + U_{g,f} + U_{s,f}, \quad U_g = mgh, \quad U_s = \frac{1}{2}k(\Delta\ell)^2.$$

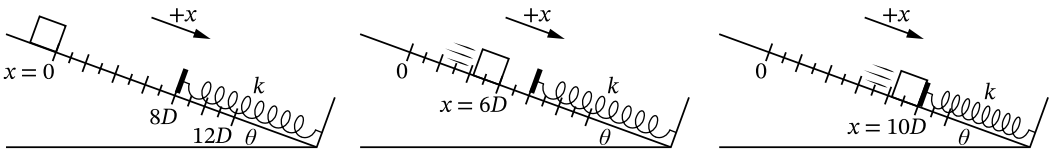
h 是相对所选零点的竖直高度，Δℓ 是相对原长的形变，不能用整个弹簧长度替代。斜坡上的路程 s 对应竖直下降 s sinθ；连接物块的不可伸长绳则使两端移动距离相同。

例题 2025 Q2 (b)

Question 2: Version J

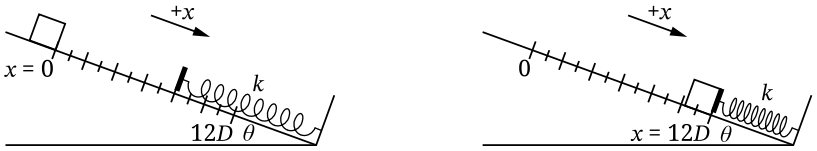
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2. A block of mass M is released from rest at position $x = 0$ near the top of a ramp. The ramp makes an angle of θ with the horizontal. The block slides down the ramp with negligible friction. At $x = 8D$ the block makes contact with an uncompressed spring with spring constant k. The spring is then compressed and the block momentarily comes to rest at $x = 12D$. Figure 1 shows the instants when the block is at $x = 0$, $x = 6D$, and $x = 10D$, respectively.



B. Figure 5 shows the block at $x = 0$ when the block is released from rest and the block at $x = 12D$ when the block momentarily comes to rest against the compressed spring.

Figure 5



Starting with conservation of energy, **derive** an equation for the spring constant k. Express your answer in terms of M, θ, D, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

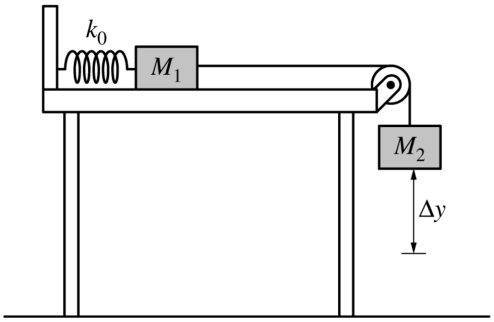
从释放点到 12D 位置，两端均静止。竖直下降量是 12Dsinθ，弹簧从 8D 位置才开始压缩，所以最终压缩量是 4D：

$$E_i = E_f, \qquad Mg(12D\sin\theta) = \frac{1}{2}k(4D)^2,$$

$$k = \frac{24MgD\sin\theta}{16D^2} = \frac{3Mg\sin\theta}{2D}.$$

这次弹簧的伸长与悬挂物块下降量相同，因为连接绳长度不变。端点仍都静止，先把同一个正位移放入重力势能减少与弹簧势能增加。

练习 2022 Q1 (a)(ii)



1. (7 points, suggested time 13 minutes)

Two blocks are connected by a string that passes over a pulley, as shown above. Block 1 is on a horizontal surface and is attached to a spring that is at its unstretched length. Frictional forces are negligible in the pulley's axle and between the block and the surface. Block 2 is released from rest and moves downward before momentarily coming to rest.

k_0 is the spring constant of the spring.
M_1 is the mass of block 1.
M_2 is the mass of block 2.
Δy is the distance block 2 moves before momentarily coming to rest.

(a)

ii. Derive an expression for the distance Δy that block 2 travels before momentarily coming to rest. Express your answer in terms of k_0, M_1, M_2, and physical constants, as appropriate.

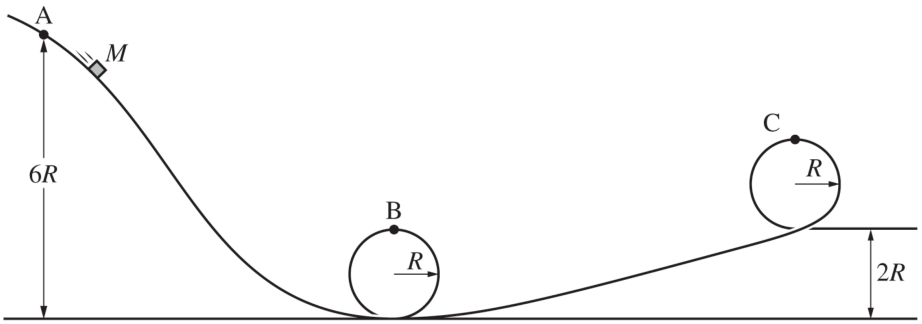
释放高度给定、终点仍在运动：用能量求速率

若末状态不是静止，不能删去它的动能。先用高度差求势能减少，再把它等于末动能；这一步只决定速率，不决定速度方向。

$$\frac{1}{2}mv^2 = mg(h_i - h_f).$$

h_i 和 h_f 是同一零点下的两个高度。质量在此式中约掉，速率取非负根。

例题 2024 Q1 (b)



1. (7 points, suggested time 13 minutes)

A block of mass M is released from rest at Point A, a height $6R$ above the horizontal. After being released, the block slides down a track, as shown. When released from Point A, the block does not lose contact with the track at any point. Points B and C are located at the highest points of their respective circular loops, both of radius R. All frictional forces are negligible.

(b) Starting with conservation of energy, **derive** an expression for the speed of the block at Point B. Express your answer in terms of R and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference book.

解：

从能量守恒开始写所选两个状态：

$$E_A = E_B, \qquad Mg(6R) = Mg(2R) + \frac{1}{2}Mv_B^2,$$

$$v_B^2 = 8gR, \qquad v_B = \sqrt{8gR}.$$

能量柱：先定零点与总高度，再分给各项

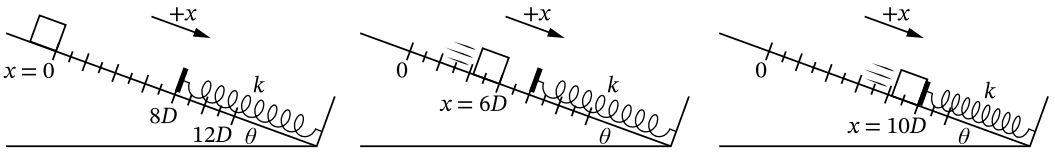
从题给能量柱读出共同的总能量，再问每个时刻是否运动、是否已经接触弹簧、离势能零点还有多高。静止使动能为零；未形变使弹簧势能为零；重力势能随竖直高度变化。

能量为零要在零线上画明确横线。每根柱都从零线开始，使用原图同一尺度。无耗散时，各状态柱高之和相同。

例题 2025 Q2 (a)

Question 2: Version J

2. A block of mass M is released from rest at position $x = 0$ near the top of a ramp. The ramp makes an angle of θ with the horizontal. The block slides down the ramp with negligible friction. At $x = 8D$ the block makes contact with an uncompressed spring with spring constant k. The spring is then compressed and the block momentarily comes to rest at $x = 12D$. Figure 1 shows the instants when the block is at $x = 0$, $x = 6D$, and $x = 10D$, respectively.



A. Figure 4 shows an energy bar chart that represents the kinetic energy K of the block, the gravitational potential energy U_g of the block-spring-Earth system, and the spring potential energy U_s of the block-spring-Earth system at the instant that the block is at $x = 10D$. The gravitational potential energy U_g of the block-spring-Earth system is defined to be zero when the block momentarily comes to rest at $x = 12D$.

Draw shaded bars that represent K, U_g, and U_s to complete the energy bar charts in Figure 2 and Figure 3 for when the block is released from rest at $x = 0$ and for when the block is at $x = 6D$, respectively.

- Shaded bars should start at the dashed line that represents zero energy.
- Represent any energy that is equal to zero with a distinct line on the zero-energy line.
- The relative heights of each shaded bar should reflect the magnitude of the respective energy consistent with the scale used in Figure 4.

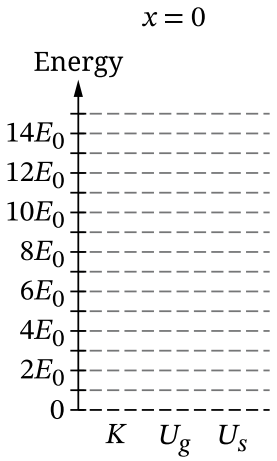


Figure 2

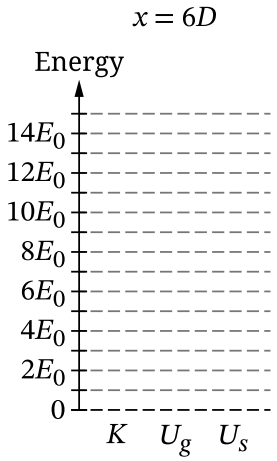


Figure 3

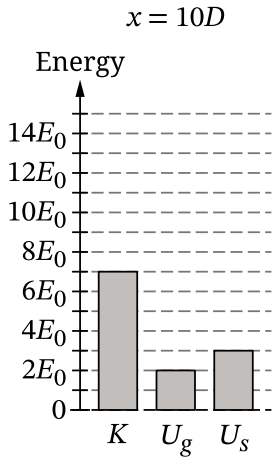


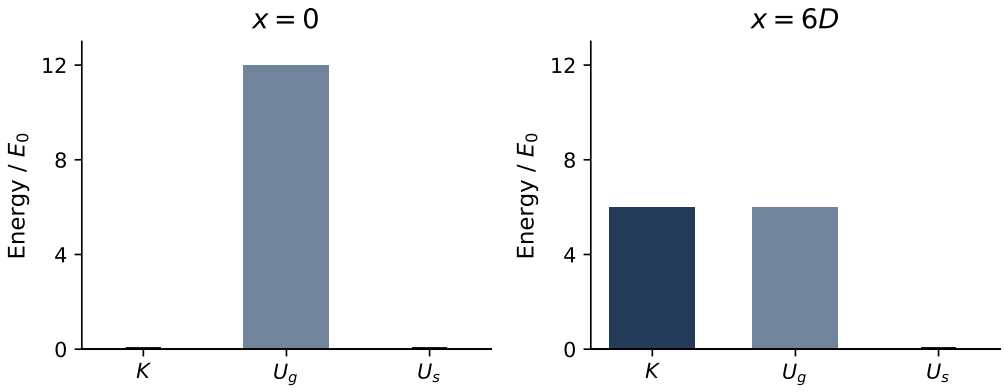
Figure 4

解：

选物块—弹簧—地球系统，忽略摩擦。沿坡坐标 x 从释放点向下增加，重力势能零点在沿坡坐标 12D 的位置。原题 10D 位置的能量柱给出总能量为 $12E_0$。

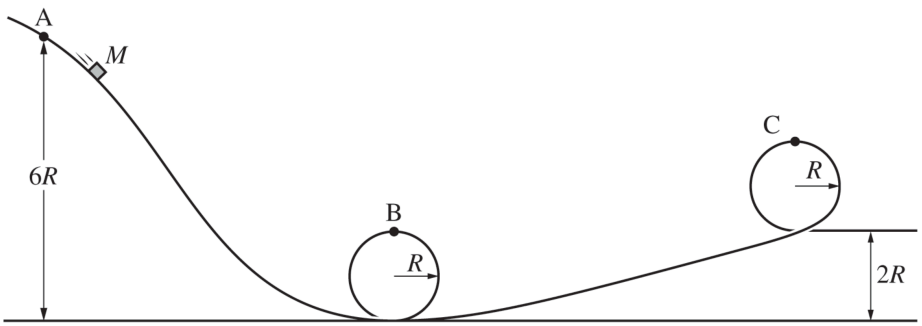
在释放点，物块静止、弹簧未压缩，动能与弹簧势能均为零，全部能量是重力势能，柱高 $12E_0$。6D 位置距离零高度沿坡还有 6D，重力势能是原来一半；弹簧尚未接触，剩余能量全为动能。

$$x = 0: \quad (K, U_g, U_s) = (0, 12E_0, 0),$$
$$x = 6D: \quad (K, U_g, U_s) = (6E_0, 6E_0, 0).$$



零能量画在零线上；两状态各柱总高度相同。

练习 2024 Q1 (a)



1. (7 points, suggested time 13 minutes)

A block of mass M is released from rest at Point A, a height $6R$ above the horizontal. After being released, the block slides down a track, as shown. When released from Point A, the block does not lose contact with the track at any point. Points B and C are located at the highest points of their respective circular loops, both of radius R. All frictional forces are negligible.

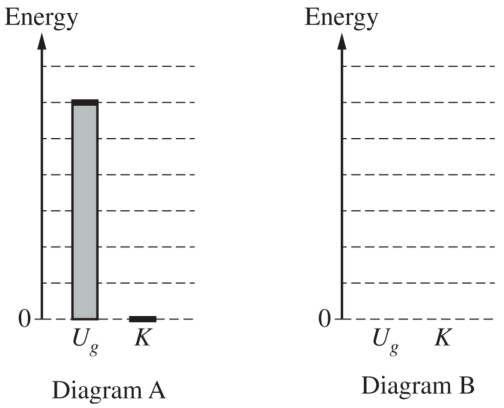


Diagram A shows an energy bar chart that represents the gravitational potential energy U_g of the block-Earth system and the kinetic energy K of the block at Point A, when the block is released from rest at height $6R$.

(a) **Draw** shaded regions in Diagram B that represent the gravitational potential energy U_g and kinetic energy K of the block-Earth system when the block is located at Point B, a height $2R$ above the horizontal.

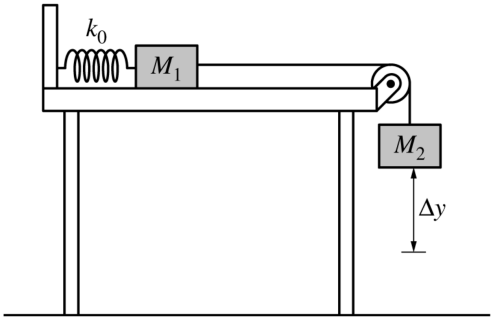
- Shaded regions should start at the dashed line that represents zero energy.
- Represent any energy that is equal to zero with a distinct line on the zero-energy line.
- The relative height of each shaded region should reflect the magnitude of the respective energy consistent with the scale shown in Diagram A.

有摩擦的能量柱：系统丢失的机械能不能画回柱内

表面没有被包含在题给系统内时，摩擦把部分机械能转移给表面，最终机械能总和小于开始。首次停止仍使动能为零，但停止位置也改变了；不能照搬无摩擦时的两个势能柱。

题目未给摩擦系数时，能判断正负与总和范围，却不能假定唯一柱高。把原题允许的关系画出来即可。

例题 2022 Q1 (c)



1. (7 points, suggested time 13 minutes)

Two blocks are connected by a string that passes over a pulley, as shown above. Block 1 is on a horizontal surface and is attached to a spring that is at its unstretched length. Frictional forces are negligible in the pulley's axle and between the block and the surface. Block 2 is released from rest and moves downward before momentarily coming to rest.

k_0 is the spring constant of the spring.
M_1 is the mass of block 1.
M_2 is the mass of block 2.
Δy is the distance block 2 moves before momentarily coming to rest.

Consider the system that includes the spring, Earth, both blocks, and the string, but not the surface. Let the initial state be when the blocks are at rest just before they start moving, and let the final state be when the blocks first come momentarily to rest. Diagram A at left below is a bar chart that represents the energies in the scenario where there is negligible friction between block 1 and the surface.

The shaded-in bars in the energy bar charts represent the potential energy of the spring and the gravitational potential energy of the blocks-Earth system, U_s and U_g, respectively, in the initial and final states. Positive energy values are above the zero-point line ("0") and negative energy values are below the zero-point line.

Diagram A: Negligible Friction

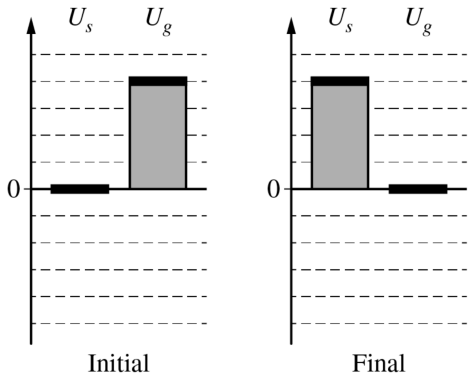
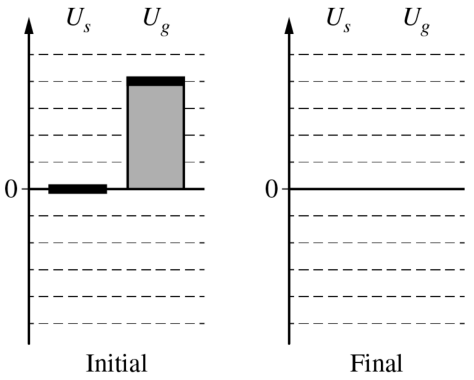


Diagram B: Nonnegligible Friction



(c) Complete diagram B (at right above) for the scenario in which friction is nonnegligible. The energies for the initial state are already provided. Shade in the energies in the final state using the same scale as in diagram A.

- Shaded regions should start at the solid line representing the zero-point line.
- Represent any energy that is equal to zero with a distinct line on the zero-point line.

解：

有摩擦时，桌面不在系统内，部分机械能转移到桌面。开始仍是 4 格重力势能；首次停止前下降量比无摩擦时小，所以最终重力势能仍为正，弹簧也已伸长，所以弹簧势能也为正。两根最终柱都从零向上画，柱高之和小于 4 格；题目没有提供摩擦系数，不能指定唯一的两柱数值。

K—U 图截距：两端都表示相同总能量

横轴是势能 U，纵轴是动能 K。总机械能 E 固定时，U 每增加一份，K 就减少一份；关系是一条下降直线。横截距处动能为零，纵截距处势能为零，两截距都等于总能量。

$$K + U = E, \qquad K = E - U.$$

是否改变质量，不直接决定这幅图是否改变；要看新运动的总振动能是否改变，特别是加入物块时的弹簧形变和水平速度。

例题 2023 Q1 (a)

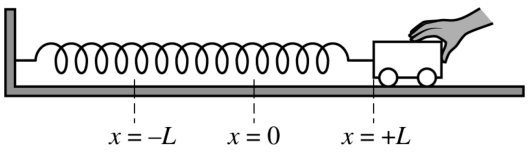


Figure 1

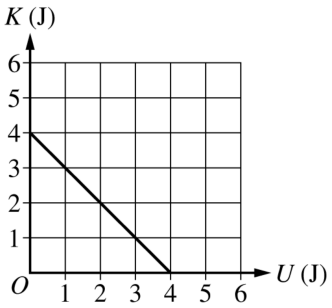


Figure 2

1. (7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions $x = +L$ and $x = -L$. Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(a) On the graph of kinetic energy K versus potential energy U shown in Figure 2, the values for the x-intercept and y-intercept are the same. Briefly explain why this is true, using physics principles.

解：

摩擦可忽略，动能与势能的总和恒定。原图纵截距为 K 的最大值、横截距为 U 的最大值，均为总能量 4 J；图线上动能是总能量减去势能，因此两截距相同。

$$K = 4\,\mathrm{J} - U.$$

物块是在小车瞬时水平静止的端点落上去的。比较加入前后这个时刻的弹簧形变与水平动能，再判断后续振动能；不要只凭总质量变大作结论。

练习 2023 Q1 (c)(i)

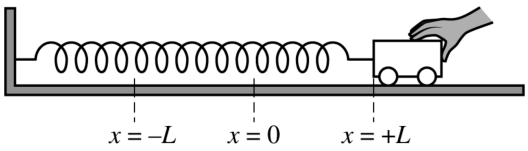


Figure 1

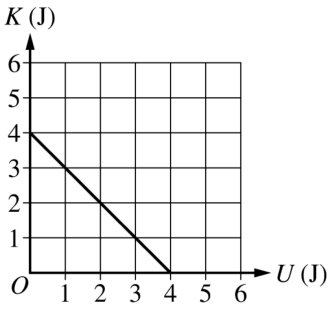


Figure 2

1. (7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions $x = +L$ and $x = -L$. Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(c) The dashed line in Figure 4 shows the kinetic energy K versus potential energy U of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

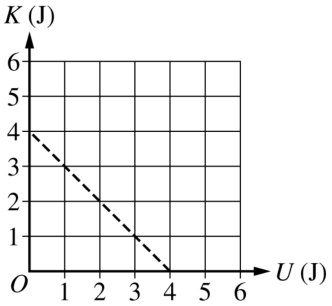


Figure 4

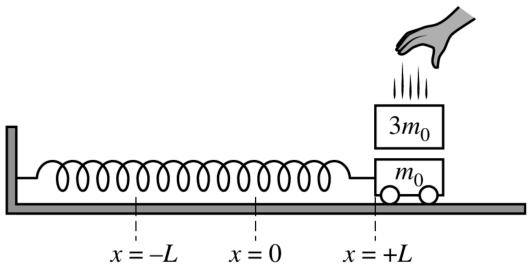


Figure 3

When the cart is at $+L$ and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between $-L$ and $+L$. The masses of the cart and the block are m_0 and $3m_0$, respectively.

i. Briefly explain why the two graphs must be the same, using physics principles.

缩小系统后重画 K—U 图：只计所选物体的动能

车与物块一起运动，速度相同，但各自的动能按质量分配。若纵轴只统计车的动能，先求车质量占总质量的比例，再用它乘总动能；弹簧仍在系统内，其势能不乘这个比例。

$$K_{\text{cart}} = \frac{m_{\text{cart}}}{m_{\text{cart}} + m_{\text{block}}} K_{\text{total}}.$$

这条比例来自两者同为 $\frac{1}{2}mv^2$。它改变纵截距，不自动改变弹簧能量的横截距。

例题 2023 Q1 (c)(ii)

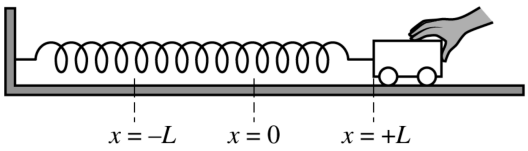


Figure 1

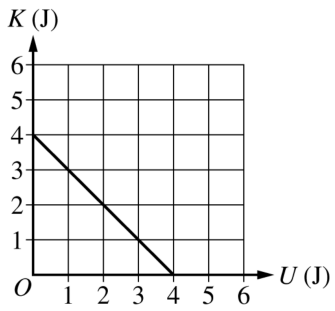


Figure 2

1. (7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions $x = +L$ and $x = -L$. Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(c) The dashed line in Figure 4 shows the kinetic energy K versus potential energy U of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

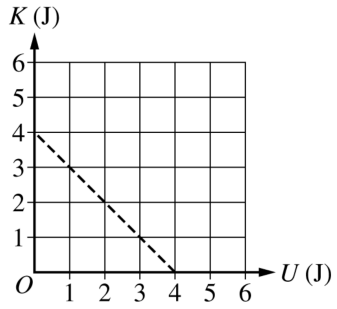


Figure 4

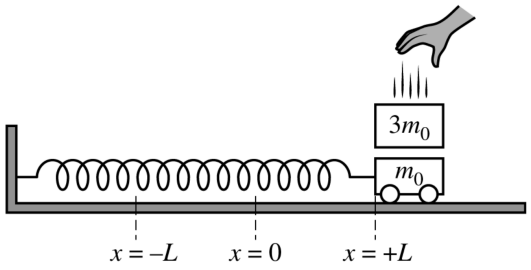


Figure 3

When the cart is at $+L$ and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between $-L$ and $+L$. The masses of the cart and the block are m_0 and $3m_0$, respectively.

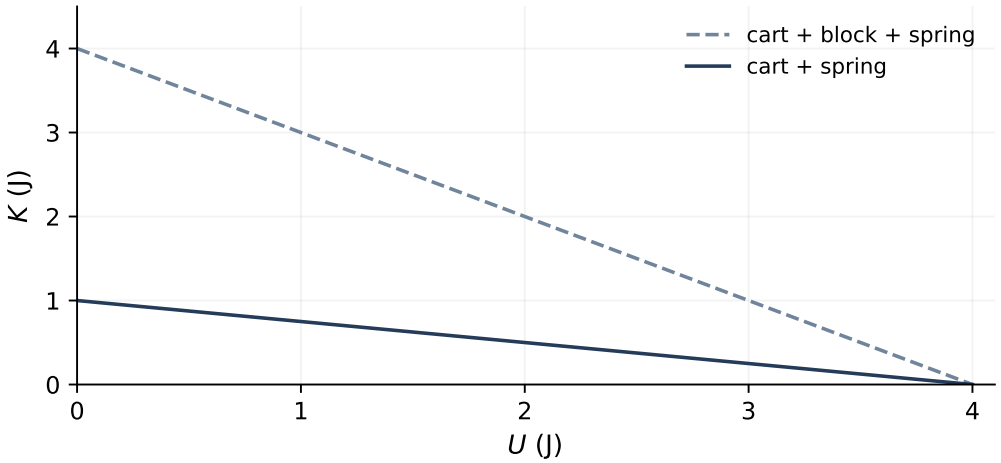
ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

解：

现在只选车与弹簧，附加物块的动能不计入纵轴。车与块速度相同，车质量占一起运动总质量的四分之一，因此车的动能只是总动能的四分之一；弹簧势能仍全部计入：

$$K_{\text{cart}} = \frac{1}{4}K_{\text{cart+block}} = \frac{1}{4}(4\,\text{J} - U).$$

画一条从纵截距 1 J 连到横截距 4 J 的实直线；横截距不变，纵截距由 4 J 降为 1 J。



虚线对应车、块与弹簧；实线只计车动能与弹簧势能。

总机械能对位置的图：守恒量画水平线

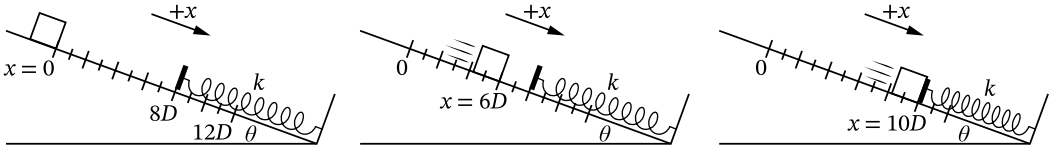
横轴换成位置后，能量守恒仍表示总机械能的数值不随位置变化。先从给定状态的各能量之和读出这条线的高度，再让它贯穿题目指定的整个位置区间。

例题 2025 Q2 (c)(i)

Question 2: Version J

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2. A block of mass M is released from rest at position $x = 0$ near the top of a ramp. The ramp makes an angle of θ with the horizontal. The block slides down the ramp with negligible friction. At $x = 8D$ the block makes contact with an uncompressed spring with spring constant k. The spring is then compressed and the block momentarily comes to rest at $x = 12D$. Figure 1 shows the instants when the block is at $x = 0$, $x = 6D$, and $x = 10D$, respectively.



C. Figure 6 shows a graph of the energy of the system as a function of the position of the block from $x = 8D$ to $x = 12D$. The spring potential energy U_s of the block-spring-Earth system is shown on the graph.

On the axes shown in Figure 6, do the following.

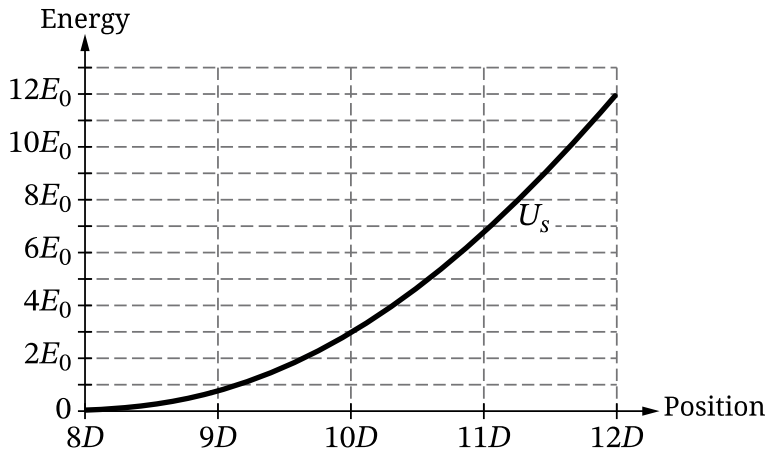


Figure 6

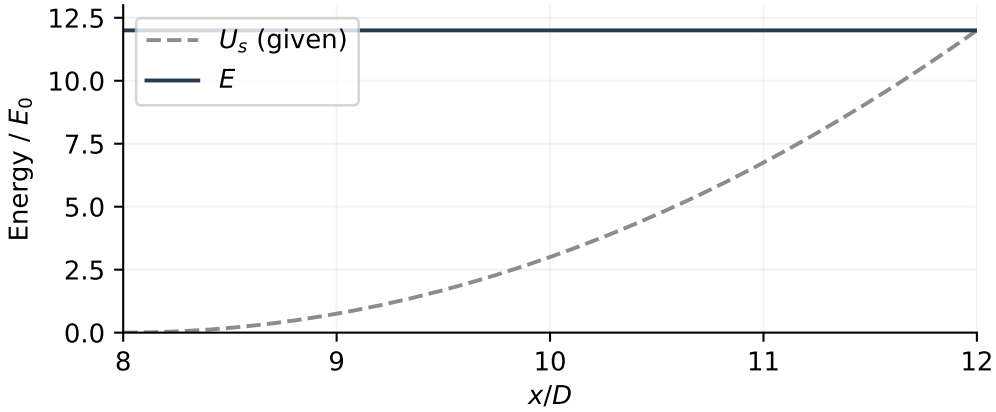
i. **Sketch** and **label** a line or curve that represents the total mechanical energy E for the block-spring-Earth system as a function of the position of the block from $x = 8D$ to $x = 12D$.

解：

摩擦可忽略，物块—弹簧—地球系统的总机械能在整段运动中保持不变。原题在

$$x = 10D$$

的三根能量柱之和为 $12E_0$，因此画一条高度为 $12E_0$ 的水平线，横跨 $8D$ 到 $12D$，标为 E。



只补本问要求的总机械能线 E；原有弹簧势能曲线保留。

重力势能对沿坡位置的图：等距离对应等高度差

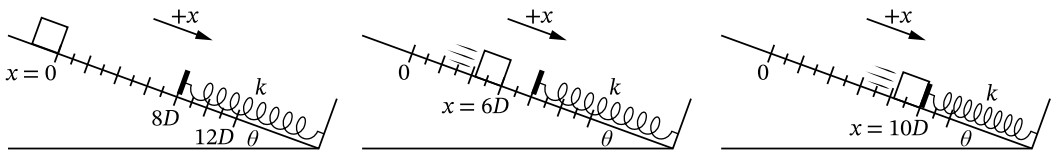
固定坡角时，沿坡每向下移动相同距离，竖直高度下降相同，重力势能等量减少。所以势能—沿坡位置图为下降直线。先标势能零点，再用另一个已知位置和能量尺度确定直线。

例题 2025 Q2 (c)(ii)

Question 2: Version J

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2. A block of mass M is released from rest at position $x = 0$ near the top of a ramp. The ramp makes an angle of θ with the horizontal. The block slides down the ramp with negligible friction. At $x = 8D$ the block makes contact with an uncompressed spring with spring constant k. The spring is then compressed and the block momentarily comes to rest at $x = 12D$. Figure 1 shows the instants when the block is at $x = 0$, $x = 6D$, and $x = 10D$, respectively.



C. Figure 6 shows a graph of the energy of the system as a function of the position of the block from $x = 8D$ to $x = 12D$. The spring potential energy U_s of the block-spring-Earth system is shown on the graph.

On the axes shown in Figure 6, do the following.

ii. **Sketch** and **label** a line or curve that represents the gravitational potential energy U_g for the block-spring-Earth system as a function of the position of the block from $x = 8D$ to $x = 12D$.

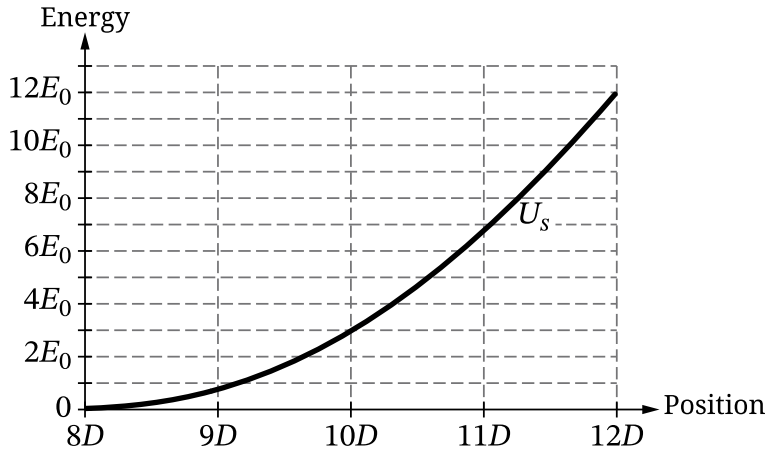


Figure 6

解：

重力势能零点在

$$x = 12D$$

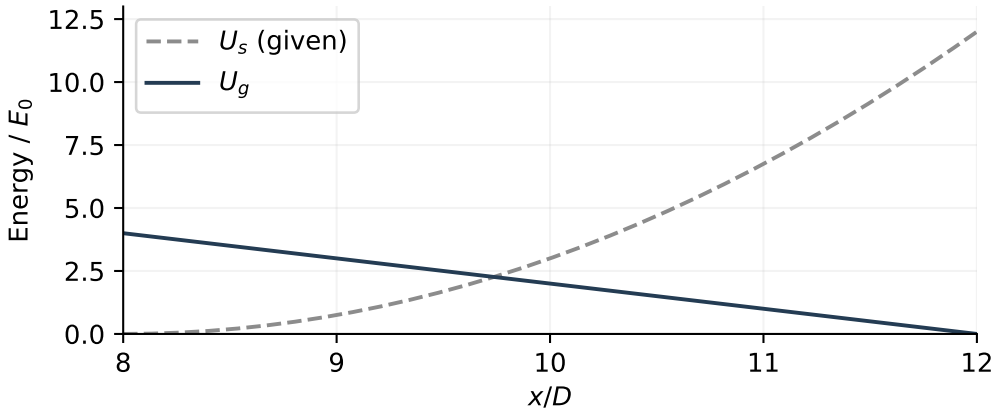
沿固定坡角向下移动相同距离时，竖直高度下降量相同，重力势能等量减少。因此它对 x 是下降直线。原图的能量尺度给出

$$U_g(8D) = 4E_0$$

终点

$$U_g(12D) = 0$$

连接这两点并标 U_g。



本问的重力势能线是下降直线；原有弹簧势能曲线保留。

由能量图比较速率：看总势能剩下的动能

同一物块质量不变时，速率越大，动能越大。由图比较两个位置，不能只看弹簧势能上升；还要同时看重力势能下降。总机械能固定，总势能越小，剩下的动能越大。

$$K = E - U_g - U_s.$$

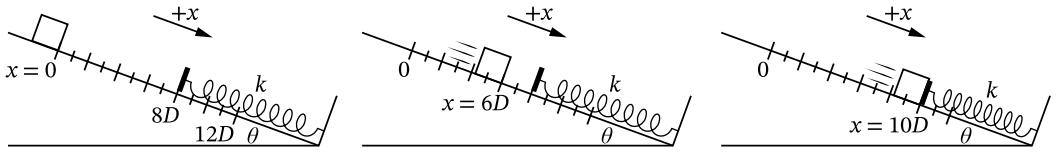
例题 2025 Q2 (d)

本问承接 2025 Q2 (c)(i)、2025 Q2 (c)(ii)。

Question 2: Version J

---

2. A block of mass M is released from rest at position $x = 0$ near the top of a ramp. The ramp makes an angle of θ with the horizontal. The block slides down the ramp with negligible friction. At $x = 8D$ the block makes contact with an uncompressed spring with spring constant k. The spring is then compressed and the block momentarily comes to rest at $x = 12D$. Figure 1 shows the instants when the block is at $x = 0$, $x = 6D$, and $x = 10D$, respectively.



C. Figure 6 shows a graph of the energy of the system as a function of the position of the block from $x = 8D$ to $x = 12D$. The spring potential energy U_s of the block-spring-Earth system is shown on the graph.

On the axes shown in Figure 6, do the following.

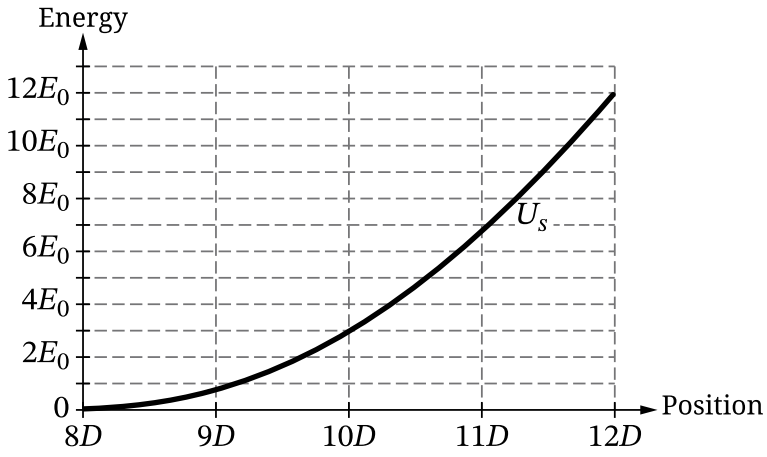


Figure 6

D. **Indicate** whether the speed v_{9D} of the block at $x = 9D$ is greater than, less than, or equal to the speed v_{8D} of the block at $x = 8D$.

_____ $v_{9D} > v_{8D}$

_____ $v_{9D} < v_{8D}$

_____ $v_{9D} = v_{8D}$

Justify how your response is consistent with the energy lines or curves you drew in Figure 6 in part C.

解：

选择 9D 处速率大于 8D 处速率。刚接触弹簧后，重力势能的下降一开始大于弹簧势能的增加，总势能降低，动能增加。用前面推导核验：一个能量单位对应下降沿坡距离 D 时重力势能的减少。8D 处动能为 $8E_0$；9D 处重力势能为 $3E_0$、弹簧势能为 $3E_0$/4，所以动能稍大。

$$K_{9D} = 12E_0 - 3E_0 - \frac{3}{4}E_0 = \frac{33}{4}E_0 > 8E_0 = K_{8D}.$$

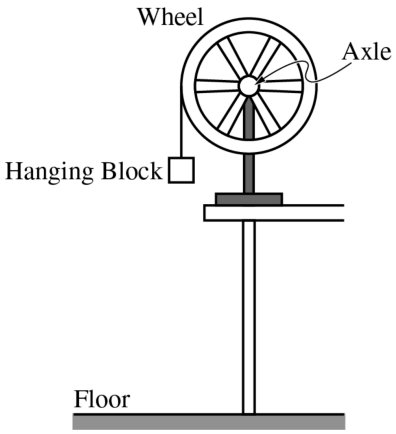
相同质量下动能越大，速率越大，和能量图的解释一致。

从总能量扣除已知部分，画剩余的转动动能

物块下落能让自身平动、也让轮转动。对包含物块、轮和地球的系统，势能减少分成这两份；把已知的平动动能从势能减少的大小中扣掉，才得到轮的动能。

原图若把重力势能的“变化”画为负值，先取它减少的正数大小，别把负值与动能直接相减。每个相同横坐标处做这次差，再判断得到的线。

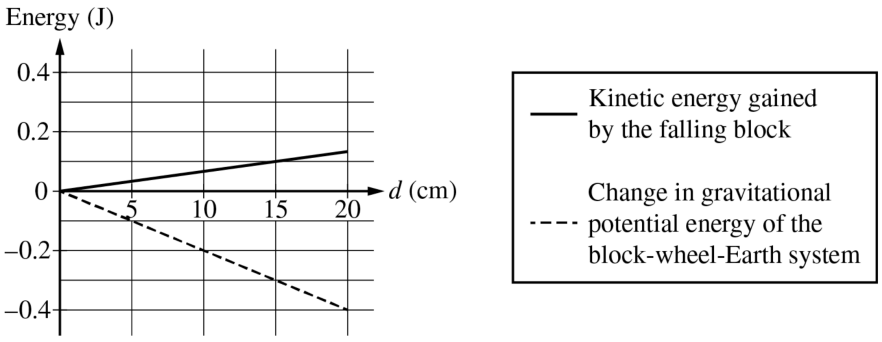
例题 2022 Q3 (c)



3. (12 points, suggested time 25 minutes)

A wheel is mounted on a horizontal axle. A light string is attached to the wheel’s rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block’s kinetic energy from when the block starts moving to immediately before it reaches the floor.



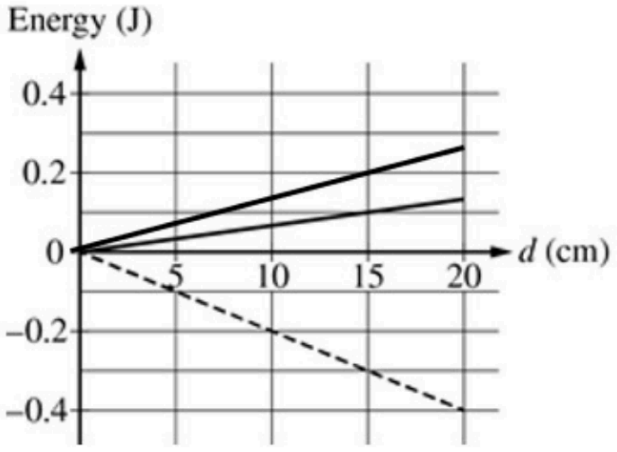
(c) The graph above represents both the change in the gravitational potential energy of the block-wheel-Earth system and the translational kinetic energy gained by the block as functions of the block's falling distance d. On the graph, draw a line or curve to represent the rotational kinetic energy of the wheel as a function of the block's falling distance d.

解：

物块—轮—地球系统能量守恒，初动能为零：

$$\Delta U_g + K_{\text{block}} + K_R = 0, \qquad K_R = -\Delta U_g - K_{\text{block}}.$$

对原图每个下降距离，用重力势能减少的大小减去平动动能，所得就是轮动能。两原线均为直线，差也为直线；画过原点、正斜率，并经过 (15 cm,0.20 J) 的线。



由质心高度判断转动动能：最低处不等于力矩最大处

重力势能由系统质心的高度决定。无摩擦转下去时，质心下降越多，势能转成的动能越多；质心最低时，动能最大。此时力矩可能瞬时为零，力矩为零只说明角速度此刻不再加快，并不把已有角速度变为零。

例题 2023 Q5 (a)(ii)

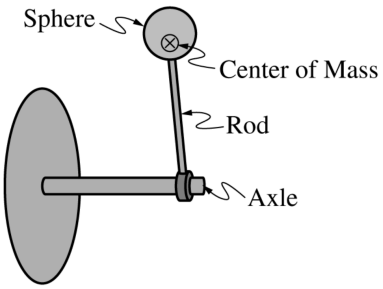


Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a ⊗, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.
Note: these frames are not equally spaced apart in time.

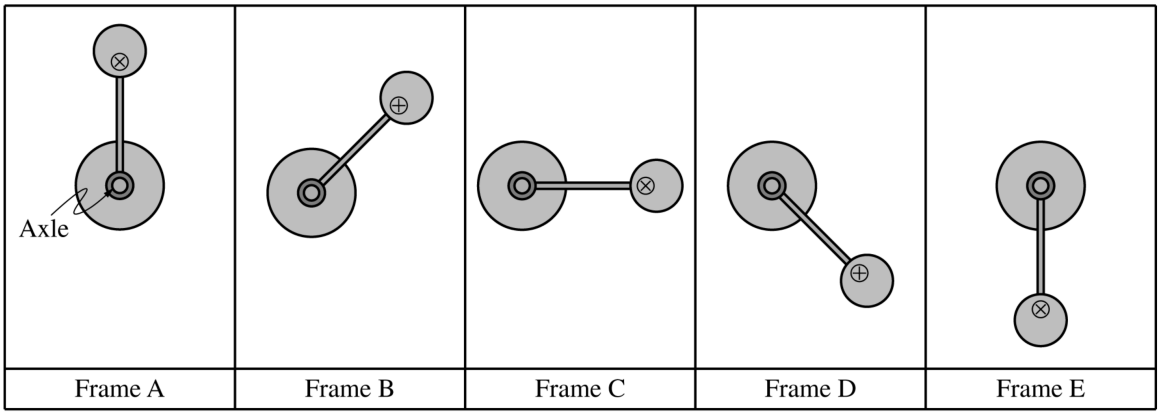


Figure 2

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

解：

Frame E。质心此时最低，重力势能减少最多，转成的转动动能最多。在落下过程中力矩与转动方向相同，杆持续加速；到最低处力矩瞬间为零，并不表示动能为零。

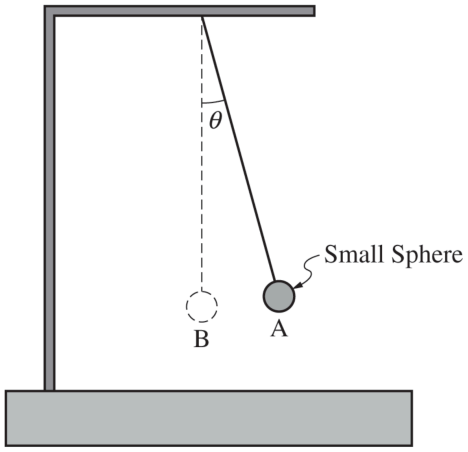
重力做功：用重量与竖直下降量比较

重力对下降物体做的功等于重力大小乘竖直下降量。比较行星上的同一段摆动时，先确认下降高度是否相同，再用行星质量和半径判断表面引力。

$$g = \frac{GM}{R^2}, \qquad W_g = mg\Delta h.$$

M、R 是施力行星的质量和半径，m 是摆球质量。不能把相同质量的行星当作表面重力一定相同。

例题 2024 Q4 (a)



4. (7 points, suggested time 13 minutes)

A simple pendulum consists of a small sphere that hangs from a string with negligible mass. The top end of the string is fixed. The sphere is pulled to Point A so that the string makes a small angle θ with the vertical, as shown. The sphere is then released from rest and swings through its lowest point at Point B. The work done on the sphere by Earth between points A and B is W_{E}.

The pendulum is then taken to Planet X. The mass of Planet X is the same as the mass of Earth, but the radius of Planet X is greater than the radius of Earth. The sphere is again brought to Point A (displaced θ from the vertical), released from rest, and swings through its lowest point at Point B. The work done on the sphere by Planet X between points A and B is W_{X}.

(a) **Justify** why W_{X} is less than W_{E}.

解：

X 行星质量与地球相同，但半径更大。表面引力随半径平方增大而减小，摆球重量因此较小。相同摆长与相同初角，使 A 到 B 的竖直下降量相同；重力做功是重量乘下降高度，所以 X 行星做功较少。

$$g_X = \frac{GM_E}{R_X^2} < \frac{GM_E}{R_E^2} = g_E, \qquad W = mg\Delta h.$$

转动系统的能量增加：下降量用质心高度差

杆和球的各点下降量不同，重力势能用整个系统质心的高度计算。先从图找质心相对轴的开始高度和结束高度，竖直下降量是二者之差；从轴上方转到轴下方时，两段高度相加。

初动能为零、无摩擦时，势能减少等于转动动能增加。质心到轴的距离不必等于整个杆长。

例题 2023 Q5 (b)(i)

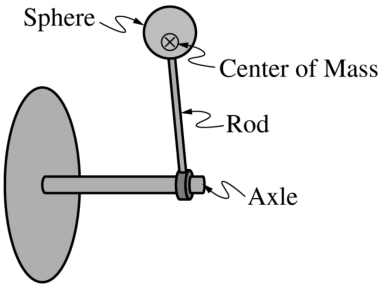


Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a ⊗, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.
Note: these frames are not equally spaced apart in time.

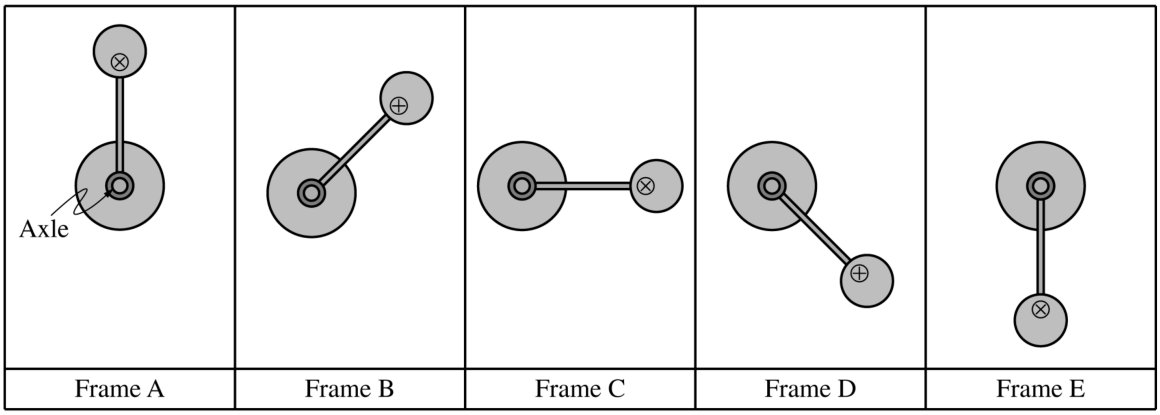


Figure 2

(b) The rod-sphere system has mass M and length L, and the center of mass is located a distance $\frac{3}{4}L$ from the axle, shown in Figure 3.

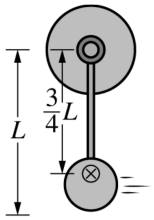


Figure 3

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of M, L, and fundamental constants, as appropriate.

解：

地球包含在系统中，从能量守恒开始：

$$K_i + U_{g,i} = K_f + U_{g,f}, \qquad \Delta K = Mg(h_{\mathrm{cm},i} - h_{\mathrm{cm},f}).$$

质心距轴 3L/4，起初位于轴上方 3L/4，最后位于轴下方 3L/4；实际下降两段相加：

$$h_{\mathrm{cm},i} - h_{\mathrm{cm},f} = \frac{3}{4}L + \frac{3}{4}L = \frac{3}{2}L,$$
$$\Delta K = \frac{3}{2}MgL.$$

地球不在系统内：用外力做功说明动能增加

同一次运动可以选不同系统。只选杆与球时，重力来自系统外的地球；它向系统输入正功。缩小系统之后，就不能又把“杆—球—地球”的重力势能当成缩小系统的内部能量。

$$\Delta K = W_{\mathrm{external}}.$$

例题 2023 Q5 (b)(ii)

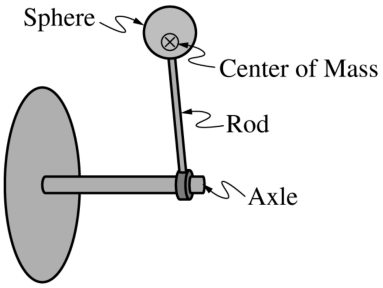


Figure 1

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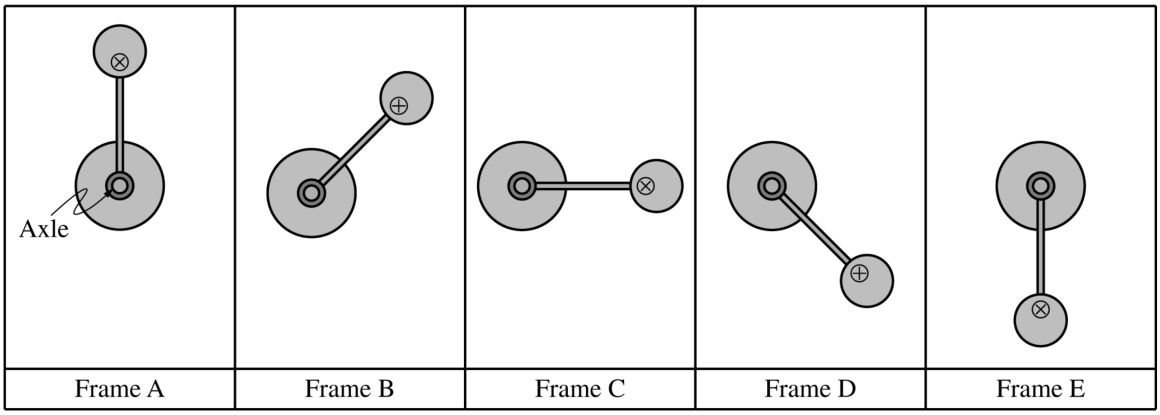


Figure 2

(b) The rod-sphere system has mass M and length L, and the center of mass is located a distance $\frac{3}{4}L$ from the axle, shown in Figure 3.

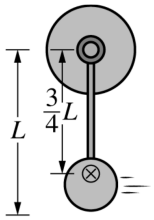


Figure 3

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

解：

若不把地球纳入系统，重力是外力。它沿质心总体下降方向做正功，向杆—球系统传递能量，所以动能增加；重力势能不再算作这个缩小系统的内部能量。

由两时刻速度求动能变化：始终末减初

先在速度图读取题目指定的两个时刻，再分别计算动能。速度的正负会在平方中消失，但时间区间不能换成下一问的区间。

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2.$$

下标 i、f 表示这道小问指定的开始和结束。负结果说明动能减少。

例题 2024 Q2 (c)(i)

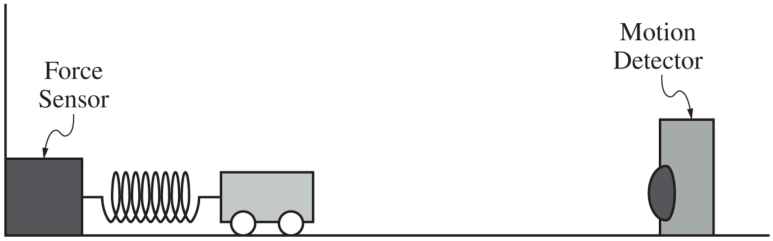
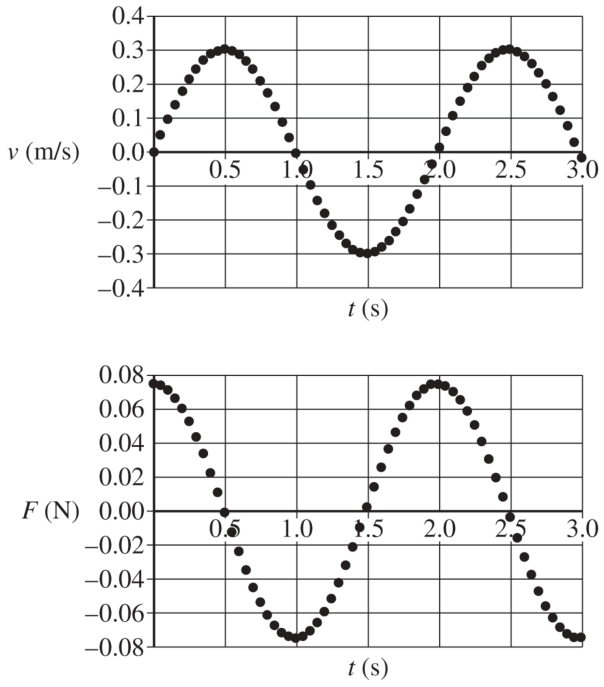


Figure 2

In a different experiment, the student attaches one end of a spring to a force sensor that is attached to a wall. The other end of the spring is attached to a cart with mass $m = 0.25$ kg. The student places a motion detector to the right of the cart, as shown in Figure 2, and pulls the cart to the right a small distance so that the spring is stretched. The student releases the cart from rest, and the cart-spring system oscillates.

The following graphs show the velocity v of the cart and the force F exerted on the cart by the spring as functions of time t.



i. Using the data in the velocity-time graph, **calculate** the change in kinetic energy of the cart from $t = 0.5$ s to $t = 2.0$ s. Show your steps and substitutions.

解：

这部分是质量 0.25 kg 的另一小车实验。原速度图在 0.5 s 的速度约为 +0.30 m/s，在 2.0 s 约为零。按末减初：

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(0.25)(0)^2 - \frac{1}{2}(0.25)(0.30)^2 = -0.01125\,\text{J}.$$

负号表示动能减少；2.0 s 与下一问的 2.5 s 不能混用。

由动能增加反求末速率：先确认初始静止

物块原来静止时，动能增加的大小就是末动能。先写 K=½mv^2，再求非负平方根；若原来已有速度，就不能把“增加量”直接当末动能。

例题 2024 Q5 (a)

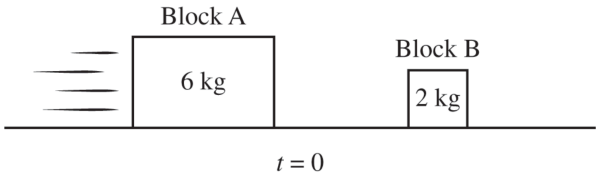


Figure 1

5. (7 points, suggested time 13 minutes)

At time $t = 0$, Block A slides along a horizontal surface toward Block B, which is initially at rest, as shown in Figure 1. The masses of blocks A and B are 6 kg and 2 kg, respectively. The blocks collide elastically at $t = 1.0$ s, and as a result, the magnitude of the change in kinetic energy of Block B is 9 J. All frictional forces are negligible.

(a) **Determine** the speed of Block B immediately after the collision.

解：

B 初始静止，动能增加 9 J 就是它的末动能。质量为 2 kg，速率取正根：

$$9 = \frac{1}{2}(2)v_B^2 \;\Rightarrow\; v_B = 3\,\mathrm{m/s}.$$

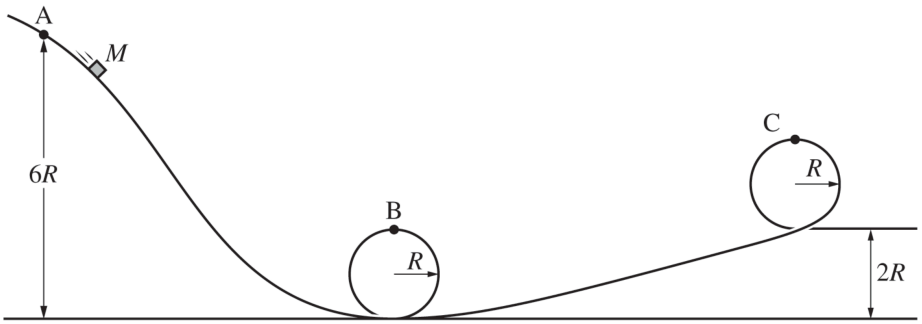
圆环顶端保持接触：高度够到不等于能通过

到达圆环顶端还需要向下的径向加速度。轨道只能接触着推物块，不能在物块离轨后拉它回轨；顶端的支持力必须非负。刚好接触时，它降到零，重力单独提供径向合力。

$$mg + N = \frac{mv^2}{r}, \qquad N \geq 0.$$

所以顶端仍须保留非零速度和动能。只把释放高度设为顶端高度，会使能量式给出顶端速度为零，和接触要求冲突。

例题 2024 Q1 (c)(ii)



1. (7 points, suggested time 13 minutes)

A block of mass M is released from rest at Point A, a height $6R$ above the horizontal. After being released, the block slides down a track, as shown. When released from Point A, the block does not lose contact with the track at any point. Points B and C are located at the highest points of their respective circular loops, both of radius R. All frictional forces are negligible.

(c)

ii. A student claims that $4R$ is the minimum height of Point A, such that the block can slide through Point C without losing contact with the track after the block is released from rest. Briefly **explain** why this claim is incorrect.

解：

4R 是 C 的高度。若由高度 4R 静止释放，能量守恒只允许物块在该高度动能为零；但顶端重力仍向下，物块会离轨道而落下，无法沿圆周继续运动。

进一步量化：顶端刚好接触时支持力为零，重力提供全部径向合力；需要保留相应动能。因此最小释放高度还要比 4R 高：

$$Mg = \frac{Mv_C^2}{R} \Rightarrow v_C^2 = gR,$$

$$Mgh_{\text{min}} = Mg(4R) + \frac{1}{2}MgR \Rightarrow h_{\text{min}} = \frac{9}{2}R.$$

原题只要求说明 4R 为什么错误；非零顶端速度与接触条件是关键理由。

动量、冲量与碰撞

判断系统水平动量是否变：只看水平外冲量

先选整个车—块系统，区分系统内部的碰撞力与来自系统外的力。内力成对等大反向，其冲量在总动量中抵消；即使车和块各自速度改变，没有水平合外冲量时，总水平动量不变。

$$p_{x,i} = p_{x,f}, \qquad p_x = \sum mv_x.$$

总水平动量—时间图因而是连续的水平线。竖直落下的物块可以有竖直速度，但它没有初始水平动量。

例题 2025 Q1 (a)(i)

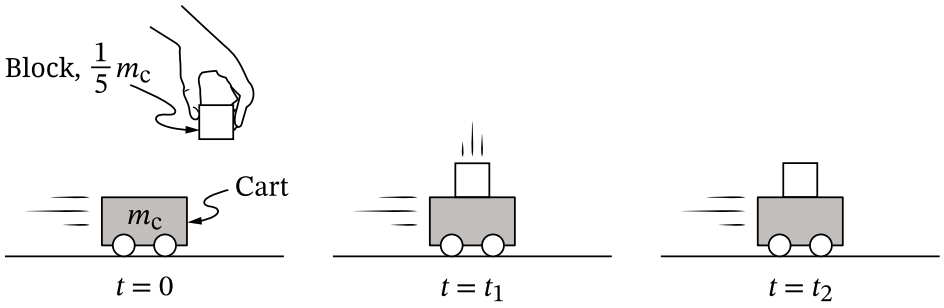
Question 1: Version J

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1. A student has a cart of mass m_c and a block of mass $\frac{1}{5}m_c$, as shown in Figure 1.

- At time $t = 0$, the cart is moving to the right across a horizontal surface with constant speed v_c, and the student releases the block from rest.
- At $t = t_1$, the block collides with and sticks to the top of the cart. The block does not slide on the cart.
- At $t = t_2$, the block-cart system continues to move to the right with constant speed v_f.

Figure 1



A.

i. On the axes shown in Figure 2, **sketch** a graph of the magnitude p_x of the *x*-component of the momentum of the block-cart system as a function of time *t* from $t = 0$ until $t > t_2$.

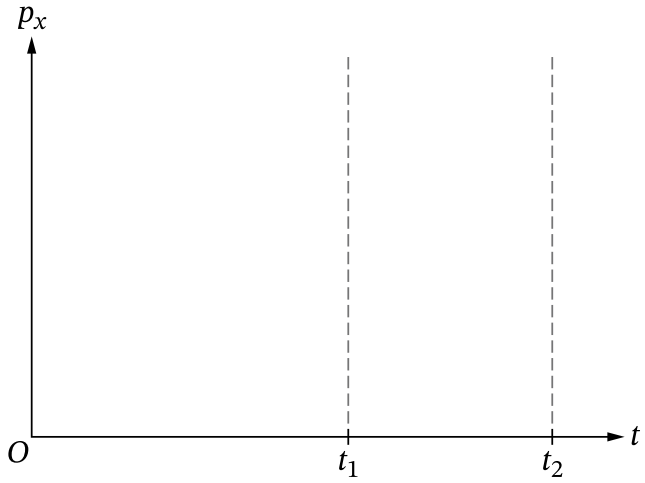
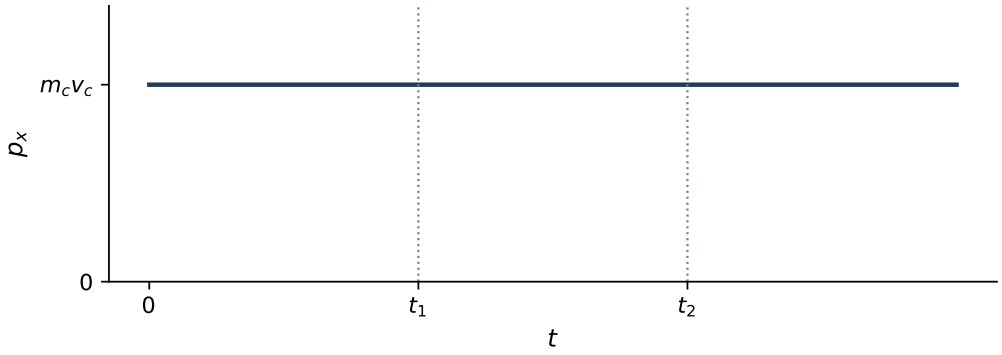


Figure 2

解：

取向右为水平正方向，系统包含小车与物块。小车质量 m_c，物块质量是车质量的五分之一。从开始计时到碰撞前，物块可能获得竖直速度，但水平速度为零，系统没有水平合外力。

总水平动量在碰撞前、中、后都保持 $m_c v_c$。画一条正值、连续的水平直线，经过 t_1、t_2 时不跳变。



系统水平动量保持不变，碰撞时刻没有突变。

练习 2025 Q1 (b)

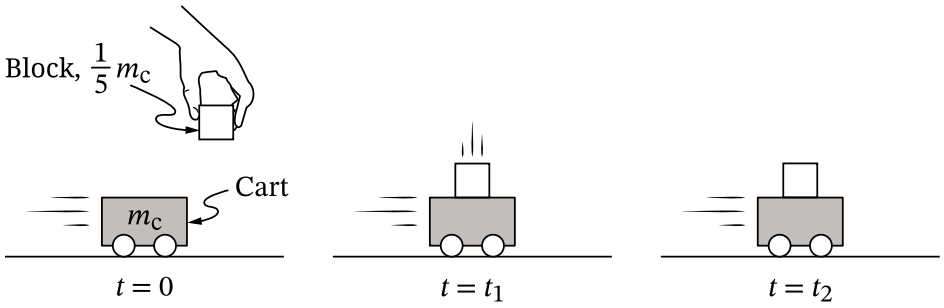
Question 1: Version J

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- At time $t = 0$, the cart is moving to the right across a horizontal surface with constant speed v_c, and the student releases the block from rest.
- At $t = t_1$, the block collides with and sticks to the top of the cart. The block does not slide on the cart.
- At $t = t_2$, the block-cart system continues to move to the right with constant speed v_f.

Figure 1



B. Consider the case where a new block is dropped and collides with the top of the cart. The new block slides along the cart during the collision but does not slide off the cart. The time interval from when the new block collides with the cart and moves together with the cart is Δt. During Δt there is a frictional force between the new block and the cart.

Indicate whether the x-component of the momentum of the new block-cart system increases, decreases, or remains constant during Δt.

_____ Increases

_____ Decreases

_____ Remains constant

Justify your response.

动量矢量图：带方向相加，再用原图尺度画箭头

水平动量是带方向的量。向右为正、向左为负，箭头长度表示大小；先求目标动量，再按题给格数换算长度。总量相同不表示每个物体的动量相同。

如果题目要求整个系统的箭头，就把内部各物体的动量先相加；如果分别要求两个物体，就分别画在对应点上。

例题 2026 Q2 (a)

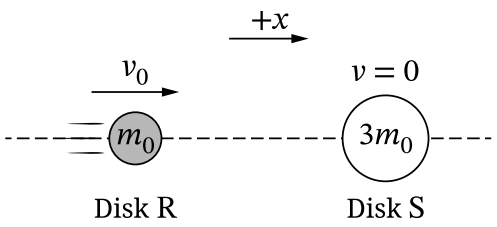
Question 2: Version J

2. Two small disks, R and S, are on a straight, horizontal track.

At time $t = 0$, Disk R of mass m_0 is located at position $x = 0$. Disk R is initially moving with speed v_0 in the +x-direction. Disk S of mass $3m_0$ is initially at rest, as shown in the top view in Figure 1. Frictional forces are negligible.

At $t = t_1$, the disks collide. Immediately after the collision, Disk R moves with speed $\frac{1}{2}v_0$ in the –x-direction.

Figure 1



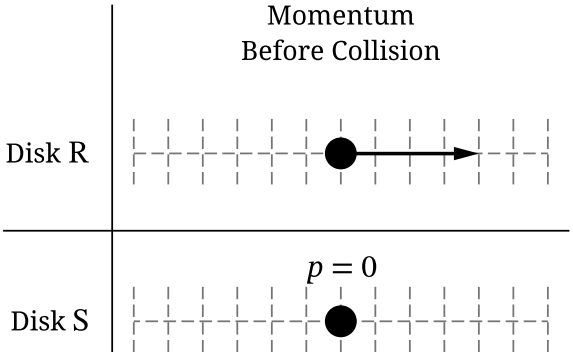
Top View

Note: Figure not drawn to scale.

Part A

The momentum-vector diagram in Figure 2 represents the momentums of Disk R and Disk S before the collision.

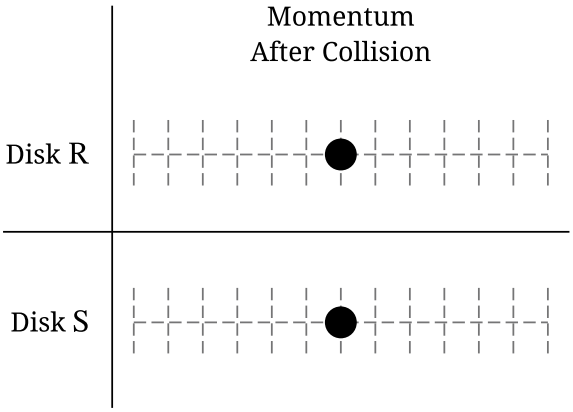
Figure 2



Draw arrows on Figure 3 to represent the momentum vectors of Disk R and Disk S after the collision.

- Each arrow must start on, and point away from, each dot.
- If the momentum of either disk is zero, write "$p = 0$" next to the dot.
- Draw the length of each arrow to represent the magnitude of each momentum, consistent with the scale used in Figure 2.

Figure 3



解：

右为正。R 初动量为 $m_0 v_0$，对应原图向右 4 格。碰后 R 速度向左、大小 $v_0/2$，故动量为 $-m_0 v_0/2$。

为 R 画向左 2 格的箭头。总动量仍是向右 4 格，为抵消 R 的负 2 格，S 必须有向右 6 格的动量箭头。

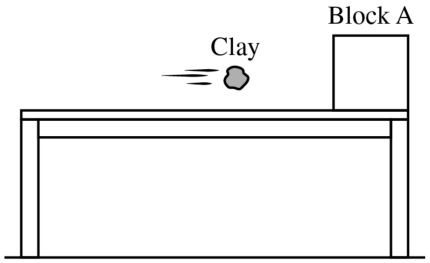
本问画的是碰后“球—块整体”的动量，不是只画物块。反弹改变系统内部的动量分配，但不能凭它增大整体动量。

练习 2022 Q4 (a)

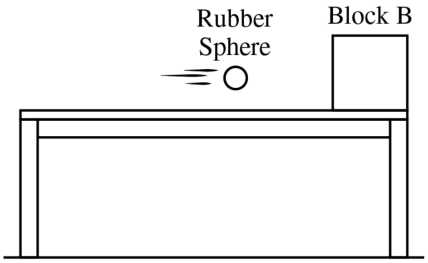
4. (7 points, suggested time 13 minutes)

A student has a piece of clay and a rubber sphere, both of the same mass. Both objects are thrown horizontally at the same speed at identical blocks that are at rest at the edge of identical tables, as shown, where friction between the blocks and the table is negligible. After the collisions, both blocks fall to the floor.

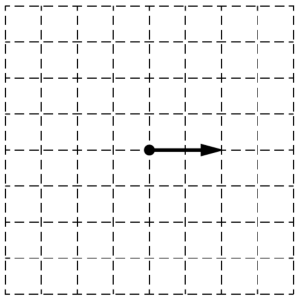
In Case A, the clay sticks to Block A after the collision. In Case B, the rubber sphere bounces off of Block B after the collision.



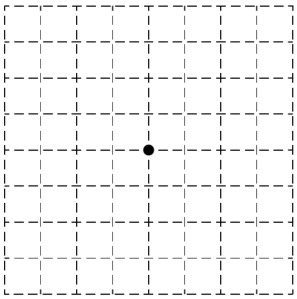
Case A: Clay and Block A
Before Collision



Case B: Rubber Sphere and Block B
Before Collision



Case A: Momentum of
Clay-Block System
Immediately After Collision



Case B: Momentum of
Sphere-Block System
Immediately After Collision

(a) In the figure at left above, the arrow represents the momentum immediately after the collision for the clay-block system in Case A. In the figure at right above, draw an arrow starting on the dot to represent the momentum of the sphere-block system immediately after the collision in Case B. If the momentum is zero, write "zero" next to the dot. The momentum, if it is not zero, must be represented by an arrow starting on, and pointing away from, the dot. The length of the vector, if not zero, should reflect the magnitude of the momentum relative to Case A.

粘在一起的末速度：总动量除以总质量

碰后粘住表示两物体速度相同；把末动量写成总质量乘共同速度。先写水平动量守恒，保留原题给的质量比例，再解速度。碰撞中机械能可能损失，不能用动能守恒替代。

例题 2025 Q1 (a)(ii)

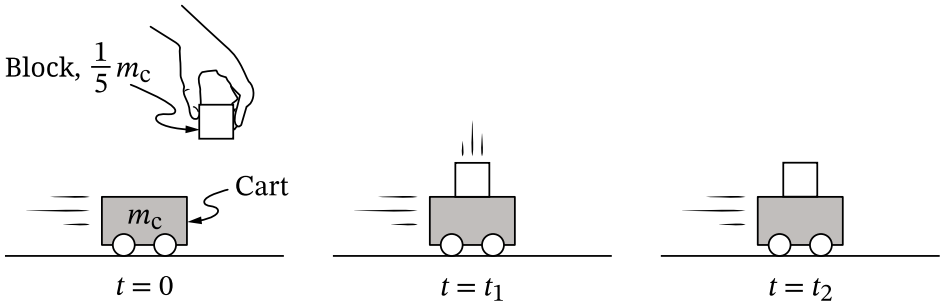
Question 1: Version J

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1. A student has a cart of mass m_c and a block of mass $\frac{1}{5}m_c$, as shown in Figure 1.

- At time $t = 0$, the cart is moving to the right across a horizontal surface with constant speed v_c, and the student releases the block from rest.
- At $t = t_1$, the block collides with and sticks to the top of the cart. The block does not slide on the cart.
- At $t = t_2$, the block-cart system continues to move to the right with constant speed v_f.

Figure 1



A.

ii. **Derive** an expression for the speed v_f of the block-cart system after time $t = t_2$ in terms of m_c, v_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

从水平动量守恒开始。粘住后总质量是 $6m_c/5$：

$$p_{x,i} = p_{x,f}, \qquad m_c v_c + \frac{m_c}{5}(0) = \left(m_c + \frac{m_c}{5}\right) v_f,$$

$$v_f = \frac{5}{6} v_c.$$

先由动量求速度，再求碰后单个物体动能

反弹物体的末速度为负时，其末动量也要带负号。动量式先求另一个物体的速度，求出后再代入这个物体自己的 $½mv^2$；动能始终非负。

例题 2026 Q2 (b)

Question 2: Version J

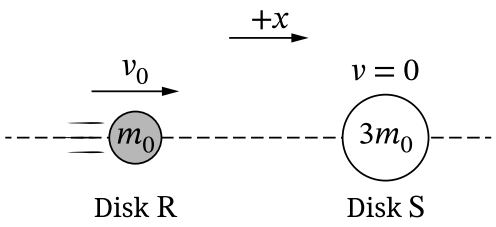
---

2. Two small disks, R and S, are on a straight, horizontal track.

At time $t = 0$, Disk R of mass m_0 is located at position $x = 0$. Disk R is initially moving with speed v_0 in the +x-direction. Disk S of mass $3m_0$ is initially at rest, as shown in the top view in Figure 1. Frictional forces are negligible.

At $t = t_1$, the disks collide. Immediately after the collision, Disk R moves with speed $\frac{1}{2}v_0$ in the –x-direction.

Figure 1



Top View

Note: Figure not drawn to scale.

Part B

Starting with conservation of linear momentum, **derive** an expression for the kinetic energy of Disk S immediately after the collision. Express your final answer in terms of m_0, v_0, and physical constants, as appropriate. Begin your derivation by writing the fundamental physics principle or an equation from the reference information.

解：

从动量守恒开始：

$$m_0 v_0 = m_0\left(-\frac{1}{2}v_0\right) + 3m_0 v_S,$$

$$v_S = \frac{1}{2}v_0, \qquad K_S = \frac{1}{2}(3m_0)\left(\frac{1}{2}v_0\right)^2 = \frac{3}{8}m_0 v_0^2.$$

粘合前后动能变化：使用题目指定的初末状态

先用已经求出的共同末速度。初动能需包括原初时刻正在运动的各物体，末动能用共同速度和总质量；最后末减初。若初时刻是“刚释放”，不能换成已落下、碰撞之前的时刻。

例题 2025 Q1 (a)(iii)

本问承接 2025 Q1 (a)(ii)。

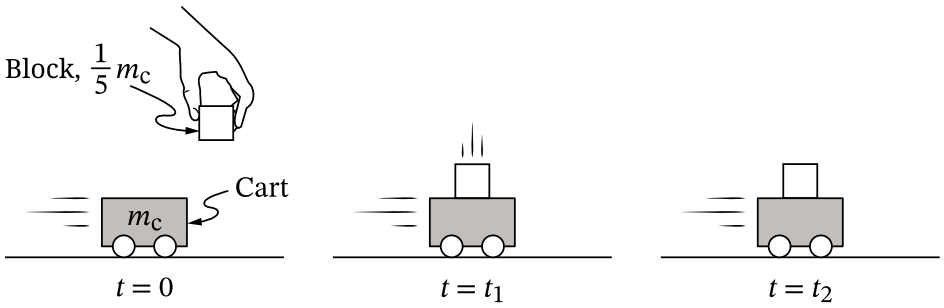
Question 1: Version J

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1. A student has a cart of mass m_c and a block of mass $\frac{1}{5}m_c$, as shown in Figure 1.

- At time $t = 0$, the cart is moving to the right across a horizontal surface with constant speed v_c, and the student releases the block from rest.
- At $t = t_1$, the block collides with and sticks to the top of the cart. The block does not slide on the cart.
- At $t = t_2$, the block-cart system continues to move to the right with constant speed v_f.

Figure 1



A.

iii. **Derive** an expression for the change in the kinetic energy ΔK in the block-cart system from $t = 0$ to $t = t_2$ in terms of m_c, v_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

题目区间从释放物块的零时刻 开始，此时物块静止，所以初始动能只来自小车。最终车与物块共同向右运动：

$$K_0 = \frac{1}{2}m_c v_c^2, \qquad K_f = \frac{1}{2}\left(\frac{6}{5}m_c\right)\left(\frac{5}{6}v_c\right)^2 = \frac{5}{12}m_c v_c^2,$$

$$\Delta K = K_f - K_0 = -\frac{1}{12}m_c v_c^2.$$

负号表示这两个指定状态之间的动能减少。不能把初状态偷偷换成物块已下落、即将碰到车顶的时刻。

反弹造成更大物块动量：用有符号总动量解释射程

相同初动量下，球反弹后带负向动量，要保持总动量，物块就须获得更大的正向动量。相同质量的物块速度因此更大。

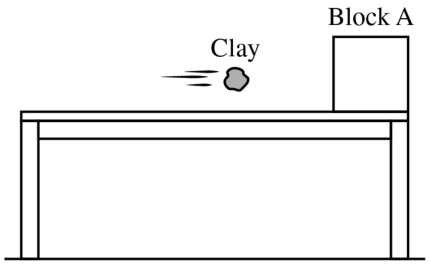
两次都从同一桌边高度水平飞出，初竖直速度相同，重力相同，落地时间相同；水平距离由水平速度乘这段时间决定。题目没有说完全弹性时，不新增能量守恒假设。

例题 2022 Q4 (b)

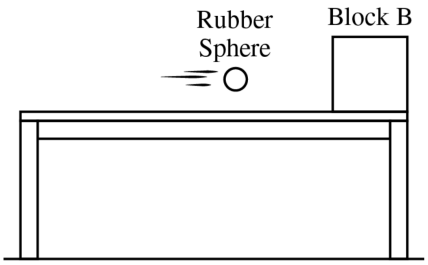
4. (7 points, suggested time 13 minutes)

A student has a piece of clay and a rubber sphere, both of the same mass. Both objects are thrown horizontally at the same speed at identical blocks that are at rest at the edge of identical tables, as shown, where friction between the blocks and the table is negligible. After the collisions, both blocks fall to the floor.

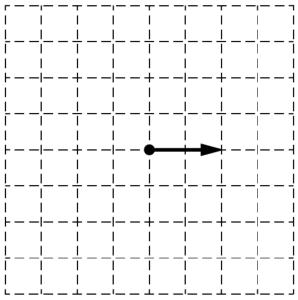
In Case A, the clay sticks to Block A after the collision. In Case B, the rubber sphere bounces off of Block B after the collision.



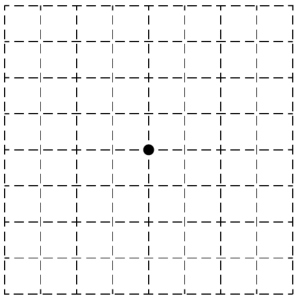
Case A: Clay and Block A
Before Collision



Case B: Rubber Sphere and Block B
Before Collision



Case A: Momentum of
Clay-Block System
Immediately After Collision



Case B: Momentum of
Sphere-Block System
Immediately After Collision

(b) After the clay and Block A collide, Block A lands a horizontal distance d_A from the edge of the table. Does Block B land on the floor at a horizontal distance from the edge of the table that is greater than, less than, or equal to d_A ? In a clear, coherent, paragraph-length response that may also contain equations and/or drawings, explain your reasoning. Neglect any frictional effects due to the table or air resistance.

解：

两次系统总动量相同，但系统内部的分配不同。A 中黏土随块一起向右，因此黏土仍带走一部分正向动量。B 中橡胶球反弹，带走负向动量；为使球—块总动量保持原来的正值，B 块必须获得更大的正向动量。同质量物块因而是 B 的速度更大。

两块都从相同高度水平离开桌边，初竖直速度为零，重力加速度相同，下落时间相同。忽略空气阻力，水平速度保持不变；速度更大的 B 在相同飞行时间内走得更远，所以 B 的水平落地距离更大。

$$t_{\text{fall}} = \sqrt{\frac{2h}{g}}, \qquad d_A = v_A t_{\text{fall}}, \quad d_B = v_B t_{\text{fall}}.$$

这段比较只使用反弹方向与动量守恒，题目没有说 B 的碰撞完全弹性，不需要额外假定机械能守恒。

碰撞后位置图：连续接线，斜率由速度决定

瞬间碰撞可以改变速度，但不能让位置跳跃。先从原位置图斜率读出碰前速度，再由动量求碰后各速度；从碰撞点连续接出直线，负速度使线向下倾。

$$v_{\text{cm}} = \frac{p_{\text{total}}}{m_1 + m_2}.$$

cm 表示系统质心。在没有水平合外力时，总动量和总质量不变，质心速度不变，所以质心位置线不因内碰撞拐弯。它是按质量加权的位置，不能总放在两物体的几何中点。

例题 2024 Q5 (b)

本问承接 2024 Q5 (a)。

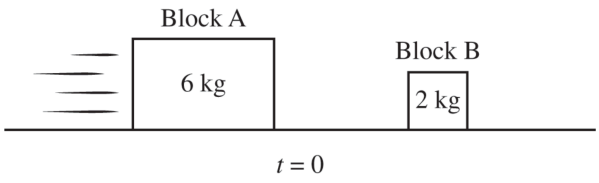


Figure 1

5. (7 points, suggested time 13 minutes)

At time $t = 0$, Block A slides along a horizontal surface toward Block B, which is initially at rest, as shown in Figure 1. The masses of blocks A and B are 6 kg and 2 kg, respectively. The blocks collide elastically at $t = 1.0$ s, and as a result, the magnitude of the change in kinetic energy of Block B is 9 J. All frictional forces are negligible.

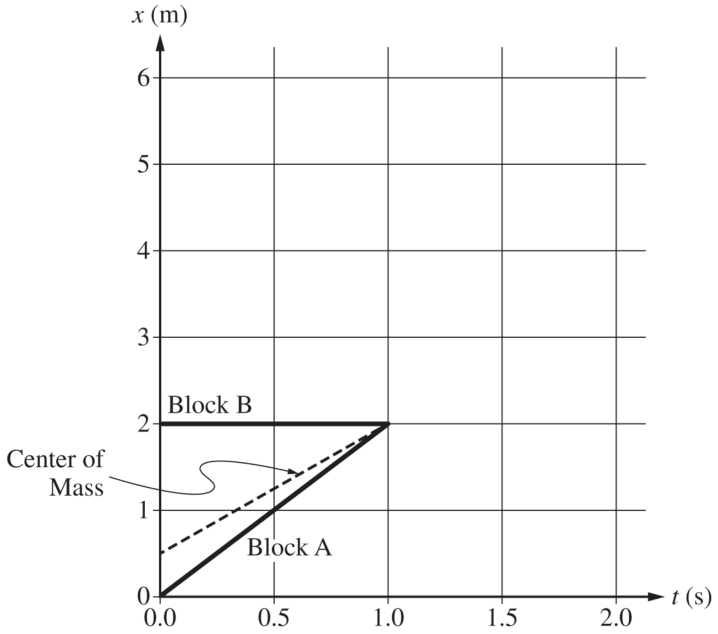


Figure 2

The graph shown in Figure 2 represents the positions x of Block A, Block B, and the center of mass of the two-block system as functions of t between $t = 0$ and $t = 1.0\,\text{s}$.

(b) On the graph in Figure 2, **draw** and **label** three lines to represent the positions of Block A, Block B, and the center of mass of the two-block system as functions of t between $t = 1.0\,\text{s}$ and $t = 2.0\,\text{s}$. Each line should be distinctly labeled.

解：

原图显示 A 在 1 s 内由 0 走到 2 m，碰前速度为 2 m/s；B 原来静止在 2 m 位置。碰前总动量为 12 kg·m/s，碰后仍相同：

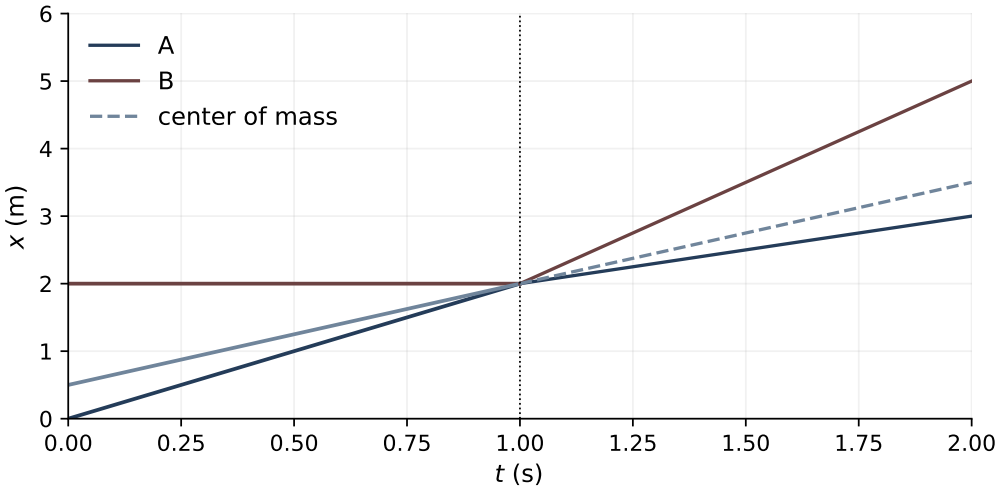
$$6(2) + 2(0) = 6v_A + 2(3) \;\Rightarrow\; v_A = 1\,\text{m/s}.$$

质心速度为总动量除总质量：

$$v_{\text{cm}} = \frac{12}{6+2} = 1.5\,\text{m/s}.$$

1 s 时两块都位于 2 m 位置，所以质心也在此处。三条线从 (1 s,2 m) 连续接出；A 斜率 1、B 斜率 3、质心斜率 1.5，均以 m/s 为单位：

$$x_A = 2 + (t-1), \qquad x_B = 2 + 3(t-1), \qquad x_{\text{cm}} = 2 + 1.5(t-1).$$



碰后物体改变斜率，质心沿碰前直线继续。

核验弹性条件：初动能为 12 J，末动能是两块的 3 J 与 9 J 相加，也为 12 J，与原题一致。

练习 2026 Q2 (c)

本问承接 2026 Q2 (b)。

Question 2: Version J

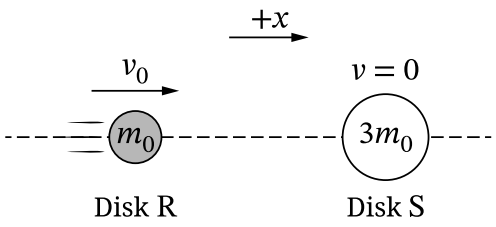
---

2. Two small disks, R and S, are on a straight, horizontal track.

At time $t = 0$, Disk R of mass m_0 is located at position $x = 0$. Disk R is initially moving with speed v_0 in the +x-direction. Disk S of mass $3m_0$ is initially at rest, as shown in the top view in Figure 1. Frictional forces are negligible.

At $t = t_1$, the disks collide. Immediately after the collision, Disk R moves with speed $\frac{1}{2}v_0$ in the –x-direction.

Figure 1



Top View

Note: Figure not drawn to scale.

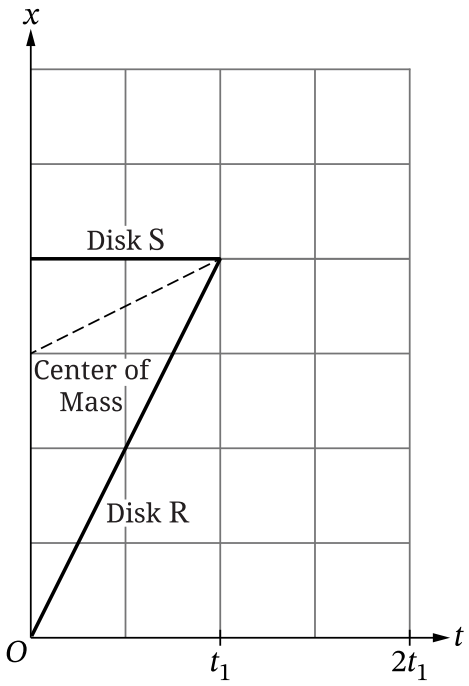
Part C

The graph shown in Figure 4 represents the positions x as functions of time t from $t = 0$ until $t = t_1$ for each of the following:

- Disk R
- Disk S
- The center of mass of the two-disk system

On the graph shown in Figure 4, **draw** three lines that represent the positions x of Disk R, Disk S, and the center of mass of the two-disk system as functions of t from $t = t_1$ to $t = 2t_1$. Distinctly **label** each line.

Figure 4



练习 2024 Q5 (c)

本问承接 2024 Q5 (b)。

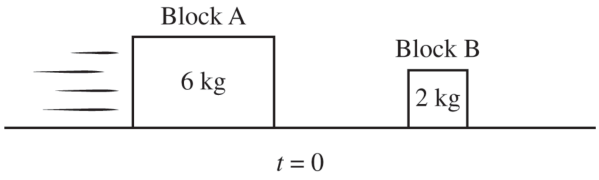


Figure 1

5. (7 points, suggested time 13 minutes)

At time $t = 0$, Block A slides along a horizontal surface toward Block B, which is initially at rest, as shown in Figure 1. The masses of blocks A and B are 6 kg and 2 kg, respectively. The blocks collide elastically at $t = 1.0$ s, and as a result, the magnitude of the change in kinetic energy of Block B is 9 J. All frictional forces are negligible.

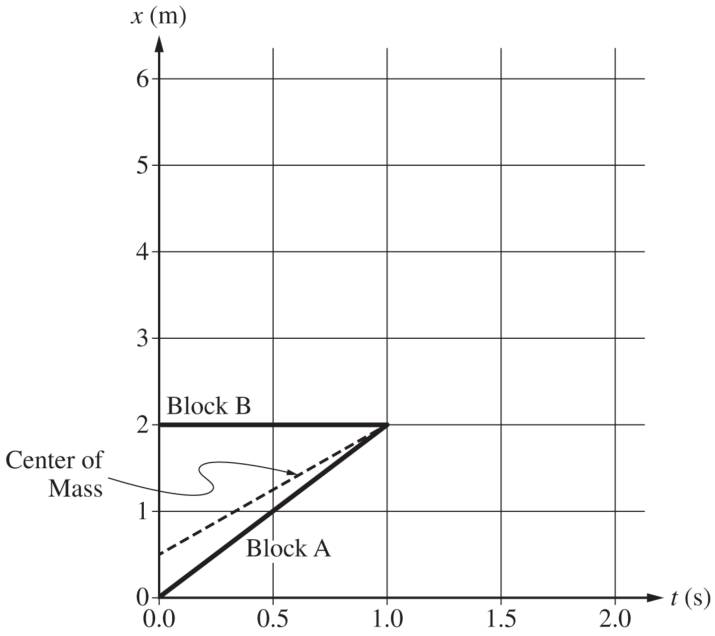


Figure 2

The graph shown in Figure 2 represents the positions x of Block A, Block B, and the center of mass of the two-block system as functions of t between $t = 0$ and $t = 1.0$ s.

(c) Consider if in the original scenario, instead of colliding elastically, the blocks collided and stuck together. **Describe** how the line drawn for the center of mass in part (b) would change, if at all. Briefly **justify** your response.

两物体的动量变化比较：同一对内力给等大反向冲量

牛顿第三定律说，两物体互相施的力在每一时刻等大反向。它们作用的时间相同，所以各自冲量也等大反向。质量不同只使速度变化不同，不能使这一对动量变化的大小不同。

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2.$$

例题 2026 Q2 (d)

Question 2: Version J

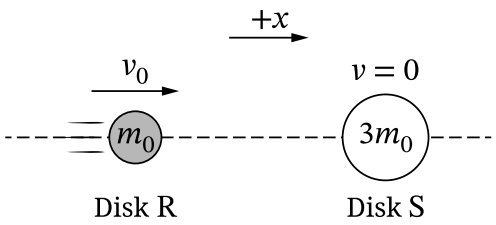
---

2. Two small disks, R and S, are on a straight, horizontal track.

At time $t = 0$, Disk R of mass m_0 is located at position $x = 0$. Disk R is initially moving with speed v_0 in the +x-direction. Disk S of mass $3m_0$ is initially at rest, as shown in the top view in Figure 1. Frictional forces are negligible.

At $t = t_1$, the disks collide. Immediately after the collision, Disk R moves with speed $\frac{1}{2}v_0$ in the –x-direction.

Figure 1



Top View

Note: Figure not drawn to scale.

Part D

During the collision, the magnitudes of the changes in momentum of Disk R and Disk S are $|\Delta p_R|$ and $|\Delta p_S|$, respectively.

Indicate whether $|\Delta p_R|$ is greater than, less than, or equal to $|\Delta p_S|$ by writing one of the following.

- $|\Delta p_R| > |\Delta p_S|$
- $|\Delta p_R| < |\Delta p_S|$
- $|\Delta p_R| = |\Delta p_S|$

Briefly **justify** your response by referencing a fundamental physics principle.

解：

两者动量改变量大小相等。碰撞内力等大反向，作用时间相同，所以两圆盘所受冲量等大反向；也可由系统动量守恒直接得到两个动量改变量之和为零。

$$\Delta p_R^{\text{signed}} + \Delta p_S^{\text{signed}} = 0 \;\Rightarrow\; |\Delta p_R| = |\Delta p_S|.$$

不能因 S 质量较大就说它的动量变化较大；相同冲量使其速度变化较小。

力—时间图估计动量变化：相反符号的面积相消

冲量就是力—时间图在指定时间区间的有符号面积。横轴上方正、下方负；先各算或估两片面积，再相加，得到动量变化。不要把正负面积的绝对值直接相加。

$$\Delta p = J \approx \sum F\,\Delta t.$$

J 是冲量，每小段 FΔt 是该段面积；曲线用小矩形或原图对称性估计。

例题 2024 Q2 (c)(ii)

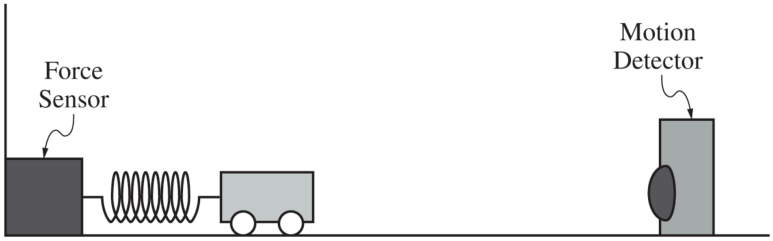
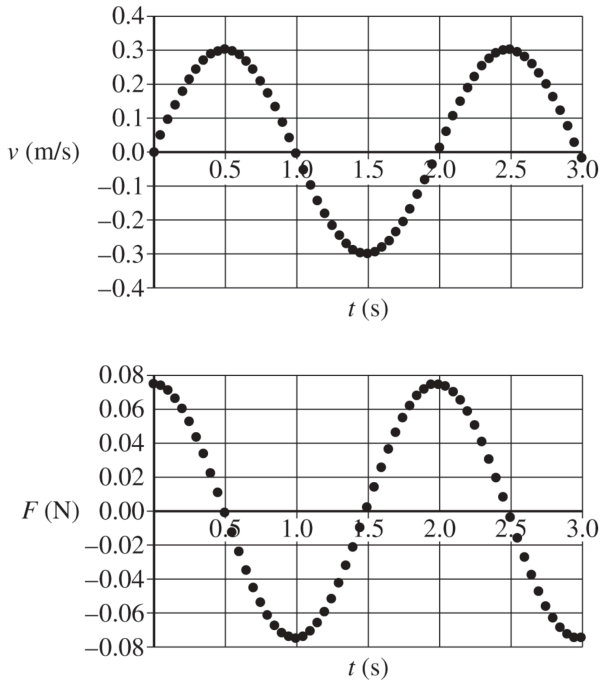


Figure 2

In a different experiment, the student attaches one end of a spring to a force sensor that is attached to a wall. The other end of the spring is attached to a cart with mass $m = 0.25$ kg. The student places a motion detector to the right of the cart, as shown in Figure 2, and pulls the cart to the right a small distance so that the spring is stretched. The student releases the cart from rest, and the cart-spring system oscillates.

The following graphs show the velocity v of the cart and the force F exerted on the cart by the spring as functions of time t.



ii. Using the data in the force-time graph, **estimate** the change in momentum of the cart from $t = 0.5$ s to $t = 2.5$ s. Briefly **explain** how you arrived at your estimation.

解：

研究区间改为 0.5 s 到 2.5 s。力—时间图在 0.5–1.5 s 的负面积与 1.5–2.5 s 的正面积大小近似相等，符号相反，总冲量约为零，因此动量变化约为零。

$$\Delta p = J \approx 0\,\mathrm{kg\,m/s}.$$

用速度图核对冲量：比较末初有符号动量

已经由力图得到动量变化后，再用速度图作独立核对：同一质量乘末速度减初速度。必须比较方向和大小；只有速率相同，速度可能仍反向。

例题 2024 Q2 (c)(iii)

本问承接 2024 Q2 (c)(ii)。

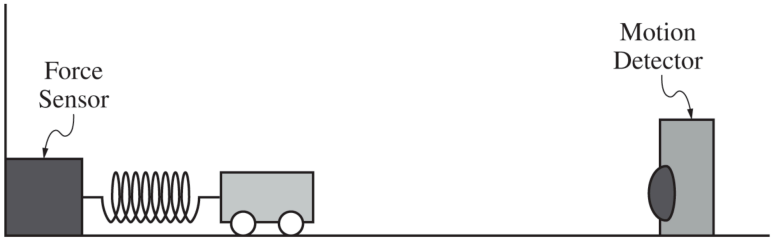
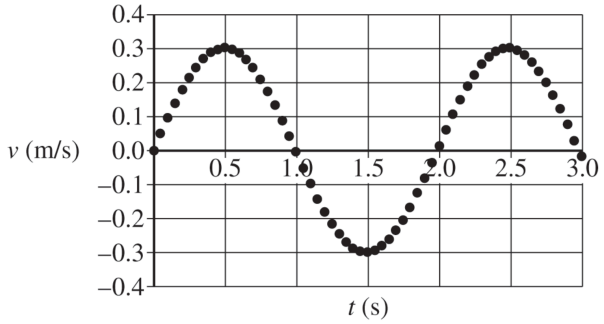
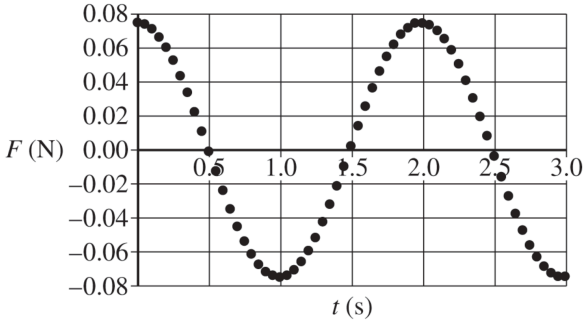


Figure 2

In a different experiment, the student attaches one end of a spring to a force sensor that is attached to a wall. The other end of the spring is attached to a cart with mass $m = 0.25$ kg. The student places a motion detector to the right of the cart, as shown in Figure 2, and pulls the cart to the right a small distance so that the spring is stretched. The student releases the cart from rest, and the cart-spring system oscillates.

The following graphs show the velocity v of the cart and the force F exerted on the cart by the spring as functions of time t.





iii. Do the data from the velocity-time graph confirm your estimation from part (c)(ii) ? Briefly **explain**.

解：

原速度图在 0.5 s 和 2.5 s 都约为 +0.30 m/s，速度大小和方向都相同；质量不变，末初动量相同：

$$\Delta p = m[v(2.5) - v(0.5)] \approx 0.25(0.30 - 0.30) = 0.$$

与力图面积估计一致。只说“速率相同”不充分；若方向反了，动量就会改变。

力矩、角加速度与转动能

角加速度在哪一帧最大：先比较力臂

力矩衡量力使物体转动的作用。力的大小相同时，转轴到该力作用线的垂直距离越大，力矩越大。重力作用在系统质心；当轴—质心连线与重力垂直时，重力力臂最大。

$$\tau = F r_{\perp}, \qquad \alpha = \frac{\tau_{\text{net}}}{I}.$$

r⊥ 是垂直力臂，τ 是力矩，I 是转动惯量，α 是角加速度。同一个刚性系统的 I 不变，比较角加速度就比较净力矩；帧间隔不相同，不能凭相邻图片转角大小代替。

例题 2023 Q5 (a)(i)

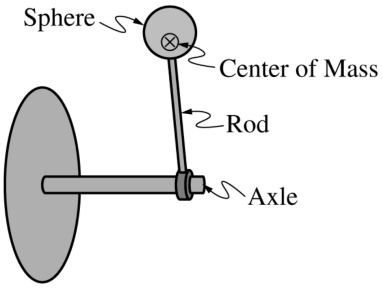


Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a ⊗, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.
Note: these frames are not equally spaced apart in time.

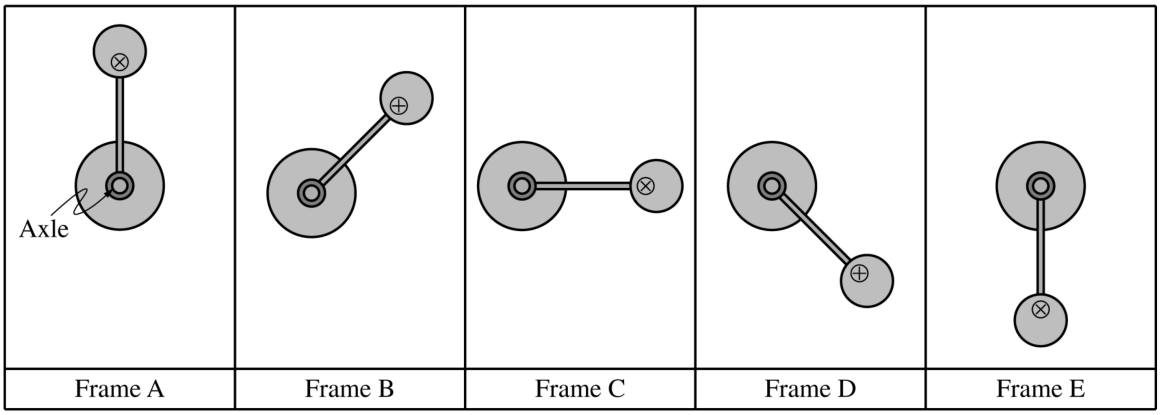


Figure 2

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

解：

Frame C。此时质心到轴的连线与竖直重力垂直，重力力臂最大，所以重力力矩最大；同一系统的惯量不变，力矩除以惯量便是角加速度，所以角加速度大小也最大。帧数间隔不是等时间，不能直接比较图片间转过多少角。

长杆的完整受力图：作用点与铰链力方向

均匀杆的重力作用于中点，张力作用于绳挂点，铰链力作用于铰链。原题要求画真实力，所以每种力用一支箭头，不把分量各当成一支力。

铰链力的方向不能猜。利用刚学过的力矩：杆静止，右端张力的竖直分量以整杆长为力臂，重力以半杆长为力臂；相反力矩平衡，先确定张力向上分量，再用竖直、水平合力都为零确定铰链力。

例题 2024 Q3 (a)

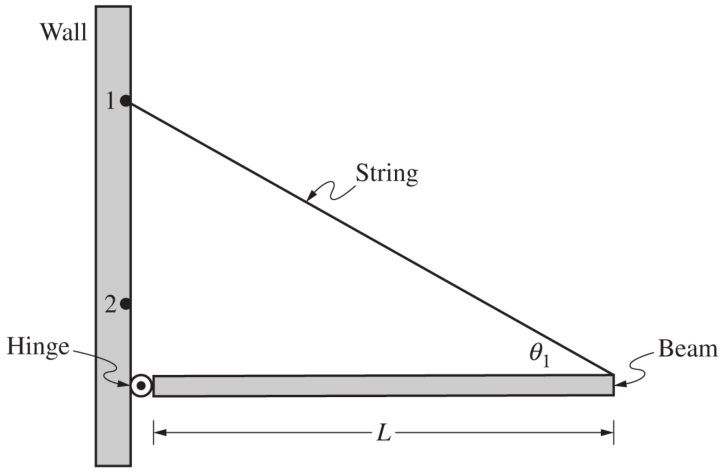


Figure 1

3. (12 points, suggested time 25 minutes)

The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H, and the angle between the beam and the string is $\theta = \theta_1$.

(a) The following rectangle represents the beam in Figure 1. On the rectangle, **draw** and **label** the forces (not components) exerted on the beam. Draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted.



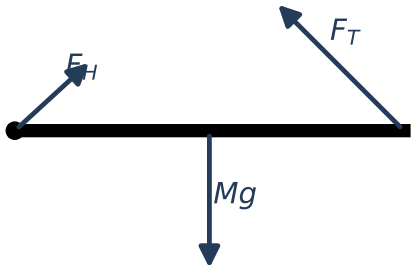
解：

杆上只画三支真实外力：杆中点向下的重力 Mg；右端沿绳向左上的张力 F_T；左端铰链处向右上的铰链力 F_H。不要把张力分量画成两个独立作用力。

铰链力的方向还要由平衡决定。杆不转动，右端张力的向上分量对轴的力矩须抵消中点重力：

$$F_{T,y}L = Mg(L/2), \qquad F_{T,y} = Mg/2.$$

张力只托住一半重量，铰链还需向上托住另一半；张力向左的分量由铰链向右的分量平衡，因此铰链的合力箭头朝右上。



绳角变小但杆仍水平：保持垂直分量而非总张力

杆始终水平静止、重量和重力力臂不变，绳须提供相同的反向力矩。绳越接近水平，它的张力中垂直杆的比例越小；若垂直分量仍相同，总张力就要改变。不要只画绳长来判断张力。

例题 2024 Q3 (b)

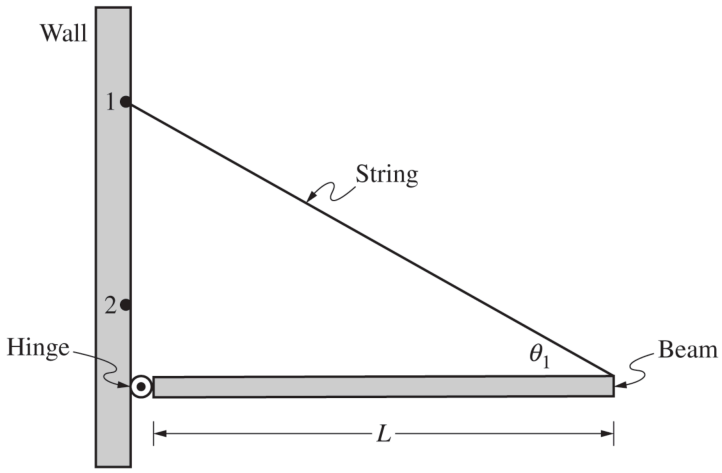


Figure 1

3. (12 points, suggested time 25 minutes)

The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H, and the angle between the beam and the string is $\theta = \theta_1$.

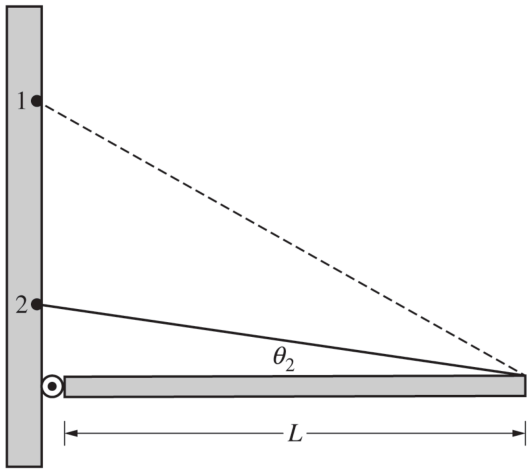


Figure 2

(b) The string is then attached lower on the wall, at Point 2, and the beam remains horizontal, as shown in Figure 2. The angle between the beam and the string is $\theta = \theta_2$. The dashed line represents the string shown in Figure 1.

The magnitude of the tension in the string shown in Figure 1 is F_{T1}. The magnitude of the tension in the string shown in Figure 2 is F_{T2}. **Indicate** which of the following correctly compares F_{T2} with F_{T1}.

_____ $F_{T2} > F_{T1}$ _____ $F_{T2} < F_{T1}$ _____ $F_{T2} = F_{T1}$

Briefly **justify** your answer, using qualitative reasoning beyond referencing equations.

解：

选择第二根绳的张力更大。杆的位置和重力力矩没有改变，绳仍须提供同样大的反向力矩。绳变平后，张力中垂直杆的比例减小；要保留同样的向上分量，总张力就必须更大。

转动平衡推导张力：取铰链为轴减少未知力

取铰链为轴时，铰链力的力臂为零，力矩式中只剩张力与重力。逆时针为正，张力在杆末端、重力在均匀杆中点；张力垂直杆的分量是 F_T sinθ。

$$\sum \tau = I\alpha = 0.$$

推导完成后，核验角度变化：θ 在 0 到 90° 内时，角度减小使 sinθ 减小；在分母中的这个量如何改变张力，必须与前面的定性解释一致。

例题 2024 Q3 (c)

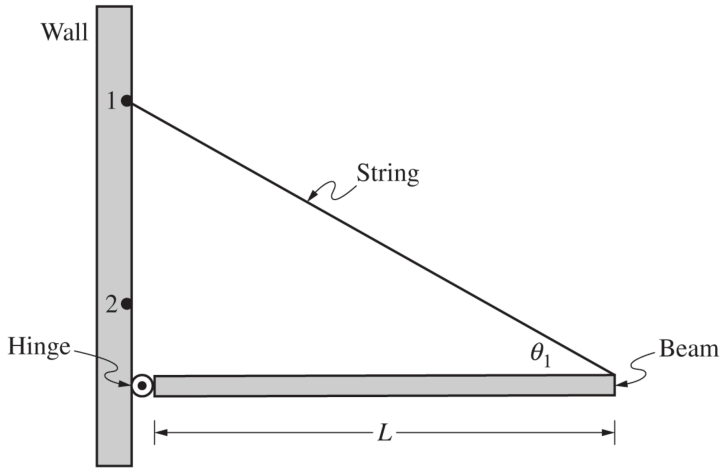


Figure 1

3. (12 points, suggested time 25 minutes)

The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H, and the angle between the beam and the string is $\theta = \theta_1$.

(c) Starting with Newton's second law in rotational form, **derive** an expression for the magnitude of the tension in the string. Express your answer in terms of M, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference book.

解：

取铰链为转轴，逆时针力矩为正。杆静止，角加速度为零。绳在右端、重力在中点：

$$\sum \tau = I\alpha = 0, \qquad F_T L \sin\theta - Mg\frac{L}{2} = 0,$$

$$F_T = \frac{Mg}{2\sin\theta}.$$

核对(a)的铰链力：竖直方向，张力的向上分量只能托住一半重量，铰链还需向上托住另一半；水平方向，铰链需要向右抵消张力的向左分量。所以铰链箭头朝右上。

练习 2024 Q3 (d)

本问承接 2024 Q3 (b)、2024 Q3 (c)。

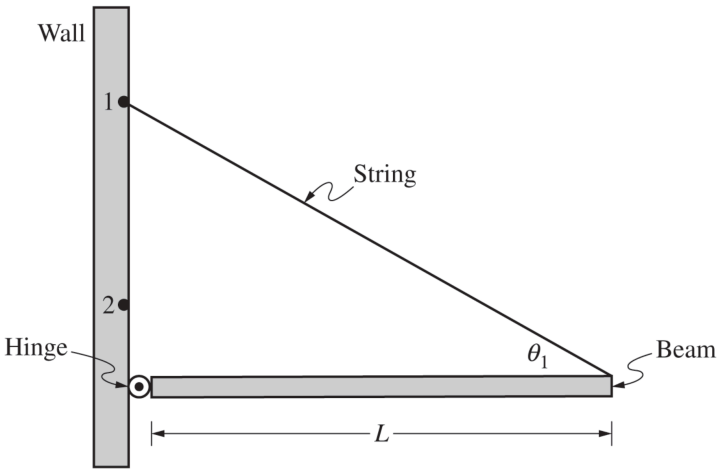


Figure 1

3. (12 points, suggested time 25 minutes)

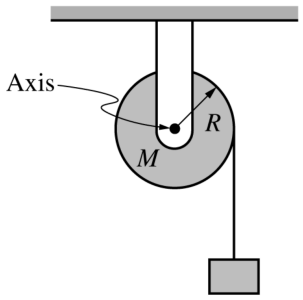
The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_{H}, and the angle between the beam and the string is $\theta = \theta_1$.

(d) Is your derived equation in part (c) consistent with your justification in part (b) ? **Explain** your reasoning.

轮缘切向拉力求角加速度：力矩除以转动惯量

绳切向拉圆盘边缘时，拉力垂直半径，力矩大小为张力乘半径。圆盘重力与轴力不绕中心产生转动力矩。先写转动第二定律，再代入题给 I；不要把悬挂物块的重力直接当圆盘上的切向力。

例题 2023 Q4 (a)



4. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_T.

(a) Determine an expression for the magnitude of the angular acceleration α_D of the disk as the block travels downward. Express your answer in terms of M, R, F_T, and physical constants as appropriate.

解：

绳在半径 R 处切向拉圆盘，力矩为 $F_T R$。轴的作用力与圆盘重力关于中心轴不产生转动力矩。代入题给圆盘转动惯量：

$$\sum \tau = I\alpha_D, \qquad F_T R = \frac{1}{2}MR^2\alpha_D,$$

$$\alpha_D = \frac{2F_T}{MR}.$$

剪绳后角速度图：角加速度是曲线斜率

剪绳后重力使杆向下转，角速度从零增大。随着杆趋近竖直，重力作用线趋近转轴，力臂变小、力矩变小、角加速度变小。角速度图仍上升，但斜率越来越小，向下凹。

接近竖直时斜率可趋零；已有角速度不因此变零。图不要画成上升后再下降。

例题 2024 Q3 (e)

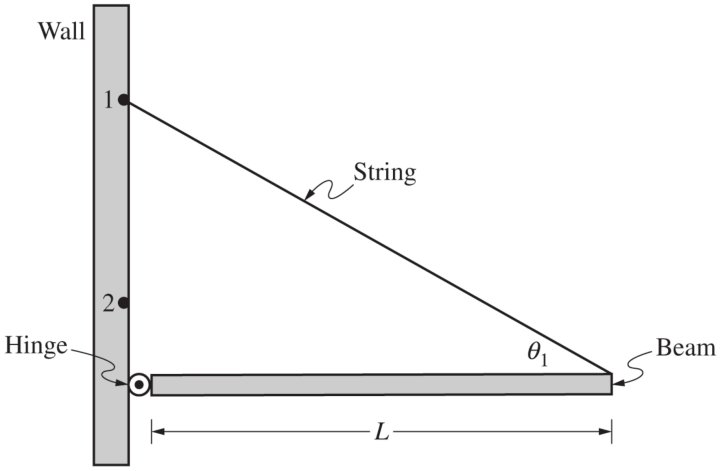
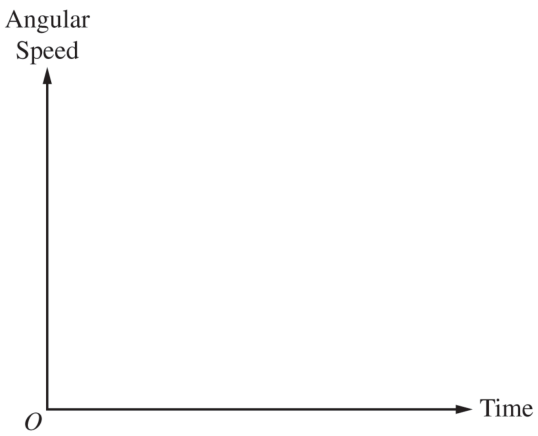


Figure 1

3. (12 points, suggested time 25 minutes)

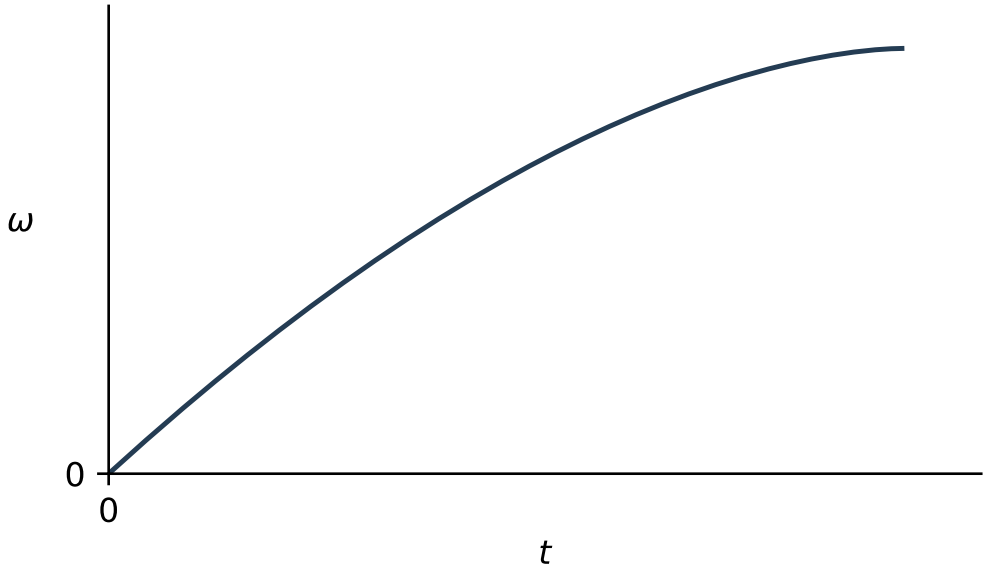
The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H, and the angle between the beam and the string is $\theta = \theta_1$.

(e) The string is cut, and the beam begins to rotate about the hinge with negligible friction. On the following axes, **sketch** the angular speed of the beam as a function of time for the time interval while the beam falls but before the beam becomes vertical.



解：

从零角速度开始画单调增加、向下凹的曲线。剪绳后重力一直使杆向下转，所以角速度增加；杆越来越接近竖直时，重力的力臂越来越小，力矩和角加速度越来越小，因此曲线斜率减小。接近竖直时斜率趋零，但角速度不会回到零。



角速度从零增加，正斜率逐渐变小；横轴 t，纵轴角速度。

相同力矩和时间：比较角冲量与转动动能

这里的转动惯量 I 对应绕轮中心的固定轴；M 是两轮相同的总质量，R 是共同半径。同一角速度下，离轴更远的质量线速更大，贡献更多动能。因此每一小块质量以“质量×距轴距离的平方”贡献转动惯量，再把各份贡献相加。

圆环的各份质量都距轴 R，所以总转动惯量等于总质量乘 R^2；原题已给出圆盘的式子。分别写出：

$$I_{\text{hoop}} = MR^2.$$

$$I_{\text{disk}} = \frac{1}{2}MR^2.$$

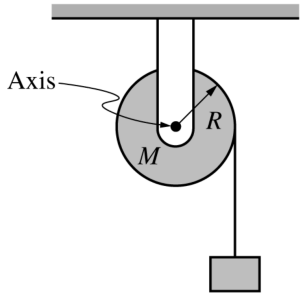
所以同质量、同半径的圆环转动惯量是圆盘的两倍；这两种形状的区别来自质量离轴的分布。

同样的轮缘半径、同样拉力给同样力矩；作用相同时间，角动量增加相同。角动量是转动惯量乘角速度，转动动能是转动惯量乘角速度平方再除二；消去角速度后，可在相同 L 下比较不同 I。

$$\Delta L = \tau \Delta t, \qquad L = I\omega, \qquad K_R = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}.$$

这里 L 表示角动量，与其他题的杆长 L 按本题区分。相同角冲量不等于相同功，因为两轮可能转过不同角度。

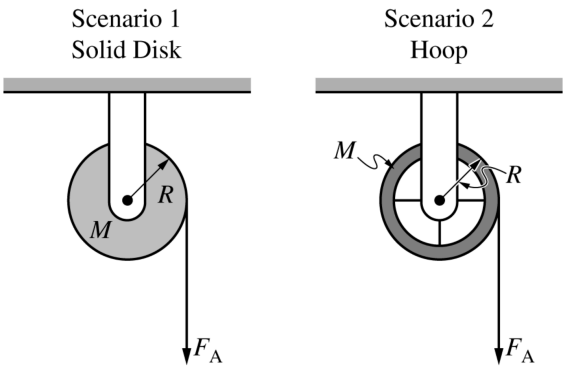
例题 2023 Q4 (b)



4. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_T.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass M and radius R as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force F_A for the same time interval Δt, causing the pulleys to rotate without the string slipping. After time interval Δt, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

(b) Consider scenarios 1 and 2 at the end of time interval Δt. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

解：

两轮所受拉力 F_A 和轮缘半径相同，故力矩相同；作用时间 Δt 也相同，所以角冲量相同，角动量增量相同。

$$\Delta L_D = \Delta L_H = F_A R \Delta t.$$

圆环质量更远离轴，其惯量是圆盘的两倍。由静止出发，两者末角动量相同，圆盘惯量更小，所以末角速度更大。以共同末角动量 L_{ang} 表示动能：

$$K_D = \frac{L_{\text{ang}}^2}{2I_D}, \qquad K_H = \frac{L_{\text{ang}}^2}{2I_H}, \qquad K_D = 2K_H.$$

因此相同角冲量并不意味着相同动能。圆盘在同时间内转过的角度更大，拉力对它所做的功也更多，与能量比较一致。

相同拉力和距离：由相同功定性比较角速度

若题目固定的是拉出同样长的绳，力做功由力乘拉出距离决定。相同功、都从静止开始，得到相同转动动能；转动惯量较小者需要较大的角速度才有相同动能。

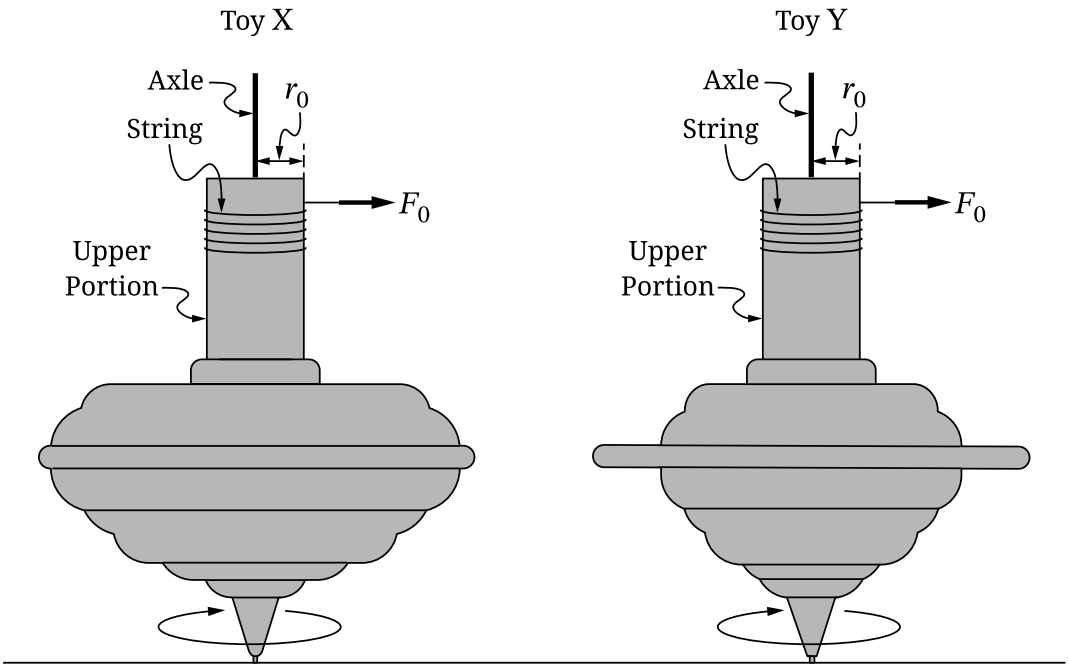
这里固定拉出距离，没有固定时间。不能照搬“相同力矩和时间给相同角动量”。

例题 2026 Q4 (a)

Question 4

4. Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius r_0, as shown in the side view in Figure 1. A string of length ℓ_0 is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is I_X, and the rotational inertia of Toy Y is $I_Y = \frac{1}{2} I_X$.

Figure 1



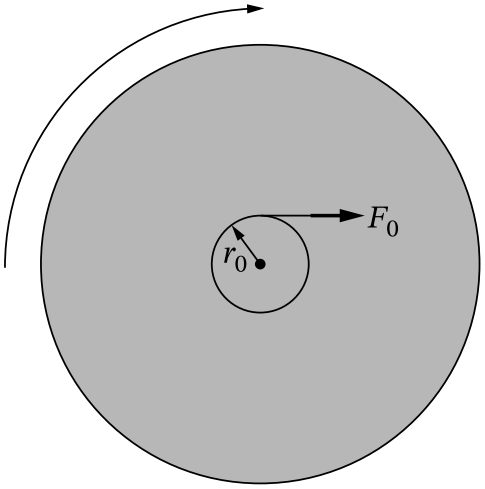
Side View

Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X.

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y.

Figure 2



Overhead View

Part A

Indicate whether ω_{Y} is greater than, less than, or equal to ω_{X} by writing one of the following.

- $\omega_{\mathrm{Y}} > \omega_{\mathrm{X}}$
- $\omega_{\mathrm{Y}} < \omega_{\mathrm{X}}$
- $\omega_{\mathrm{Y}} = \omega_{\mathrm{X}}$

Justify your answer using qualitative reasoning beyond referencing equations.

解：

选择 Y 的末角速度更大。两根绳长度相同，恒拉力相同，因此拉力对两玩具做的功相同；由静止开始，转动动能增加也相同。Y 的转动惯量较小，要有相同动能，它必须有更大的角速度。这里没有说拉绳所需时间相同。

拉出整根绳的末角速度：功—能或转动运动学推导

拉力恒定、绳全部拉出时，沿力方向的位移就是绳长。先写外力做功等于转动动能增加，再用初始静止解出末角速度。

$$W = \Delta K_R, \qquad K_R = \frac{1}{2} I \omega^2.$$

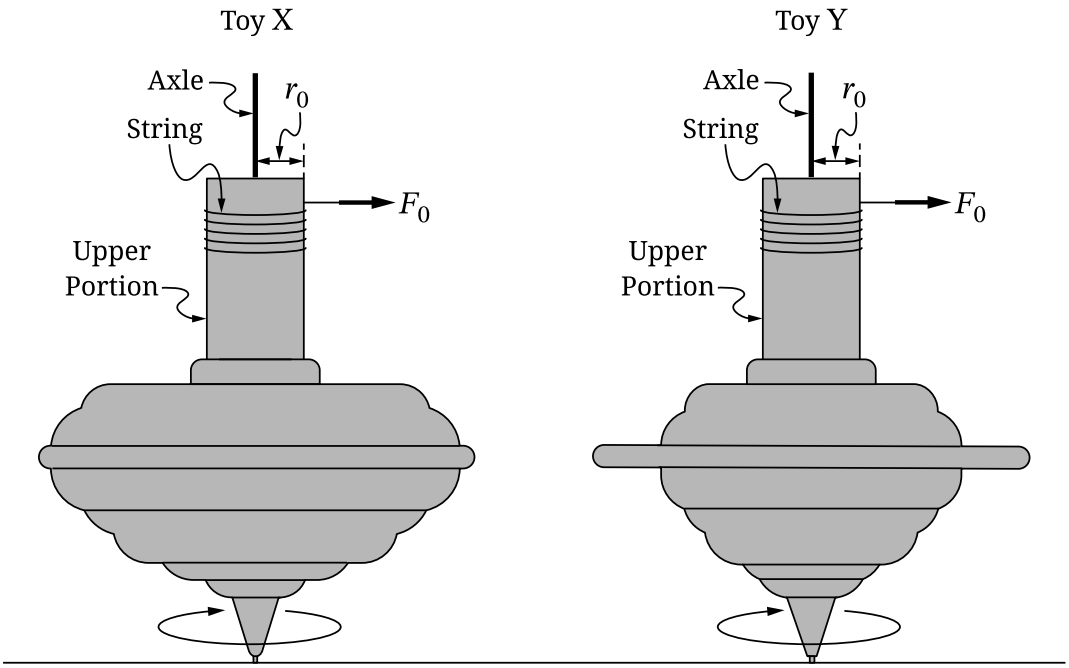
另一条路线从 τ=Iα 开始，绳不滑动使拉出长度等于半径乘转角，恒角加速度再把转角连到末角速度。推导后用转动惯量的比例核验定性比较。

例题 2026 Q4 (b)

Question 4

4. Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius r_0, as shown in the side view in Figure 1. A string of length ℓ_0 is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is I_X, and the rotational inertia of Toy Y is $I_Y = \frac{1}{2} I_X$.

Figure 1

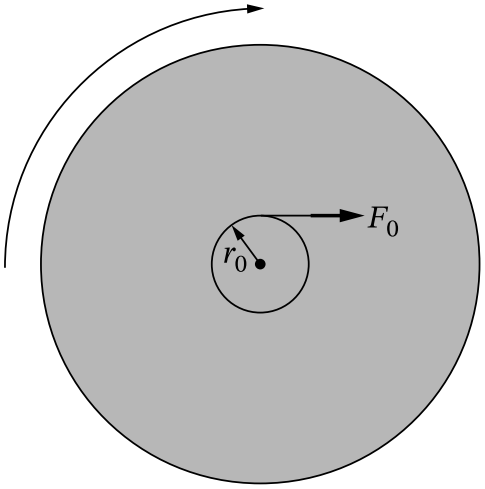


Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X.

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y.

Figure 2



Overhead View

Part B

Starting with the work-energy theorem or Newton's second law in rotational form, **derive** an expression for the angular speed ω_{X} of Toy X at the instant the string loses contact with the axle. Express your final answer in terms of ℓ_0, I_{X}, F_0, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

从功—能定理开始。全部绳被拉出时，拉力做功 $F_0\ell_0$，初动能为零：

$$W = \Delta K_R, \qquad F_0\ell_0 = \frac{1}{2} I_X \omega_X^2 - 0,$$

$$\omega_X = \sqrt{\frac{2F_0\ell_0}{I_X}}.$$

也可用转动第二定律得到恒定角加速度，再由不滑动的绳长得到转角：

$$\alpha_X = \frac{F_0 r_0}{I_X}, \qquad \Delta\theta = \frac{\ell_0}{r_0}, \qquad \omega_X^2 = 2\alpha_X \Delta\theta.$$

两式联立后 r_0 抵消，得到同一个角速度结果。

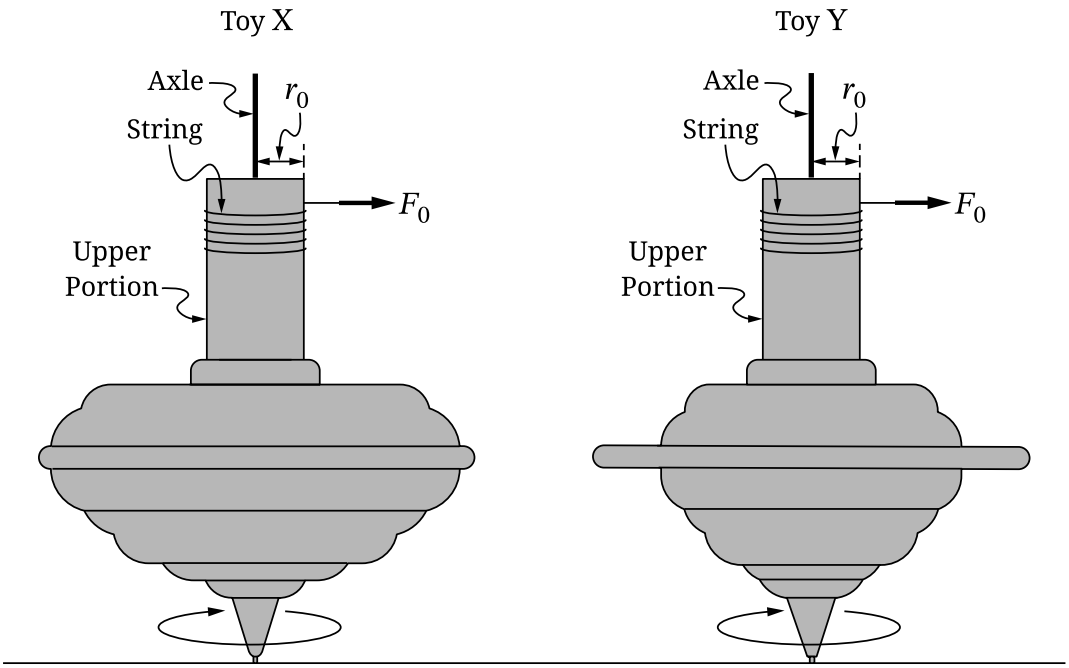
练习 2026 Q4 (c)

本问承接 2026 Q4 (a)、2026 Q4 (b)。

Question 4

4. Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius r_0, as shown in the side view in Figure 1. A string of length ℓ_0 is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is I_X, and the rotational inertia of Toy Y is $I_Y = \frac{1}{2} I_X$.

Figure 1



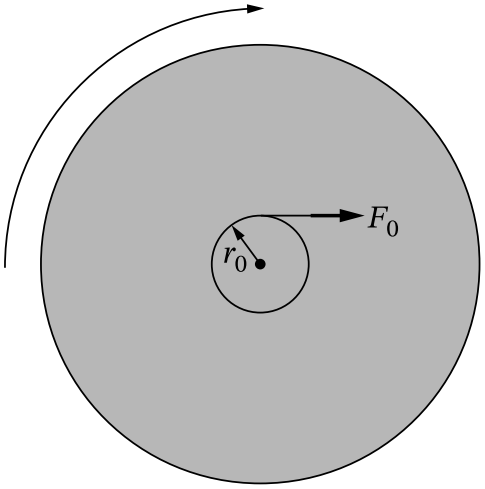
Side View

Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X.

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y.

Figure 2



Overhead View

Part C

Justify how your derived equation in part B is or is not consistent with your reasoning in part A.

弹簧振动与摆

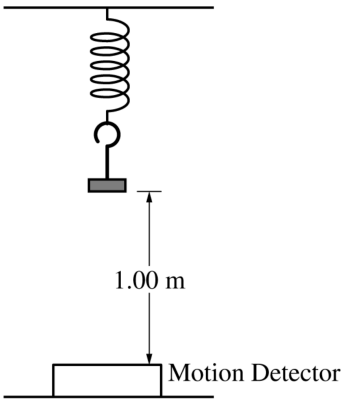
由周期求弹簧劲度：读完整重复周期

周期 T 是回到同一位置并沿同一方向运动的完整一轮时间，不能只读到相对的另一端。挂同一质量的小振动，弹簧越硬，周期越短。

$$T = 2\pi\sqrt{\frac{m}{k}}, \qquad k = \frac{4\pi^2 m}{T^2}.$$

静态平衡也可核对：重量与弹簧拉力平衡，伸长等于重量除以劲度。高度图的平衡高度与未挂物块的端点高度之差，才是静态伸长。

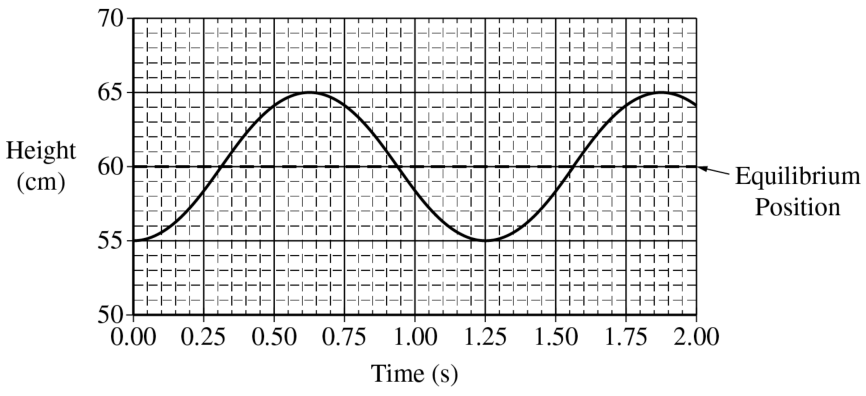
例题 2022 Q5 (a)



5. (7 points, suggested time 13 minutes)

A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.



(a) Using the information given and information taken from the graph, calculate the spring constant.

解：

原高度图中心约为 60 cm，振幅约为 5 cm，相邻两次同状态的间隔为 1.25 s。由质量 0.50 kg 与周期求劲度：

$$T = 2\pi\sqrt{m/k_0} \;\Rightarrow\; k_0 = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.50)}{1.25^2} \approx 12.6\,\mathrm{N/m}.$$

也可用静态伸长：未挂物块的端点为 1.00 m，平衡后为 0.60 m，伸长 0.40 m，把重量除以伸长量，约为 12.3 N/m（重力加速度取 9.8 m/s^2）或 12.5 N/m（取 10 m/s^2）。图上读数有限，两种方法近似一致。

改变质量比较频率：同一弹簧用比值约去常量

频率 f 是每秒完整振动次数，与周期互为倒数。同一弹簧中，增加共同振动的总质量会降低频率；用两次频率之比可以约去相同劲度与 2π。

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

例题 2023 Q1 (b)

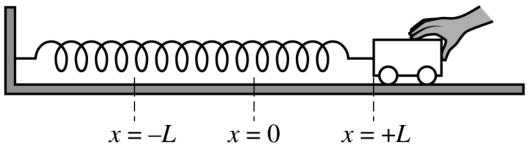


Figure 1

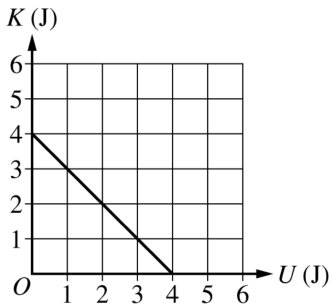


Figure 2

1. (7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions $x = +L$ and $x = -L$. Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

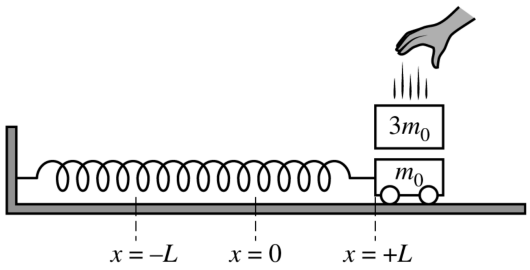


Figure 3

When the cart is at $+L$ and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between $-L$ and $+L$. The masses of the cart and the block are m_0 and $3m_0$, respectively.

(b) The frequency of oscillation before the block is dropped onto the cart is f_1. The frequency of oscillation after the block is dropped onto the cart is f_2. Calculate the numerical value of the ratio $\frac{f_2}{f_1}$.

解：

添加后一起运动的质量是原车质量的四倍，弹簧劲度不变。用频率关系相除，公共常数与劲度约掉：

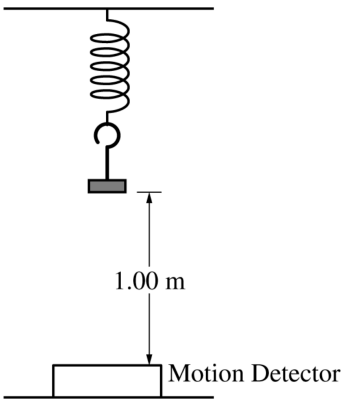
$$\frac{f_2}{f_1} = \frac{\frac{1}{2\pi}\sqrt{k/(4m_0)}}{\frac{1}{2\pi}\sqrt{k/m_0}} = \frac{1}{2}.$$

高度图比较能量：先比切线斜率大小

高度图的切线斜率是竖直速度。两个时刻切线斜率的绝对值相等，表示速率相等，同一质量的动能相等。高度本身不是速度。

对于无耗散的物块—弹簧—地球系统，K+U 总和固定；若 K 相等，总势能 U 也相等。总势能是弹簧势能与重力势能之和，不要求两部分各自相等。

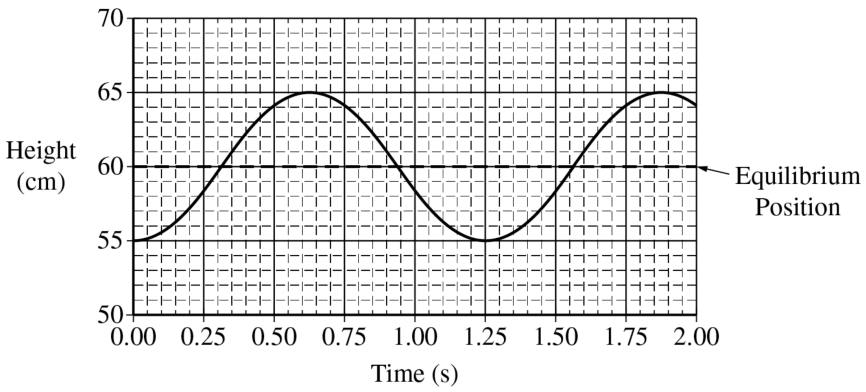
例题 2022 Q5 (b)(i)



5. (7 points, suggested time 13 minutes)

A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.



(b) At time 0.75 s, the object-spring-Earth system has a total kinetic energy K_0 and a total potential energy U_0. At 1.13 s, the object-spring-Earth system again has a total kinetic energy K_0 and a total potential energy U_0.

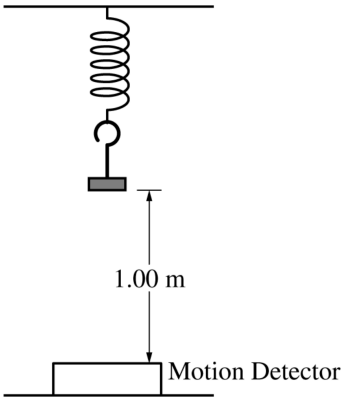
i. Explain how a feature of the graph indicates that the total kinetic energy of the system is the same at these two times.

解：

0.75 s 与 1.13 s 两处高度图切线斜率大小相同，速率相同；质量相同，所以动能相同。不能用高度值直接当速度。

练习 2022 Q5 (b)(ii)

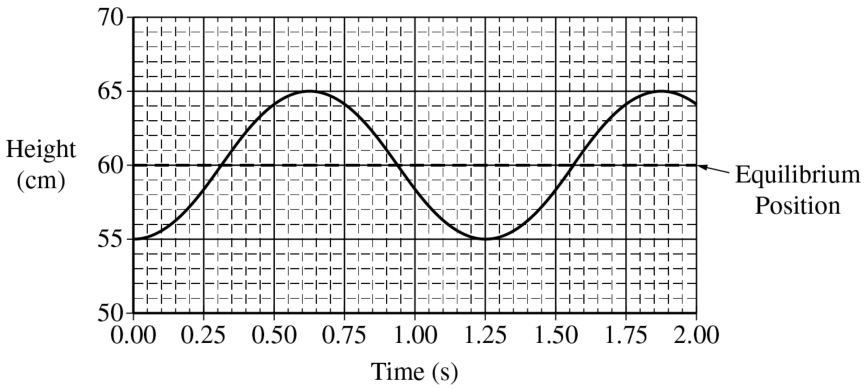
本问承接 2022 Q5 (b)(i)。



5. (7 points, suggested time 13 minutes)

A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.



(b) At time 0.75 s, the object-spring-Earth system has a total kinetic energy K_0 and a total potential energy U_0. At 1.13 s, the object-spring-Earth system again has a total kinetic energy K_0 and a total potential energy U_0.

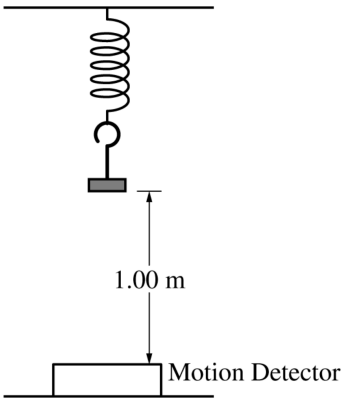
ii. Briefly explain why the total potential energy of the system is the same at these two times.

竖直弹簧的新平衡高度：先改静态伸长

平衡时速度和加速度均为零，弹簧向上拉力等于向下重量。质量不变、劲度乘四，伸长除四；把新伸长从原题未挂重物的端点高度扣除，才得到新平衡高度。

$$k\Delta\ell = mg.$$

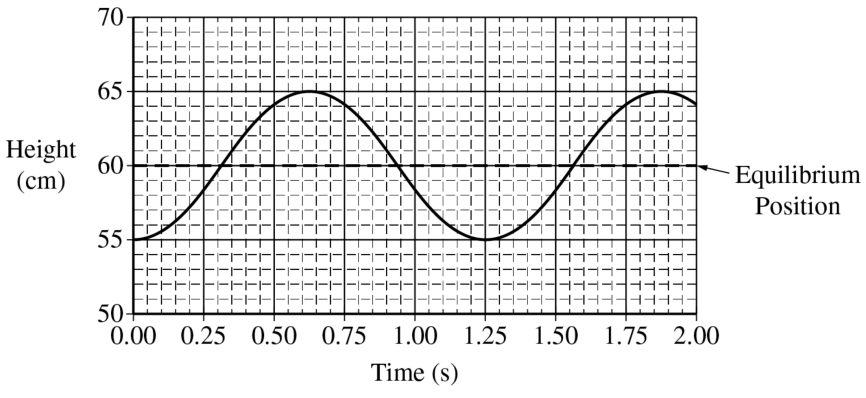
例题 2022 Q5 (c)(i)



5. (7 points, suggested time 13 minutes)

A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.



(c) The experiment is repeated with a spring of spring constant $4k_0$ and that has the same length as the original spring. The 0.50 kg object is hung from the new spring and allowed to come to rest at a new equilibrium position.

i. Determine the new equilibrium position above the motion detector.

解：

新弹簧劲度为 $4k_0$，挂同质量物块，静态伸长减到原来的四分之一，由 40 cm 变为 10 cm。未挂物块的端点高度仍 100 cm，故新平衡高度为 90 cm。

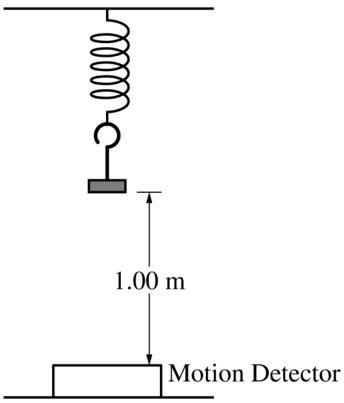
更硬弹簧的高度曲线：分别改中心、振幅与周期

振动曲线有三个不同参数：中心是平衡高度；振幅是从平衡位置拉开的距离；周期由质量与劲度决定。劲度变大不自动改变题目规定的拉开距离。

先标新中心与上下极值，再按新周期放置最小值和最大值的时刻。由最低处静止释放，曲线从最低点以水平切线起步。

例题 2022 Q5 (c)(ii)

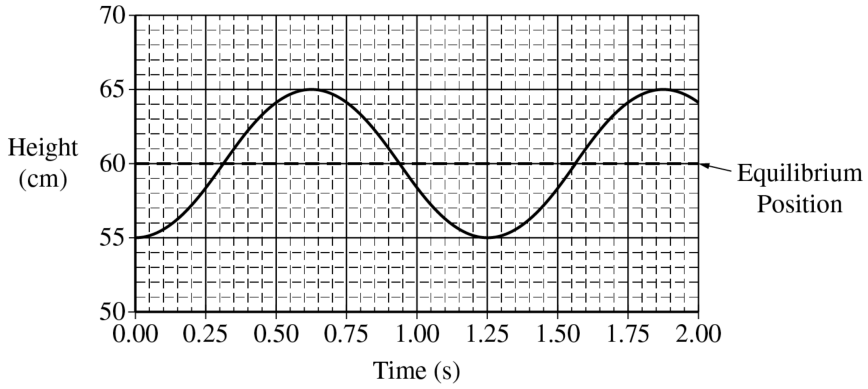
本问承接 2022 Q5 (c)(i)。



5. (7 points, suggested time 13 minutes)

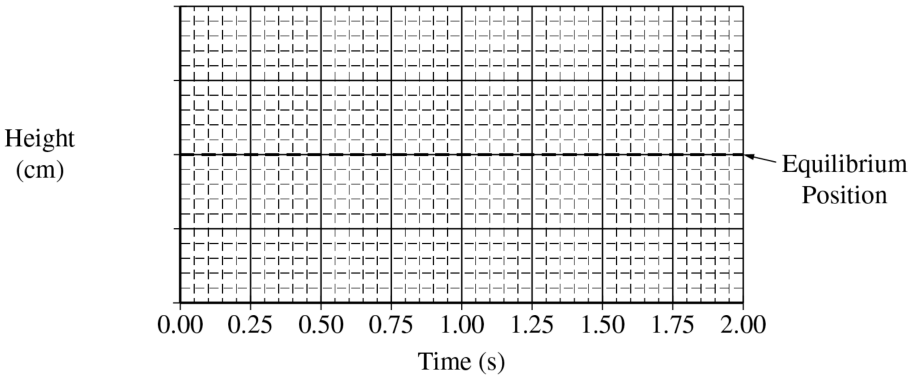
A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.



(c) The experiment is repeated with a spring of spring constant $4k_0$ and that has the same length as the original spring. The $0.50\,\text{kg}$ object is hung from the new spring and allowed to come to rest at a new equilibrium position.

ii. The object is again pulled down the same distance d_0 from the equilibrium position and released. On the following graph, draw a curve representing the motion of the object after it is released. Label the vertical axis with an appropriate numerical scale. A grid for scratch (practice) work is also provided.



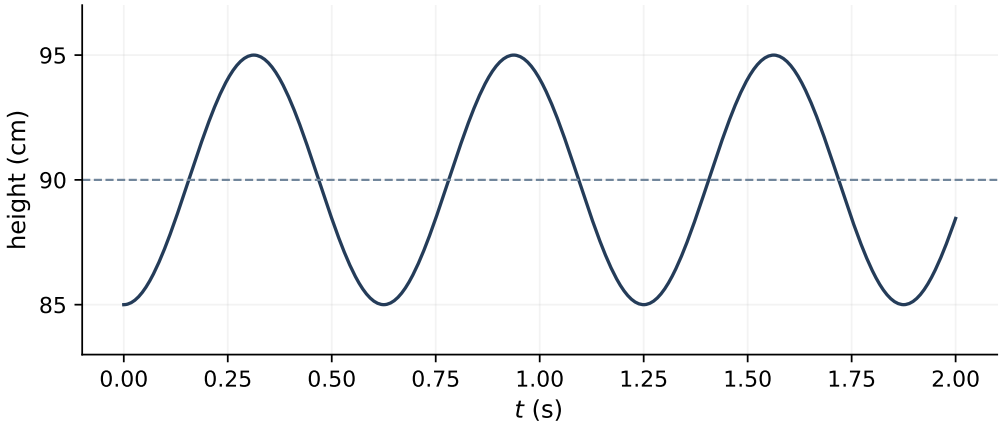
解：

仍从平衡位置向下拉原来的 d_0，振幅保持 5 cm；周期变为一半：

$$T_{\text{new}} = 2\pi\sqrt{\frac{m}{4k_0}} = \frac{1}{2}T = 0.625\,\text{s}.$$

新曲线围绕 90 cm，最低 85 cm、最高 95 cm，从零时刻的最低位置开始；0.3125 s 后到最高位置，0.625 s 后回到最低位置。可用下式画出：

$$h(t) = 90\,\text{cm} - 5\,\text{cm}\cos\left(\frac{2\pi t}{0.625\,\text{s}}\right).$$



换弹簧后要同时更新中线和周期，振幅保持 5 cm。

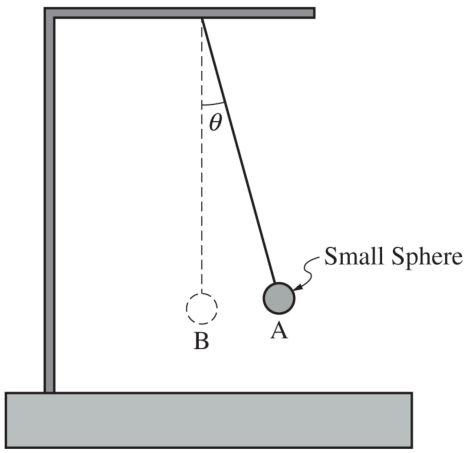
弹性摆绳与更大重力同时变化：比较两个相反趋势

小角度、有效摆长近似固定时，摆越长周期越大，重力越强周期越小。绳可伸长时，更强重力又可能增加摆长，这两个作用方向相反。

$$T \approx 2\pi\sqrt{\frac{\ell}{g}}.$$

ℓ 是有效摆长。题目没有给绳的伸长规律时，不能把 ℓ 固定，也不能凭空补胡克定律。应分别说明哪个作用占优势时周期更长或更短。

例题 2024 Q4 (b)



4. (7 points, suggested time 13 minutes)

A simple pendulum consists of a small sphere that hangs from a string with negligible mass. The top end of the string is fixed. The sphere is pulled to Point A so that the string makes a small angle θ with the vertical, as shown. The sphere is then released from rest and swings through its lowest point at Point B. The work done on the sphere by Earth between points A and B is W_E.

The pendulum is then taken to Planet X. The mass of Planet X is the same as the mass of Earth, but the radius of Planet X is greater than the radius of Earth. The sphere is again brought to Point A (displaced θ from the vertical), released from rest, and swings through its lowest point at Point B. The work done on the sphere by Planet X between points A and B is W_X.

A new pendulum is made by hanging the same small sphere from a different string with negligible mass. The new string is slightly elastic, and the length of the string may increase or decrease depending on the tension applied to the string. On Earth, when the sphere is again displaced θ from the vertical and released from rest, the new pendulum oscillates with period T_E.

The new pendulum is then taken to a different planet, Planet Y. The radius of Planet Y is the same as the radius of Earth, but the mass of Planet Y is larger than the mass of Earth. On Planet Y, when the sphere is again displaced from the vertical and released from rest, the new pendulum oscillates with period T_Y.

(b) In a clear, coherent paragraph-length response that may also contain drawings, **explain** how T_Y could be larger than T_E but also could be smaller than T_E.

解：

Y 行星与地球半径相同而质量更大，表面重力加速度更大。若只考虑重力增大，小角度摆的回复作用增强，周期倾向于缩短。可伸长的绳又会因更大的张力变长；更长的摆倾向于使周期增大。

这两个因素方向相反：如果重力增大的作用占优势，Y 上周期较短；如果有效摆长增加的作用占优势，Y 上周期较长。题干未提供绳的定量伸长规律，不能固定长度，也不能补入线性弹性公式来替它决定唯一次序。

在有效摆长近似恒定的小角度模型内，可用比例看清两项竞争；该式只作条件比较，不假装解出了题干未给出的伸长量：

$$\frac{T_Y}{T_E} \approx \sqrt{\frac{\ell_Y/\ell_E}{g_Y/g_E}}.$$

有效摆长的相对增幅大于重力加速度的相对增幅时，周期较长；较小时，周期较短。

浮力、流量与喷水

同一浸没物体在两流体中：比较浮力再比较加速度

完全浸没时，排开体积等于物体体积。浮力大小是被排开流体的重量；物体相同，所以重量、质量、排开体积相同，流体密度是改变的量。

$$F_b = \rho V g.$$

ρ 是流体密度，V 是排开体积。松手初时先找重力与浮力的方向，再比较向上合力，不凭“较稠”这种印象判断。

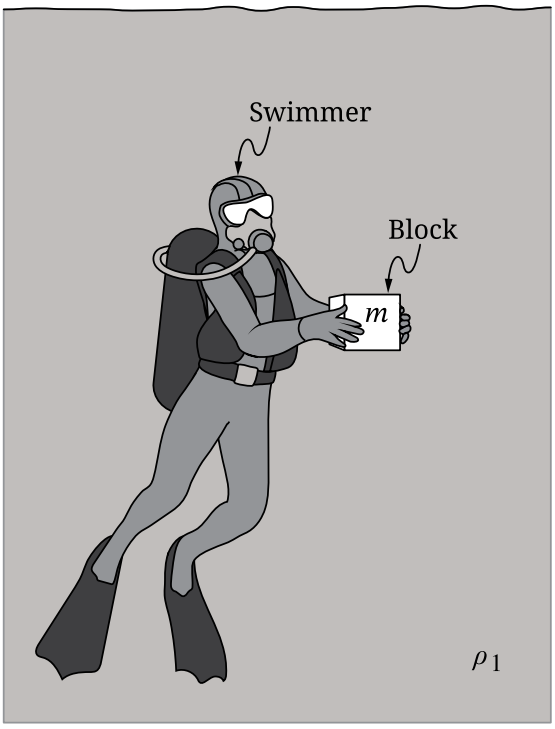
例题 2025 Q4 (a)

Question 4

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4. In Scenario 1, a swimmer holds a block of mass m at rest in a tank of freshwater with density ρ_1, as shown in Figure 1. The block is released from rest and accelerates upward with an initial acceleration a_1. All frictional forces are negligible.

Figure 1



In Scenario 2, the swimmer holds the same block at rest in a tank of salt water with density ρ_2, where $\rho_2 > \rho_1$. The swimmer again releases the block from rest, and the block accelerates upward with initial acceleration a_2. All frictional forces are negligible.

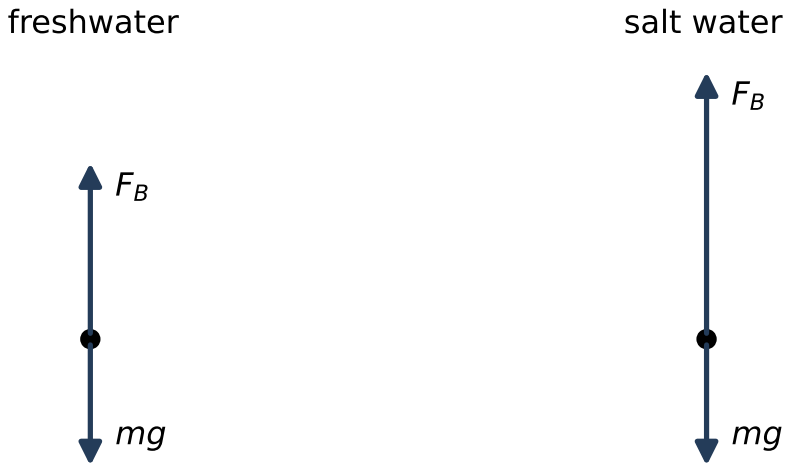
A. **Indicate** whether a_1 is greater than, less than, or equal to a_2 by writing one of the following in your answer booklet.

- $a_1 > a_2$
- $a_1 < a_2$
- $a_1 = a_2$

Justify your answer in terms of ALL forces exerted on the block in each scenario. Use qualitative reasoning beyond referencing equations.

解：

选择盐水中的初始加速度更大。松手后两种情况都只有向下重力和向上浮力。物块相同，重力相同、排水体积相同；盐水密度更大，浮力更大，所以向上合力和初始向上加速度更大。



相同重力下，盐水中的浮力更大。

浮力减重量求初始加速度，并检验密度变化

取向上为正，浮力向上为正、重量向下为负。把两真实力相加写入 ma，再把质量移到分母；完全浸没才可以把排开体积直接写成物块体积。

$$F_{\text{net}} = ma.$$

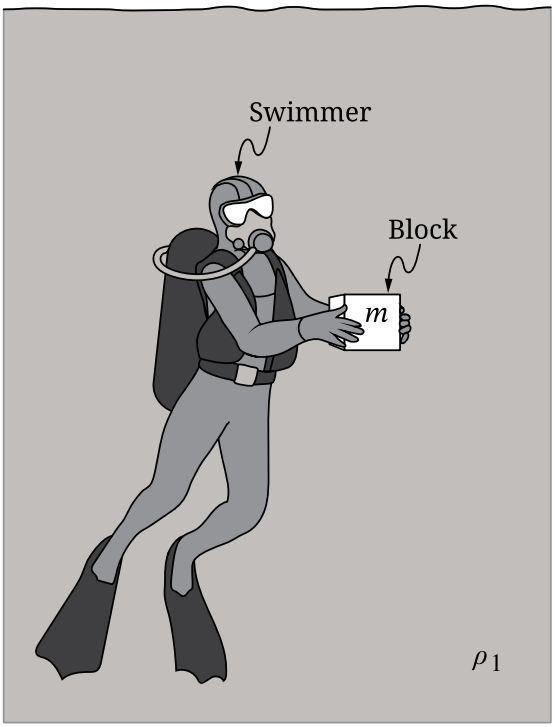
检验密度变化时，固定同一个物块的 V、m 与 g，观察密度只进入哪一项；有非零截距的线性关系不能叫正比例。

例题 2025 Q4 (b)

Question 4

4. In Scenario 1, a swimmer holds a block of mass m at rest in a tank of freshwater with density ρ_1, as shown in Figure 1. The block is released from rest and accelerates upward with an initial acceleration a_1. All frictional forces are negligible.

Figure 1



In Scenario 2, the swimmer holds the same block at rest in a tank of salt water with density ρ_2, where $\rho_2 > \rho_1$. The swimmer again releases the block from rest, and the block accelerates upward with initial acceleration a_2. All frictional forces are negligible.

B. Consider the general case where a block of mass m and volume V is completely submerged in a fluid of density ρ.

Starting with Newton's second law, **derive** an expression for the initial upward acceleration a of the block when the block is released from rest. Express your answer in terms of m, V, ρ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

取向上为正，从牛顿第二定律开始。物块完全浸没，浮力为 ρVg：

$$\sum F_y = ma, \qquad F_B - mg = ma, \qquad \rho V g - mg = ma,$$

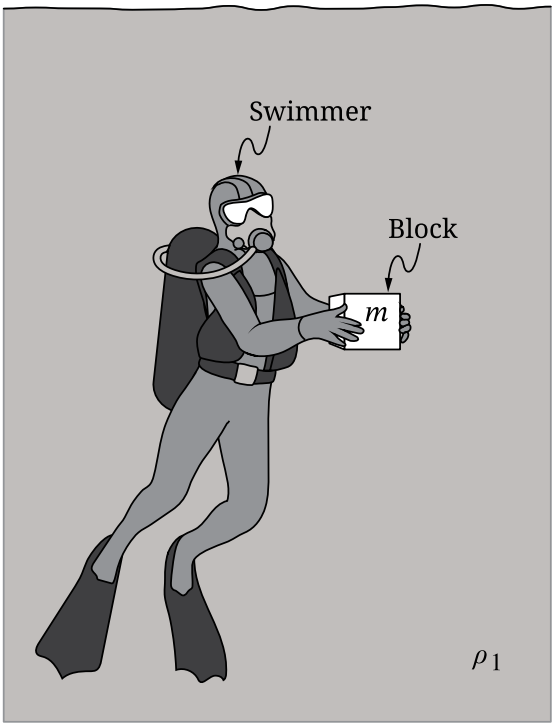
$$a = \frac{\rho V g}{m} - g.$$

练习 2025 Q4 (c)

Question 4

4. In Scenario 1, a swimmer holds a block of mass m at rest in a tank of freshwater with density ρ_1, as shown in Figure 1. The block is released from rest and accelerates upward with an initial acceleration a_1. All frictional forces are negligible.

Figure 1



In Scenario 2, the swimmer holds the same block at rest in a tank of salt water with density ρ_2, where $\rho_2 > \rho_1$. The swimmer again releases the block from rest, and the block accelerates upward with initial acceleration a_2. All frictional forces are negligible.

C. **Indicate** whether the expression for the acceleration a you derived in part B is or is not consistent with the claim made in part A. Briefly **justify** your answer by referencing your derivation in part B.

体积流量：单位时间水柱体积等于截面积乘速率

喷口截面越大，每秒可通过的水柱面积越大；速率越大，每秒通过的水柱长度越大。两者相乘就是每秒水体积 Q，单位为 m^3/s。

$$Q = Av, \qquad A = \pi r^2.$$

A 是垂直于水流的喷口面积，v 是喷口总速率，不能只用竖直分量。要用上一小问从喷水高度推导的速率替换 v。

例题 2026 Q1 (a)(iii)

本问承接 2026 Q1 (a)(ii)。

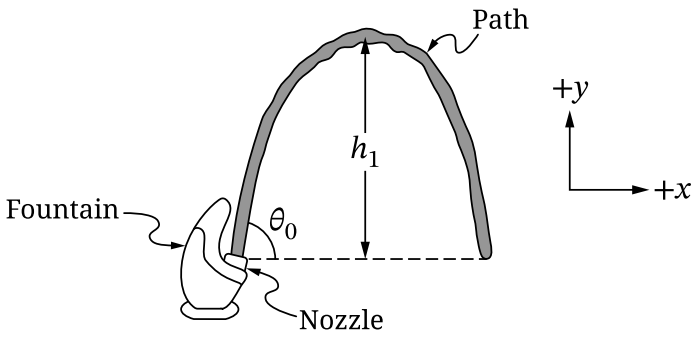
Question 1: Version J

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1. Water exits the nozzle of a fountain at an angle θ_0 above the horizontal.

At time $t = 0$, a droplet of water exits the nozzle and follows the path shown in Figure 1. The droplet reaches a maximum height h_1 above the nozzle.

Figure 1



Note: Figure not drawn to scale.

At $t = t_f$, the droplet returns to the same height at which the droplet exited the nozzle.

Part A

iii. The nozzle has a circular cross-section with radius r_0. **Derive** an expression for the volume flow rate of the water exiting the nozzle. Express your final answer in terms of θ_0, h_1, r_0, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

体积流量 Q 是每秒通过喷口的水体积。截面积为 πr_0^2，一秒内水柱沿喷射方向通过长度 v_0，所以体积等于面积乘长度。代入 A(ii) 的出口速率：

$$Q = Av_0 = \pi r_0^2 \frac{\sqrt{2gh_1}}{\sin\theta_0}.$$

流量不变而喷口变小：先比较速率，再比较最高点

流量 Q 保持相同，每秒必须通过同样体积的水；截面积减小时，通过长度增大，即喷口速率增大。喷射角固定，竖直分量随总速率同向改变。

最高点由竖直初速度决定，先写清面积→速率→竖直分量→最高高度的因果链，别把流量不变说成速率不变。

例题 2026 Q1 (b)

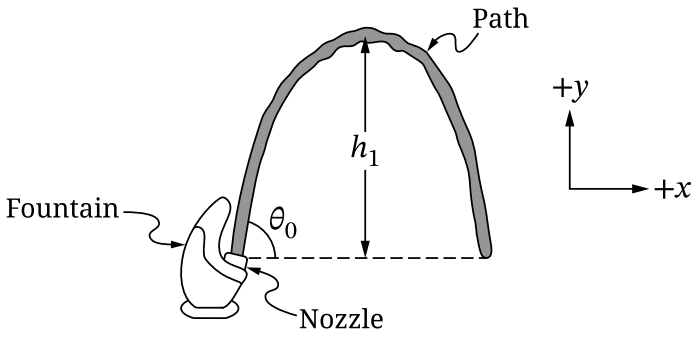
Question 1: Version J

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1. Water exits the nozzle of a fountain at an angle θ_0 above the horizontal.

At time $t = 0$, a droplet of water exits the nozzle and follows the path shown in Figure 1. The droplet reaches a maximum height h_1 above the nozzle.

Figure 1



Note: Figure not drawn to scale.

At $t = t_f$, the droplet returns to the same height at which the droplet exited the nozzle.

Part B

The fountain nozzle is replaced with a new nozzle with a radius smaller than r_0. The water exits the new nozzle at the same angle θ_0 above the horizontal. The volume flow rate of the water exiting the new nozzle is equal to the volume flow rate of the water exiting the original nozzle. A water droplet exiting the new nozzle reaches a maximum height h_2 above the nozzle.

Indicate whether h_2 is greater than, less than, or equal to h_1 by writing one of the following.

- $h_2 > h_1$
- $h_2 < h_1$
- $h_2 = h_1$

Justify your answer. In your justification, include qualitative reasoning beyond mathematical derivations or expressions.

解：

选择新喷水最大高度更大。流量不变，新喷口截面积较小，同一秒内要通过同样体积的水，水流必须更快。仰角不变，竖直初速度随总速率增大，水滴能克服重力上升到更高位置。

用已推导的关系核验：Q 不变意味着 v 与面积成反比；θ_0 不变意味着最大高度与喷口速率平方成正比，因此缩小喷嘴使最大高度增加。

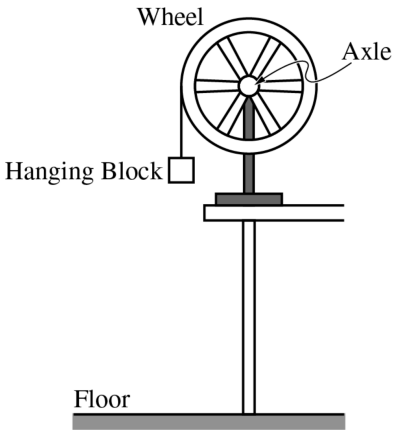
实验测量、作图与模型核验

用下落数据比较势能与动能：列可测量量和仪器

实验方案必须连起实际读数和目标能量。势能减少需要质量与竖直下降距离；末动能需要质量与末速率。若用时间和路程反求末速率，要说明从静止开始、近似恒加速度的依据。

写清释放、开始计时、停止计时、读距离的端点。每种量对应可用仪器；重复相同条件、平均读数，再换条件得到多组数据。

例题 2022 Q3 (a)



3. (12 points, suggested time 25 minutes)

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

(a) Design an experimental procedure that the students could use to compare the increase in the block's translational kinetic energy with the decrease in the gravitational potential energy of the block-Earth system as the block falls.

In the table, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity and list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

In the space to the right of the table, describe the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table.

If needed, you may include a simple diagram of the setup with your procedure.

Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement

解：

用天平测物块质量 m，用米尺测由释放点到地面的下降距离 d，用秒表测下降时间 t。绳先绷紧，从静止松手同时开始计时，到落地停表。每个 d 重复至少三次并平均时间，换几个不同 d 再重复；较长距离有助于减小反应时间在总时间中的占比。

在轮惯量、有效半径和质量保持不变的近似恒加速度模型下，由时间和距离得到末速度；也可用光电门直接测落地前速度，但要与所列仪器和程序匹配。用 m、d 计算重力势能减少 mgd，用 m 与末速度计算末平动动能。

只给秒表和秤的弹簧实验：用周期代替伸长

没有尺时，不能编造测静态伸长的步骤。秤能给质量，秒表能给振动时间；已经学过的周期—质量关系能够反求劲度。

测多轮完整振动的总时间，再除以轮数，能减小一次起停反应时间占比；保持小振幅和同一弹簧，换质量，重复并平均。

例题 2024 Q2 (a)

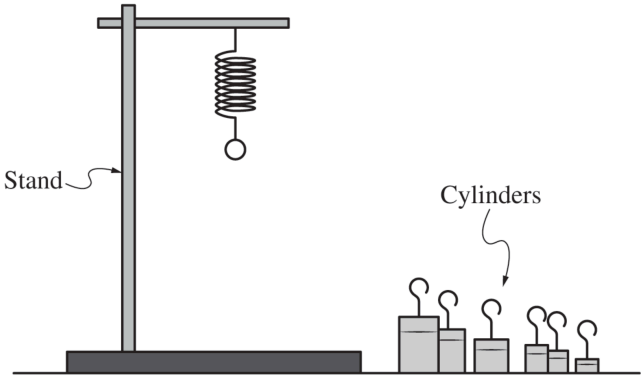


Figure 1

2. (12 points, suggested time 25 minutes)

A student hangs a spring of unknown spring constant k vertically by attaching one end to a stand, as shown in Figure 1. The other end of the spring has a small loop from which small cylinders can be hung. In addition to the spring, the student has access only to a variety of cylinders of unknown masses, a stopwatch, and a digital scale.

(a) Design an experimental procedure the student could use to determine the spring constant k of the spring.

In the following table, list the quantities that would be measured using only the provided equipment in your experiment. Define a symbol to represent each quantity.

In the space below the table, **describe** the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table. If needed, you may include a simple diagram of the setup with your procedure.

Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement
		Stopwatch
		Digital scale

解：

用数字秤称圆柱质量 m。把圆柱挂到弹簧上，待平衡后向下轻拉再由静止松手，使其振动。秒表记录十次完整往返的时间 t_{10}，总时间除以十便得到周期；同质量重复几次平均，再换不同圆柱或组合重复。保持小振幅、不碰支架、不明显横摆。提供的仪器没有尺，程序不依赖静态伸长测量。

未知物块不能直接挂秤：用力矩平衡量力和力臂

原题禁止把未知物块直接挂弹簧秤，就测它保持米尺水平时所需的另一处力。米尺中点作支点时，米尺自身重力不产生关于支点的力矩。

改变物块到支点的距离，记录另一端弹簧秤读数；另一端的力臂固定。为使数据可比较，每次都保持水平、秤先归零，并重复读数。

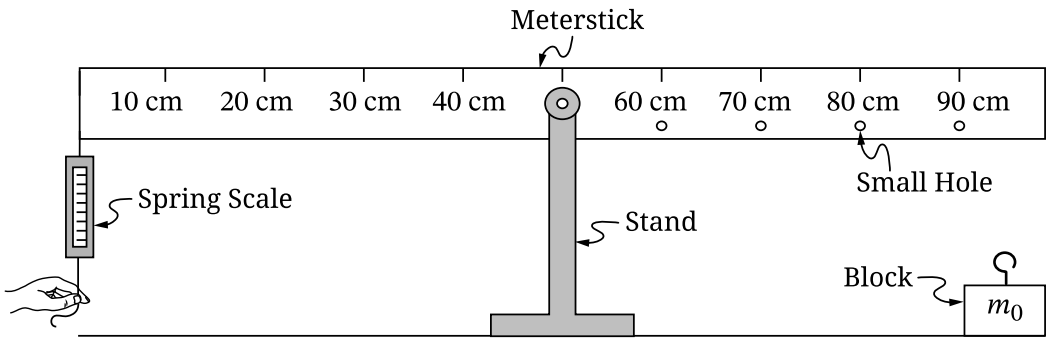
例题 2025 Q3 (a)

Question 3: Version J

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3. Students are investigating balancing systems using the following setup. The students have a spring scale of negligible mass that is fixed to one end of a uniform meterstick. The center of the meterstick is attached to a stand on which the meterstick can pivot. There is a hook of negligible mass fixed to the top of a block of mass m_0. The hook can be attached to the meterstick through one of the small holes in the meterstick, as shown in Figure 1. The students do not have a direct way to measure the mass of the block. The block cannot be attached to the spring scale.

Figure 1



The students are asked to take measurements that will allow the students to create a linear graph whose slope could be used to determine the mass m_0 of the block.

A. Describe an experimental procedure to collect data that would allow the students to determine m_0. Include any steps necessary to reduce experimental uncertainty.

解：

支点位于均匀米尺的中点，尺的重力对该支点不产生力矩。把未知物块挂在支点一侧的孔上，记录物块到支点的距离 d；另一端用弹簧秤保持米尺水平，记录读数 F。改变挂孔，得到多组 (d,F)；每个挂孔重复读数并平均，检查秤归零、米尺水平。秤到支点距离固定为 0.50 m。

只有米尺的摩擦实验：选择能量关系中的两种长度

未知质量不能称量时，先看目标关系是否能约掉质量。物块由高度释放、最终在水平粗糙面停住：开始的势能转为克服摩擦所需的能量。

可测量的是竖直释放高度和水平停止距离。两种距离的起止点不同，不能把曲面路径长替代竖直高度。

例题 2026 Q3 (a)(i)

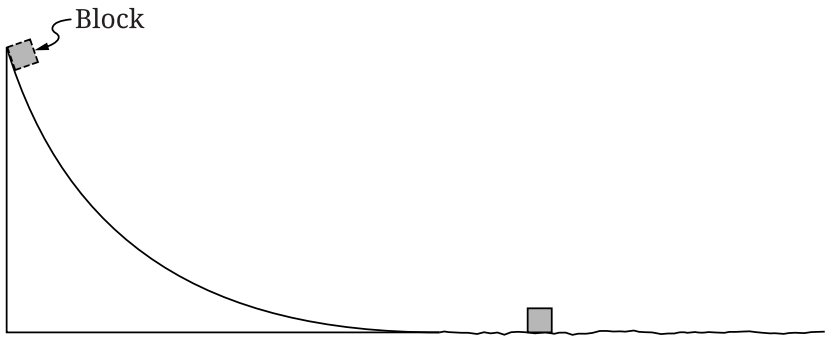
Question 3: Version J

The following information applies to parts A and B.

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3. A group of students are investigating friction using the following procedure. The students release a block of unknown mass near the top of a curved ramp. The block slides down the ramp. The block then transitions to a horizontal surface, as shown in Figure 1. Frictional forces are negligible between the block and the curved ramp, but there is friction between the block and the horizontal surface.

Figure 1



The students are asked to perform an experiment in which a single quantity is varied in order to collect data that could be graphed to determine the value μ_k of the coefficient of kinetic friction between the block and the horizontal surface. The students have access to only a meterstick.

Part A

i. **Indicate** quantities that could be measured by the students that would allow them to determine μ_k using a linear graph.

解：

只用米尺：测释放点相对水平面的竖直高度 h，以及物块在粗糙水平面上从坡口到停止位置的滑行距离 d。释放高度是竖直量，不是曲面上量出的路径长。

减小长度读数的不确定性：重复相同条件并平均

不确定性是同一条件下测量的散布，不是“数据越多越好”的口号。对同一释放高度多次从静止放开，记录每次停止距离，再用平均值代表该高度；测量基准和表面保持相同。

例题 2026 Q3 (a)(ii)

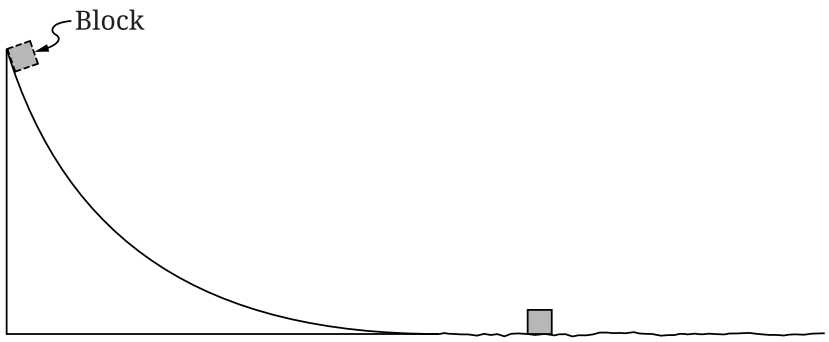
Question 3: Version J

The following information applies to parts A and B.

---

3. A group of students are investigating friction using the following procedure. The students release a block of unknown mass near the top of a curved ramp. The block slides down the ramp. The block then transitions to a horizontal surface, as shown in Figure 1. Frictional forces are negligible between the block and the curved ramp, but there is friction between the block and the horizontal surface.

Figure 1



The students are asked to perform an experiment in which a single quantity is varied in order to collect data that could be graphed to determine the value μ_k of the coefficient of kinetic friction between the block and the horizontal surface. The students have access to only a meterstick.

Part A

ii. Briefly **describe** a method to reduce experimental uncertainty for the measured quantities.

解：

对同一高度重复释放多次，记录并平均停止距离；再选多个高度重复实验。每次从静止松手，保持同一物块、同一水平表面。

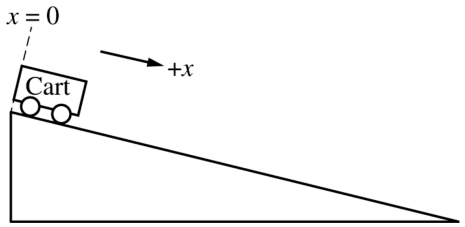
时间或速度的平方关系：把平方量作为一个坐标变量

要得到直线，先把物理式写成“一个量=固定系数×另一个量”。恒加速度由静止出发时，位移不是与时间成正比，而是与时间平方成正比。

$$x = \frac{1}{2}at^2.$$

把整个 t^2 当作横轴一个变量，x 作纵轴，每个原时间先平方。弹簧周期式平方后给 T^2 与 m 的一次关系；原式若已经给 v^2 与距离的一次关系，直接把 v^2 作纵变量。对每个选择都按实际公式说明。

例题 2023 Q2 (a)(i)



2. (12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration a of a cart. The cart is released from rest at the top of the ramp at time $t = 0$ and moves down the ramp. The x-axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position x (m)	Time t (s)	
	0.06	0.39	
	0.14	0.59	
	0.24	0.77	
	0.37	0.96	
	0.55	1.20	

i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration a of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

Vertical axis: _____ Horizontal axis: _____

解：

沿坡向下是 x 正方向，开始位置与速度均为零，恒加速度下位移与时间平方成正比，所以横轴选 $t^2(s^2)$、纵轴选 x(m)。

$$t^2 = 0.1521,\ 0.3481,\ 0.5929,\ 0.9216,\ 1.4400 \quad (\mathrm{s}^2).$$

先把已经学过的弹簧周期式两边平方，使质量只出现一次。选坐标量时写完整单位，T 的单位 s 与 T^2 的单位 s^2 不相同。

练习 2024 Q2 (b)(i)

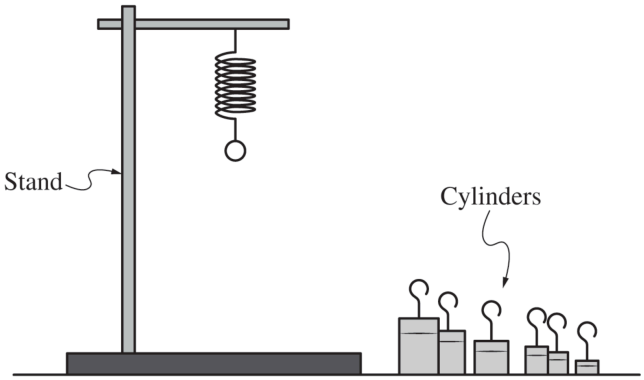


Figure 1

2. (12 points, suggested time 25 minutes)

A student hangs a spring of unknown spring constant k vertically by attaching one end to a stand, as shown in Figure 1. The other end of the spring has a small loop from which small cylinders can be hung. In addition to the spring, the student has access only to a variety of cylinders of unknown masses, a stopwatch, and a digital scale.

(b)

i. **Indicate** the quantities that could be plotted to produce a linear graph whose slope can be used to determine the spring constant k of the spring.

Vertical axis:_____ Horizontal axis:_____

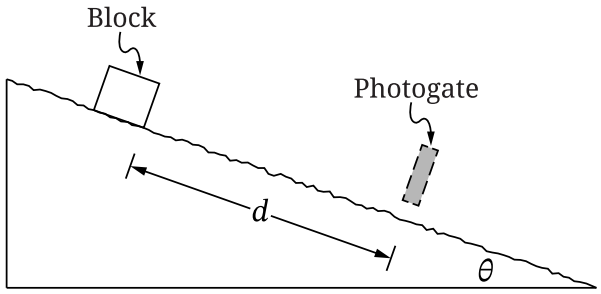
这次原卷已经给出 v^2 等于一段固定系数乘 d，并固定横轴为 d；用整个平方速度作为一个坐标量，不再对 d 平方。

练习 2026 Q3 (c)(i)

The following information applies to parts C and D.

In a different experiment, the students release a block from rest on a rough ramp. The block is released a distance d from a photogate. The ramp is inclined at an angle θ above the horizontal, as shown in Figure 2. The block slides down the ramp and passes through the photogate, which is positioned near the bottom of the ramp. The photogate determines the speed v of the block as it passes through the photogate. The unknown coefficient of kinetic friction between the block and the ramp is μ_k.

Figure 2



The experiment is repeated several times with different release distances d for the same block. Table 1 shows the measured values of d and v.

Table 1

d (m)	v (m/s)
0.20	0.29
0.30	0.38
0.40	0.41
0.50	0.49
0.60	0.52

The students correctly determine that the relationship between d and v is given by

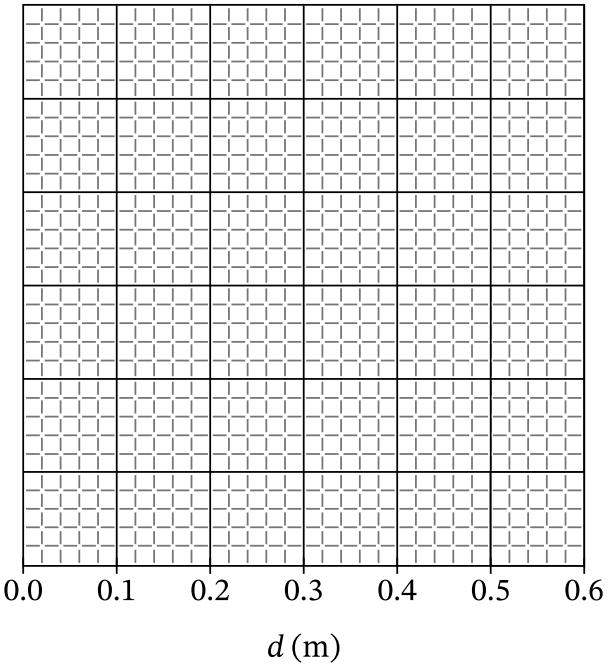
$$v^2 = [2g(\sin\theta - \mu_k \cos\theta)]d,$$

where $\theta = 30^\circ$. The students want to determine μ_k. The students create a graph with d plotted on the horizontal axis.

Figure 3

Quantity (units, if appropriate)

_____ _____



i. **Label** the vertical axis of Figure 3 with a measured or calculated quantity. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine μ_k.

长度之间的一次关系：选择直接两轴

从能量关系写出释放高度 h 与水平停止距离 d 的联系。两边包含相同质量和 g，约掉后只剩一个固定摩擦系数与两种长度；因此可以直接用两种长度作坐标，不需平方。

例题 2026 Q3 (b)(i)

本问承接 2026 Q3 (a)(i)。

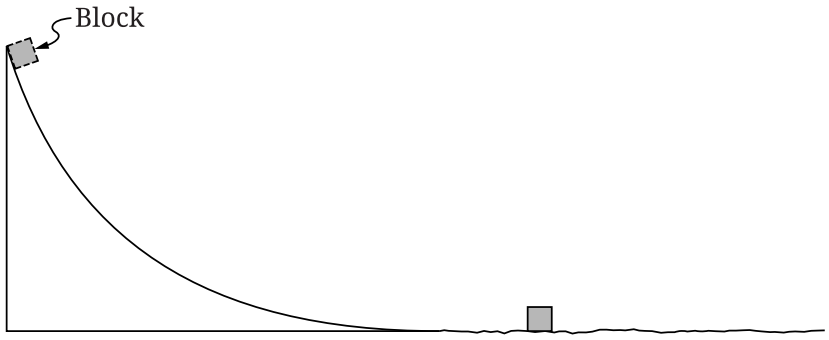
Question 3: Version J

The following information applies to parts A and B.

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3. A group of students are investigating friction using the following procedure. The students release a block of unknown mass near the top of a curved ramp. The block slides down the ramp. The block then transitions to a horizontal surface, as shown in Figure 1. Frictional forces are negligible between the block and the curved ramp, but there is friction between the block and the horizontal surface.

Figure 1



The students are asked to perform an experiment in which a single quantity is varied in order to collect data that could be graphed to determine the value μ_k of the coefficient of kinetic friction between the block and the horizontal surface. The students have access to only a meterstick.

Part B

i. **Indicate** what quantities the students could graph on the horizontal and vertical axes to create a linear graph that can be used to determine μ_k. Clearly state which quantity will be graphed on each axis.

解：

从无摩擦曲面释放到水平面停止，开始、结束均无动能，释放势能用于克服水平摩擦：

$$mgh = \mu_k mgd \;\Rightarrow\; h = \mu_k d.$$

横轴标 d(m)，纵轴标 h(m)。释放高度与水平停止距离之间的线性关系给出适用的两轴。

横轴已经指定倒数三角量：看谁与它成一次关系

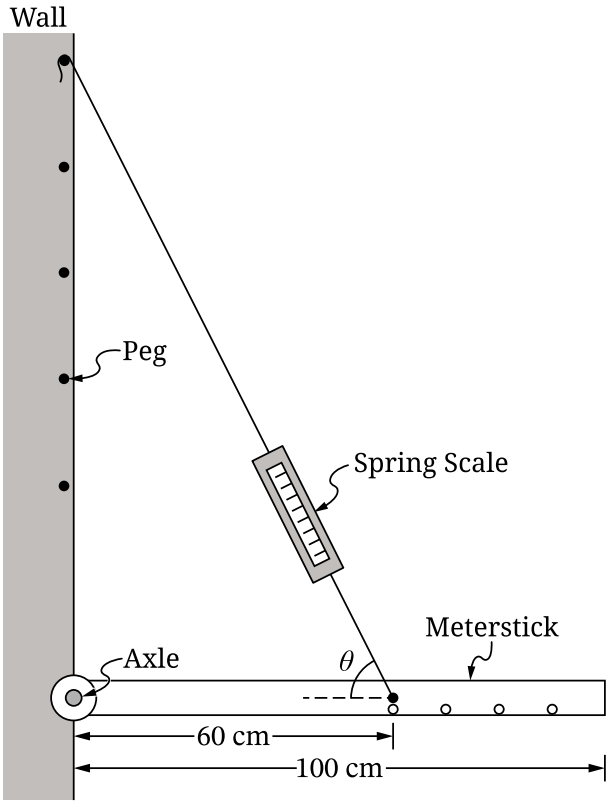
题给横轴为 1/sinθ 时，把它看作一个完整横变量。再看原物理式，张力等于固定系数乘这个横变量；所以张力适合作纵轴，单位仍为 N。

用角度计算横坐标时，计算器须按原表的角度单位；1/sinθ 没有单位。

例题 2025 Q3 (c)(i)

> The students have an identical meterstick of mass M that is now attached to an axle that is fixed to a wall. The meterstick is free to rotate with negligible friction about the axle. The meterstick is suspended horizontally by a string that is connected to a spring scale of negligible mass, as shown in Figure 2.

Figure 2



The angle θ that the string makes with the meterstick can be varied by attaching the string to one of the pegs located along the wall. The students use the spring scale to measure the tension F_T required to hold the meterstick horizontal. Table 1 shows the measured values of θ and F_T.

Table 1

θ (degrees)	F_T (N)
22	21
31	17
36	13
45	12
80	8

The students correctly determine that the relationship between F_T and θ is given by

$$F_T = \frac{5Mg}{6\sin\theta}.$$

The students create a graph with $\frac{1}{\sin\theta}$ plotted on the horizontal axis.

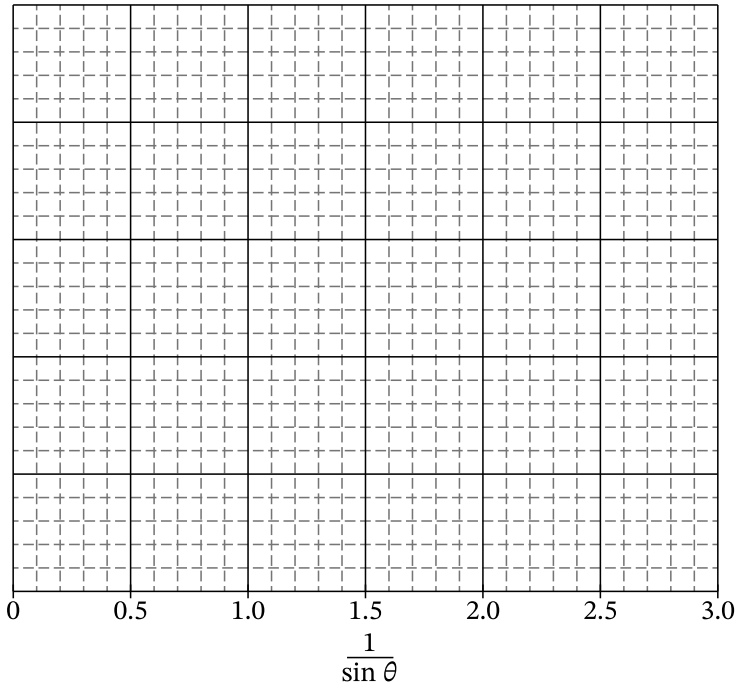


Figure 3

i. **Indicate** what measured or calculated quantity could be plotted on the vertical axis to yield a linear graph whose slope can be used to calculate an experimental value for the mass M of the meterstick.

Vertical axis: _____ Horizontal axis: $\frac{1}{\sin\theta}$

解：

题干指定横轴为 $1/\sin\theta$，而原式

$$F_T = (5Mg/6)(1/\sin\theta)$$

只有这个横变量的一次幂。选择纵轴为实测张力 F_T，单位 N；M 是本次米尺的质量，固定不变。

由转动平衡选择两轴并解释斜率

把测出的力和变化的力臂代入力矩平衡：弹簧秤的力×固定力臂=物块重量×变化力臂。整理成“纵变量=固定系数×横变量”，这个系数就是斜率。

用自己选的坐标方向解释斜率。若交换两轴，斜率就变为倒数，未知质量的求法也必须改。

例题 2025 Q3 (b)

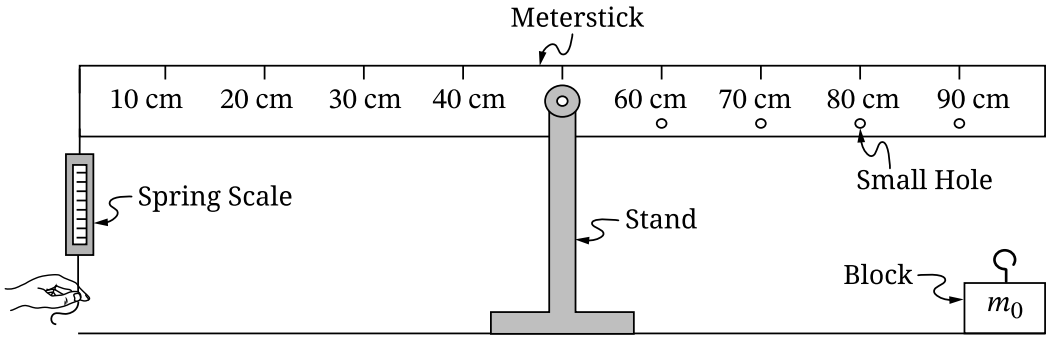
本问承接 2025 Q3 (a)。

Question 3: Version J

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3. Students are investigating balancing systems using the following setup. The students have a spring scale of negligible mass that is fixed to one end of a uniform meterstick. The center of the meterstick is attached to a stand on which the meterstick can pivot. There is a hook of negligible mass fixed to the top of a block of mass m_0. The hook can be attached to the meterstick through one of the small holes in the meterstick, as shown in Figure 1. The students do not have a direct way to measure the mass of the block. The block cannot be attached to the spring scale.

Figure 1



The students are asked to take measurements that will allow the students to create a linear graph whose slope could be used to determine the mass m_0 of the block.

B. **Describe** how the data collected in part A could be graphed and how that graph would be analyzed to determine m_0.

解：

关于支点，两个力矩平衡：

$$F(0.50\,\mathrm{m}) = m_0 g d \;\Rightarrow\; F = \frac{m_0 g}{0.50\,\mathrm{m}} d.$$

横轴 d(m)、纵轴 F(N) 的最佳拟合线斜率记 s；由它反求质量的关系见下式。

$$m_0 = \frac{(0.50\,\mathrm{m})s}{g}.$$

作散点：逐行变换，同一行的两读数必须配对

先确定两轴，再对每一行需要的量做变换；不要把排序后的另一列与原顺序错配。坐标刻度等间距，标物理量、单位和数字，使五点覆盖网格主要区域。

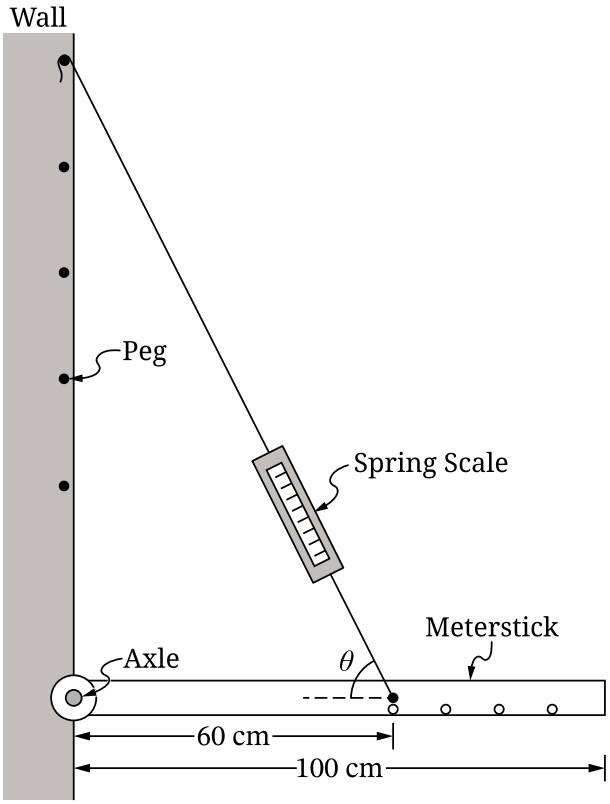
这一小问只要求作点时，先完成点图；拟合线属于下一动作。

例题 2025 Q3 (c)(ii)

本问承接 2025 Q3 (c)(i)。

> The students have an identical meterstick of mass M that is now attached to an axle that is fixed to a wall. The meterstick is free to rotate with negligible friction about the axle. The meterstick is suspended horizontally by a string that is connected to a spring scale of negligible mass, as shown in Figure 2.

Figure 2



The angle θ that the string makes with the meterstick can be varied by attaching the string to one of the pegs located along the wall. The students use the spring scale to measure the tension F_T required to hold the meterstick horizontal. Table 1 shows the measured values of θ and F_T.

Table 1

θ (degrees)	F_T (N)
22	21
31	17
36	13
45	12
80	8

The students correctly determine that the relationship between F_T and θ is given by

$$F_T = \frac{5Mg}{6\sin\theta}.$$

The students create a graph with $\frac{1}{\sin\theta}$ plotted on the horizontal axis.

ii. On the blank grid provided, create a graph of the quantities indicated in part C (i) that can be used to determine M.

- Use Table 2 to record the data points or calculated quantities that you will plot.
- Clearly **label** the vertical axis, including units as appropriate.
- **Plot** the points you recorded in Table 2.

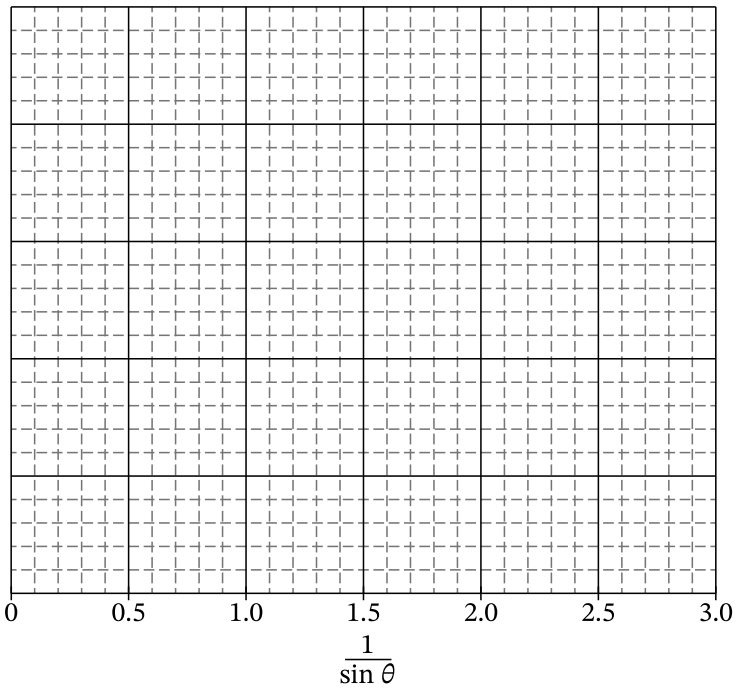


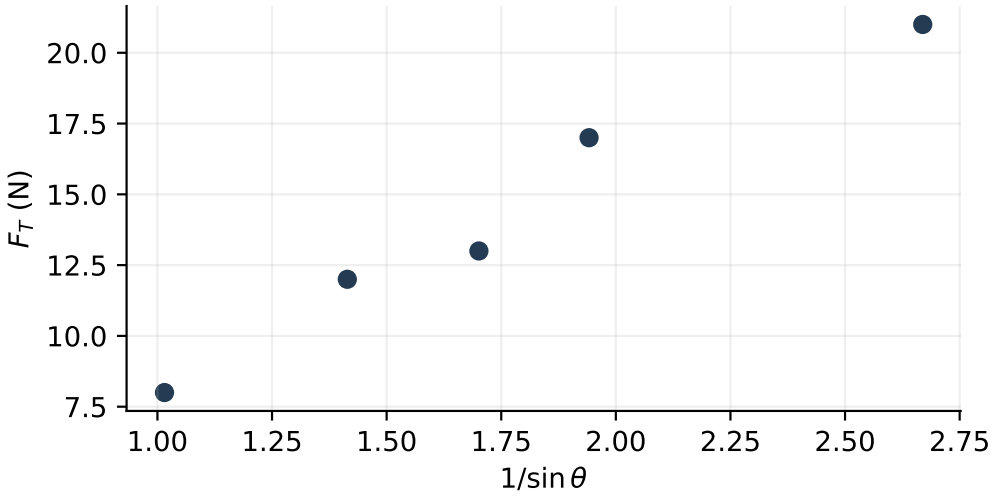
Figure 3

解：

先按每个角度计算 $1/\sin\theta$，再把它与同一行的原测量张力配成一对：

$$(1/\sin\theta, F_T) \approx (2.669, 21), (1.942, 17), (1.701, 13), (1.414, 12), (1.015, 8).$$

横轴为无单位的 $1/\sin\theta$，纵轴为 F_T(N)；选均匀刻度，使五点占据大部分网格。



由五组角度和张力作出的点，本问不加拟合线。

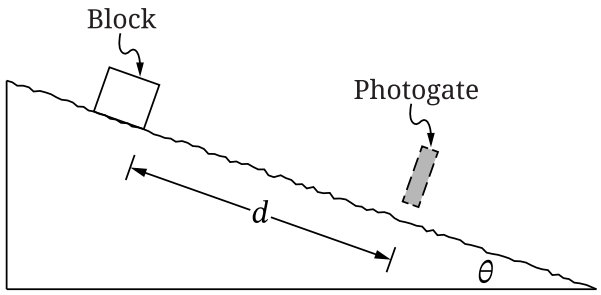
练习 2026 Q3 (c)(ii)

本问承接 2026 Q3 (c)(i)。

The following information applies to parts C and D.

In a different experiment, the students release a block from rest on a rough ramp. The block is released a distance d from a photogate. The ramp is inclined at an angle θ above the horizontal, as shown in Figure 2. The block slides down the ramp and passes through the photogate, which is positioned near the bottom of the ramp. The photogate determines the speed v of the block as it passes through the photogate. The unknown coefficient of kinetic friction between the block and the ramp is μ_k.

Figure 2



The experiment is repeated several times with different release distances d for the same block. Table 1 shows the measured values of d and v.

Table 1

d (m)	v (m/s)
0.20	0.29
0.30	0.38
0.40	0.41
0.50	0.49
0.60	0.52

The students correctly determine that the relationship between d and v is given by

$$v^2 = [2g(\sin\theta - \mu_k \cos\theta)]d,$$

where $\theta = 30^\circ$. The students want to determine μ_k. The students create a graph with d plotted on the horizontal axis.

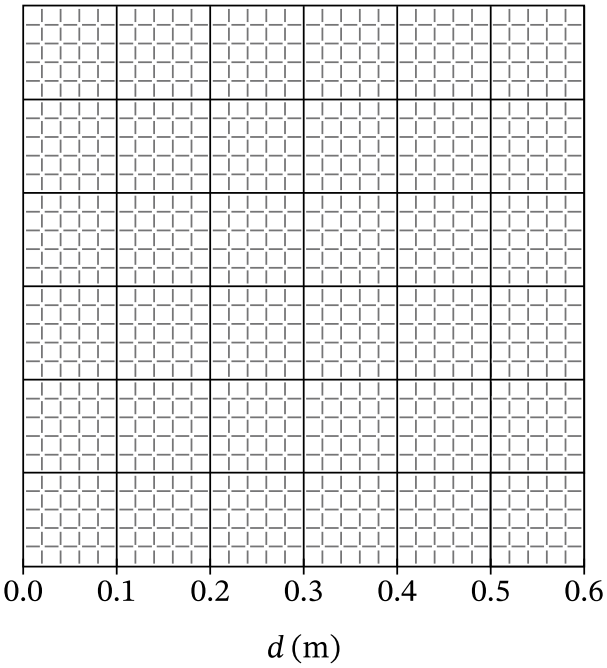
ii. On the grid provided in Figure 3, create a graph that can be used to determine μ_k.

- Clearly **label** the vertical axis with a numerical scale.
- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work and will not be scored.

Figure 3

Quantity (units, if appropriate)

_____ _____



作最佳拟合线：通过点群中央，不逐点连接

测量点存在散布，不能用折线把每个点连起来。画一条表示整体趋势的直线，让直线上下的偏离大致平衡，尽量覆盖数据范围。

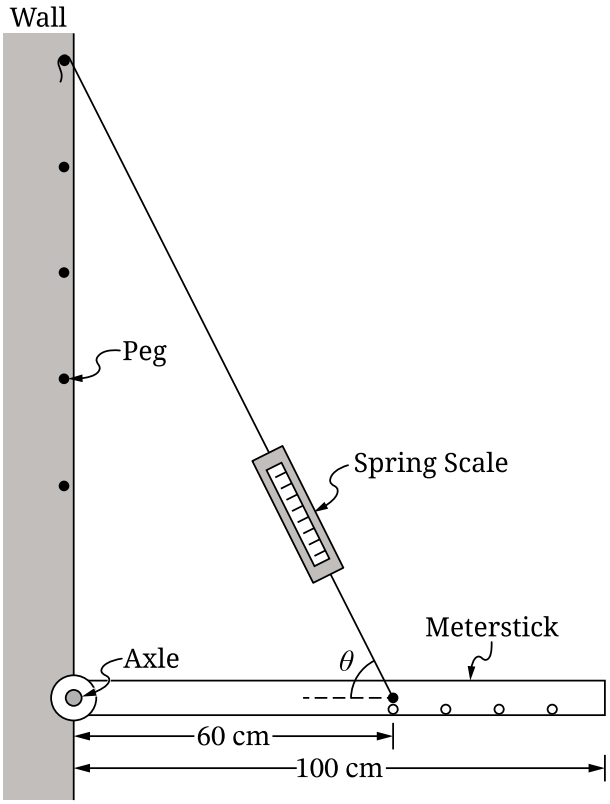
原物理模型截距可能为零，但不能在题目没有要求时强迫测量拟合线穿过原点。拟合线也不必穿过任何指定测量点。

例题 2025 Q3 (c)(iii)

本问承接 2025 Q3 (c)(ii)。

> The students have an identical meterstick of mass *M* that is now attached to an axle that is fixed to a wall. The meterstick is free to rotate with negligible friction about the axle. The meterstick is suspended horizontally by a string that is connected to a spring scale of negligible mass, as shown in Figure 2.

Figure 2



The angle θ that the string makes with the meterstick can be varied by attaching the string to one of the pegs located along the wall. The students use the spring scale to measure the tension F_T required to hold the meterstick horizontal. Table 1 shows the measured values of θ and F_T.

Table 1

θ (degrees)	F_T (N)
22	21
31	17
36	13
45	12
80	8

The students correctly determine that the relationship between F_T and θ is given by

$$F_T = \frac{5Mg}{6\sin\theta}.$$

The students create a graph with $\frac{1}{\sin\theta}$ plotted on the horizontal axis.

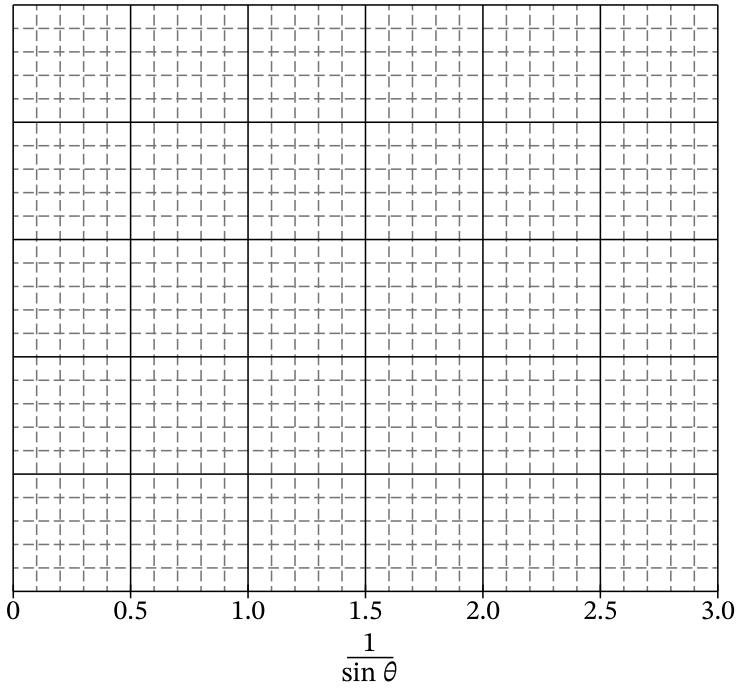


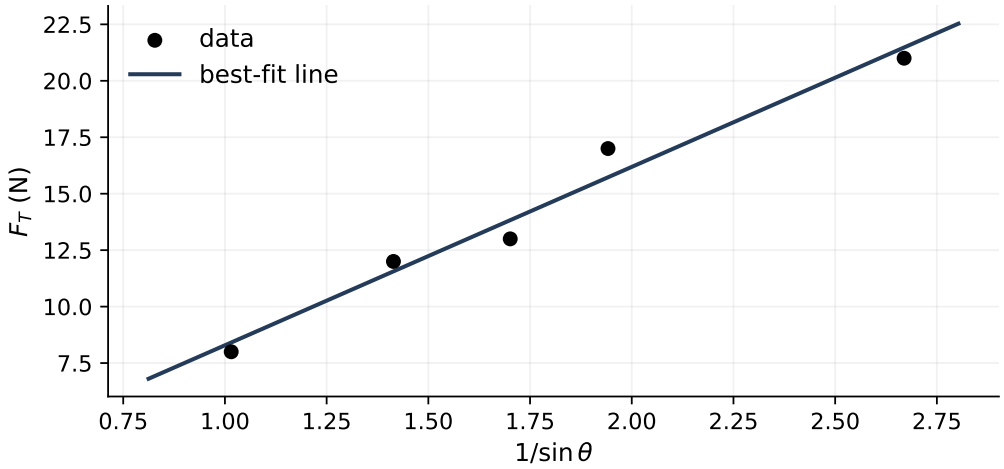
Figure 3

iii. **Draw** a straight best-fit line for the data graphed in part C (ii).

解：

在 C(ii) 的点群中画一条直线，不逐点折线连接；让数据偏离直线的正负趋势大致平衡。一条与原表相符的线为：

$$F_T \approx (7.903\,\mathrm{N})(1/\sin\theta) + 0.506\,\mathrm{N}.$$



拟合以原表五组角度和张力为依据。

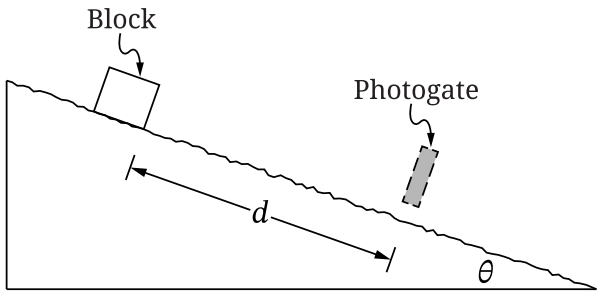
练习 2026 Q3 (c)(iii)

本问承接 2026 Q3 (c)(ii)。

The following information applies to parts C and D.

In a different experiment, the students release a block from rest on a rough ramp. The block is released a distance d from a photogate. The ramp is inclined at an angle θ above the horizontal, as shown in Figure 2. The block slides down the ramp and passes through the photogate, which is positioned near the bottom of the ramp. The photogate determines the speed v of the block as it passes through the photogate. The unknown coefficient of kinetic friction between the block and the ramp is μ_k.

Figure 2



The experiment is repeated several times with different release distances d for the same block. Table 1 shows the measured values of d and v.

Table 1

d (m)	v (m/s)
0.20	0.29
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The students correctly determine that the relationship between d and v is given by

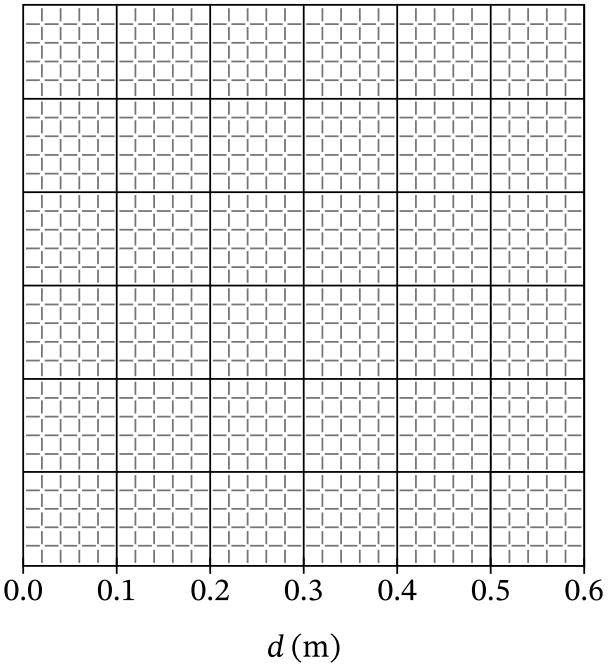
$$v^2 = [2g(\sin\theta - \mu_k \cos\theta)]d,$$

where $\theta = 30^\circ$. The students want to determine μ_k. The students create a graph with d plotted on the horizontal axis.

Figure 3

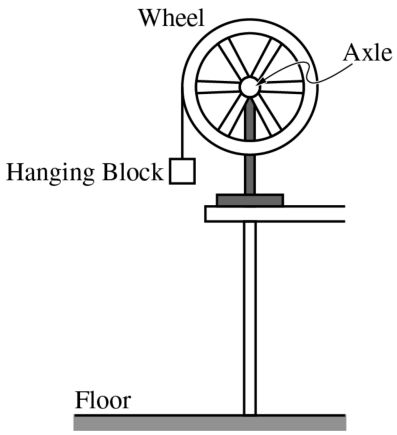
Quantity (units, if appropriate)

_____ _____



iii. **Draw** a best-fit line for the data plotted in part C (ii).

练习 2022 Q3 (d)(i)

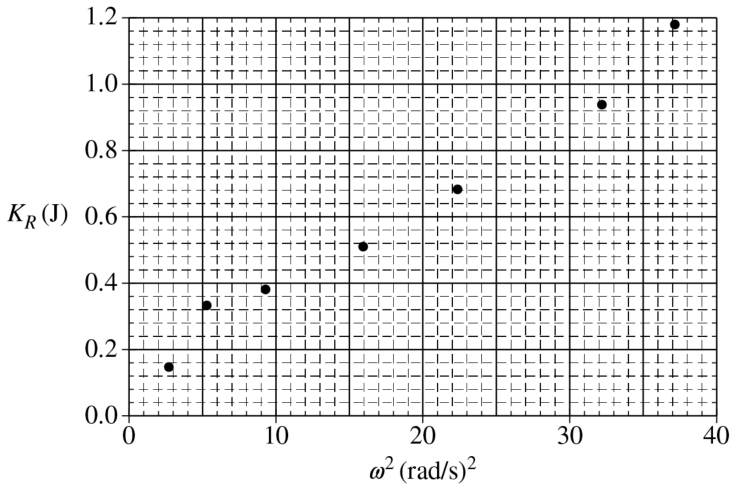


3. (12 points, suggested time 25 minutes)

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

(d) The students also measure the angular velocity ω of the wheel as the block falls and determine the rotational kinetic energy K_R of the wheel. The students then make a graph of K_R as a function of ω^2, as shown.



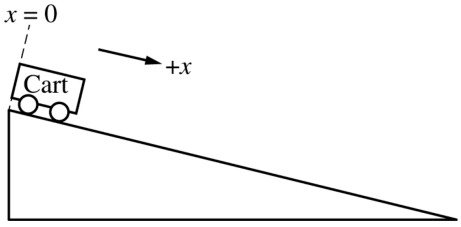
i. On the above graph, draw a straight line that best represents the data.

同一小问要求作点和拟合：把两步完整执行

先依选定变量变换数据、标轴、作点，再画点群趋势的直线。两步不能互相代替。此前分别练过作点与拟合，本问按原题把它们连起来完成。

例题 2023 Q2 (a)(ii)

本问承接 2023 Q2 (a)(i)。

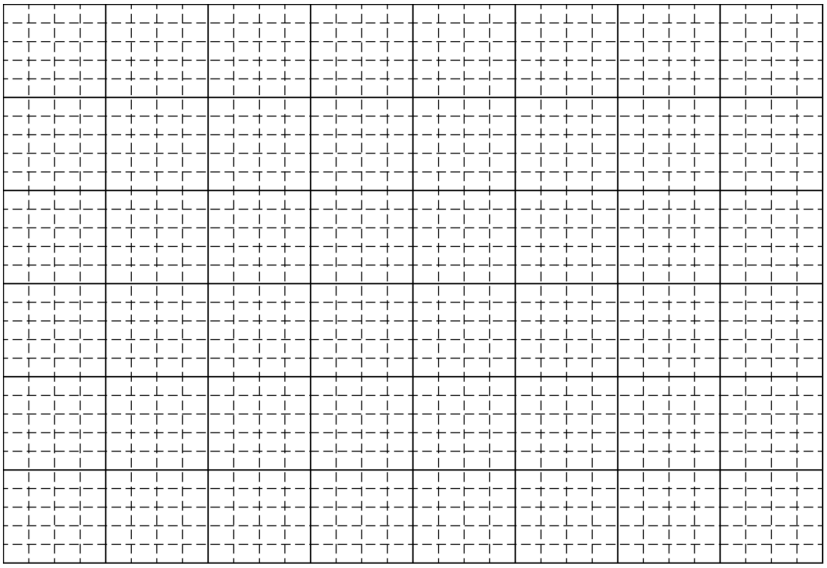


2. (12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration a of a cart. The cart is released from rest at the top of the ramp at time $t = 0$ and moves down the ramp. The x-axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position x (m)	Time t (s)	
	0.06	0.39	
	0.14	0.59	
	0.24	0.77	
	0.37	0.96	
	0.55	1.20	

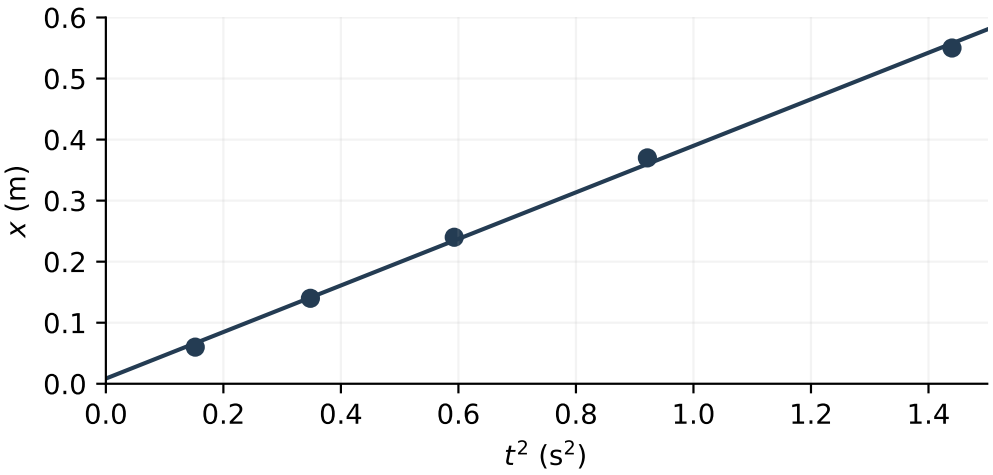
ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration a of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



解：

将上述平方时间与原表五个位置对应作点，两轴用均匀刻度、标量与单位，数据覆盖大半图面。合理拟合直线可为：

$$x \approx (0.3813\,\mathrm{m/s^2})t^2 + 0.0085\,\mathrm{m}.$$



原表的平方时间和位移作点，并画一条合理拟合线。

由拟合线斜率反求比例参数：先对应物理系数

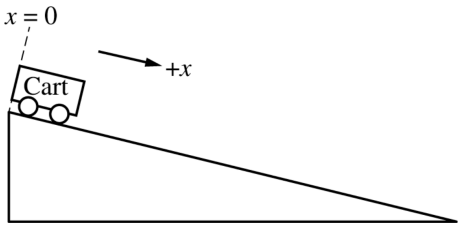
从拟合线上选两个相距较远的点，用纵差除以横差计算斜率 s。不能随便选两个散点当作拟合线。然后把 s 对应到物理式中横变量前的系数。

$$s = \frac{y_2 - y_1}{x_2 - x_1}.$$

若物理系数是 a/2，a 就等于 2s；若系数是 I/2，I 也等于 2s。这些 2 来自相应物理关系，不是所有斜率一律乘二。质量的对应系数还含 g 和力臂，应先写对应等式再移项。

例题 2023 Q2 (a)(iii)

本问承接 2023 Q2 (a)(ii)。



2. (12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration a of a cart. The cart is released from rest at the top of the ramp at time $t = 0$ and moves down the ramp. The x-axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position x (m)	Time t (s)	
	0.06	0.39	
	0.14	0.59	
	0.24	0.77	
	0.37	0.96	
	0.55	1.20	

iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration a of the cart as it rolls down the ramp.

解：

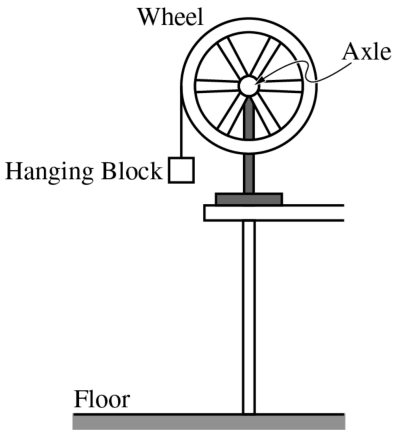
这条线斜率约为 0.3813 m/s^2，对应加速度的一半，因此加速度为它的两倍。原官方手绘示例约为 0.75 m/s^2；合理读线差异可接受，须与自己所画线一致。

$$a = 2s \approx 0.763\,\mathrm{m/s^2}.$$

这里纵量是轮的转动动能，横量是角速度的平方。先从 ½$I\omega^2$ 找横变量前的系数，再用自己上一问的拟合线斜率求 I。

练习 2022 Q3 (d)(ii)

本问承接 2022 Q3 (d)(i)。

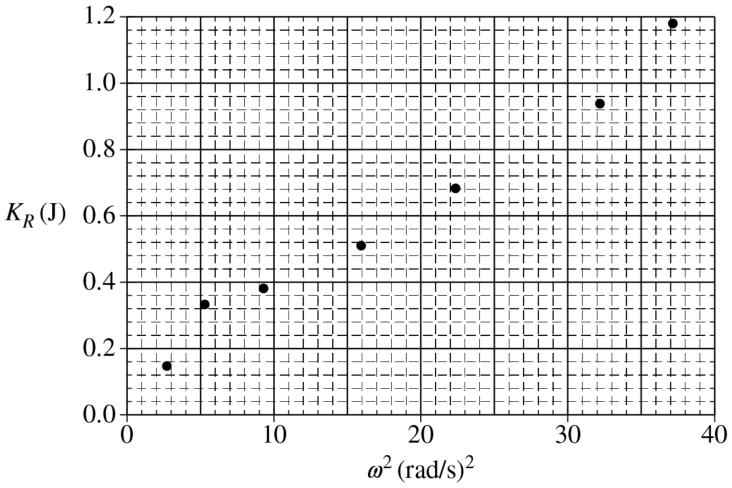


3. (12 points, suggested time 25 minutes)

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

(d) The students also measure the angular velocity ω of the wheel as the block falls and determine the rotational kinetic energy K_R of the wheel. The students then make a graph of K_R as a function of ω^2, as shown.



ii. Using the line you drew for part (d)(i), calculate an experimental value for the rotational inertia of the wheel.

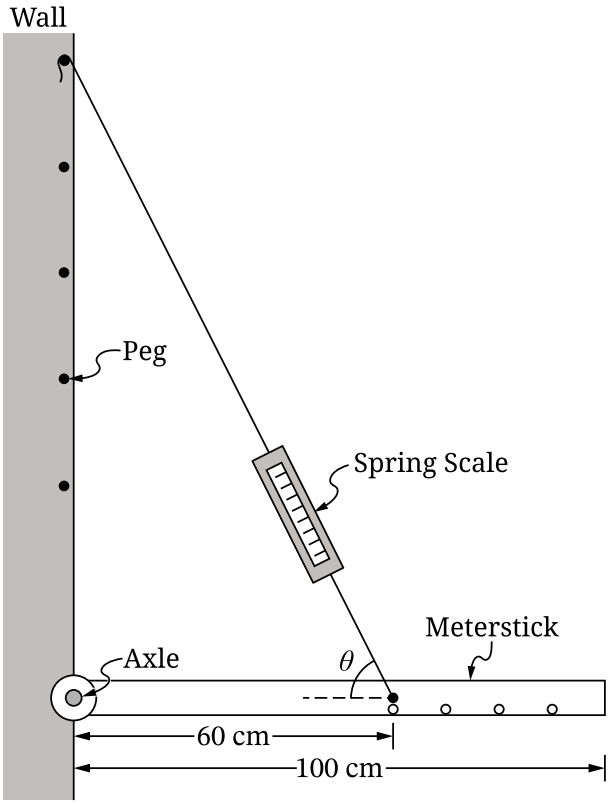
本题原式已经给 F_T 与 1/sinθ 的关系；横变量前的系数含 5、6、M 和 g。先把斜率等同这个系数，再解 M。

练习 2025 Q3 (d)

本问承接 2025 Q3 (c)(iii)。

> The students have an identical meterstick of mass *M* that is now attached to an axle that is fixed to a wall. The meterstick is free to rotate with negligible friction about the axle. The meterstick is suspended horizontally by a string that is connected to a spring scale of negligible mass, as shown in Figure 2.

Figure 2



The angle θ that the string makes with the meterstick can be varied by attaching the string to one of the pegs located along the wall. The students use the spring scale to measure the tension F_T required to hold the meterstick horizontal. Table 1 shows the measured values of θ and F_T.

Table 1

θ (degrees)	F_T (N)
22	21
31	17
36	13
45	12
80	8

The students correctly determine that the relationship between F_T and θ is given by

$$F_T = \frac{5Mg}{6\sin\theta}.$$

The students create a graph with $\frac{1}{\sin\theta}$ plotted on the horizontal axis.

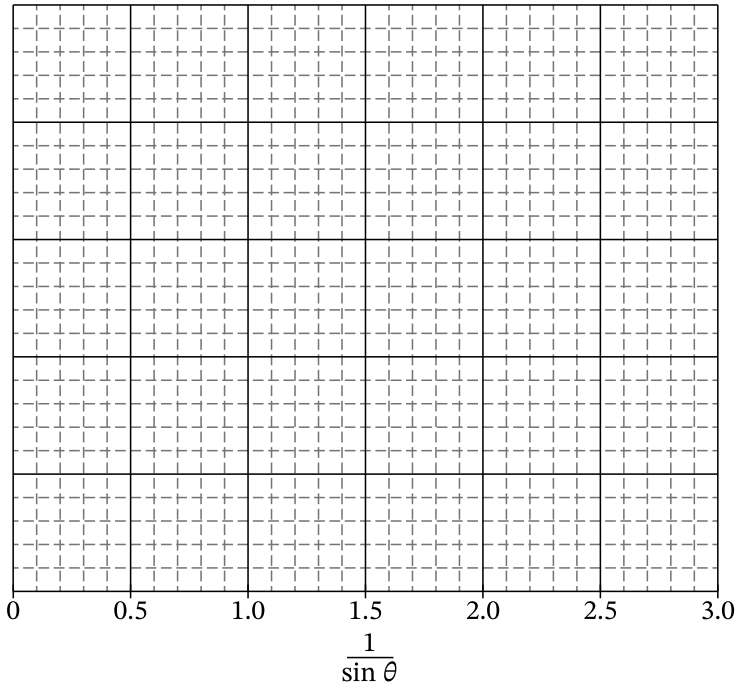


Figure 3

D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for the mass M of the meterstick.

沿用 B(i) 实际选的两轴，从 h 与 d 的物理式辨认斜率，不把“直线”本身当作摩擦系数。

练习 2026 Q3 (b)(ii)

本问承接 2026 Q3 (b)(i)。

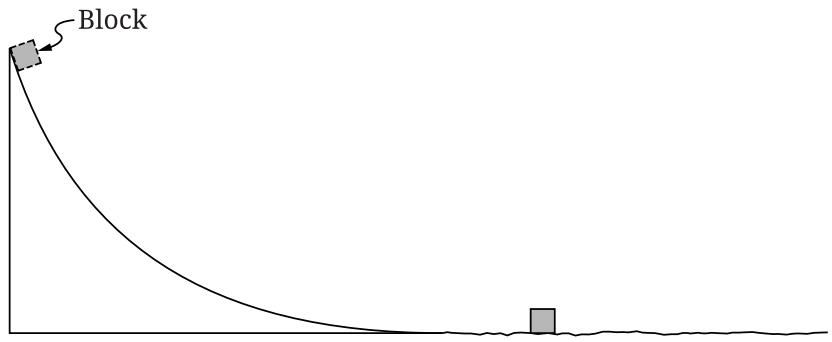
Question 3: Version J

The following information applies to parts A and B.

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3. A group of students are investigating friction using the following procedure. The students release a block of unknown mass near the top of a curved ramp. The block slides down the ramp. The block then transitions to a horizontal surface, as shown in Figure 1. Frictional forces are negligible between the block and the curved ramp, but there is friction between the block and the horizontal surface.

Figure 1



The students are asked to perform an experiment in which a single quantity is varied in order to collect data that could be graphed to determine the value μ_k of the coefficient of kinetic friction between the block and the horizontal surface. The students have access to only a meterstick.

Part B

ii. Briefly **describe** the relationship between μ_k and a feature of the graph from part B (i). Your answer may include an equation that relates μ_k and the chosen feature of the graph.

未知量在斜率系数的分母：由斜率倒推劲度

弹簧周期式平方后，T^2 对质量的斜率是 $4\pi^2/k$。斜率不是 k，也不是只差一个固定乘数；先写斜率与劲度的对应等式，再两边乘 k、两边除以 s，得到劲度。

$$T^2 = \frac{4\pi^2}{k}m, \qquad k = \frac{4\pi^2}{s}.$$

例题 2024 Q2 (b)(ii)

本问承接 2024 Q2 (b)(i)。

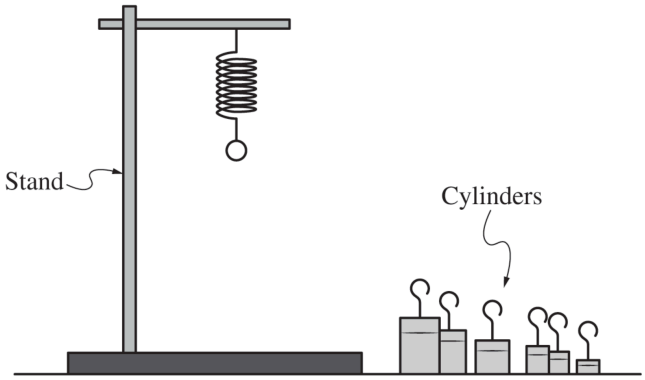


Figure 1

2. (12 points, suggested time 25 minutes)

A student hangs a spring of unknown spring constant k vertically by attaching one end to a stand, as shown in Figure 1. The other end of the spring has a small loop from which small cylinders can be hung. In addition to the spring, the student has access only to a variety of cylinders of unknown masses, a stopwatch, and a digital scale.

(b)

ii. Briefly **describe** how the slope of the graph would be analyzed to determine the spring constant k of the spring.

解：

斜率记为 s，先与上式中质量前的系数对应，再把未知劲度移到等号一边：若改画 m 对 T^2，斜率对应 k/($4\pi^2$)，应乘 $4\pi^2$，不能沿用除法。

$$s = \frac{4\pi^2}{k}, \qquad k = \frac{4\pi^2}{s}.$$

斜率含两个作用的差：解出摩擦系数再代数值

粗糙斜坡的速度平方—距离式中，系数同时含向下重力分量和反向摩擦。先让斜率等于整个 d 前的系数，再依次除以 2g、移项、除以 cosθ，把摩擦系数单独留下。

$$s = 2g(\sin\theta - \mu_k \cos\theta), \qquad \mu_k = \frac{\sin\theta - s/(2g)}{\cos\theta}.$$

θ 是固定坡角，μ_k 是无单位的动摩擦系数。斜率的单位来自 v^2 除以 d，为 m/s^2；不能把它直接当 μ_k。

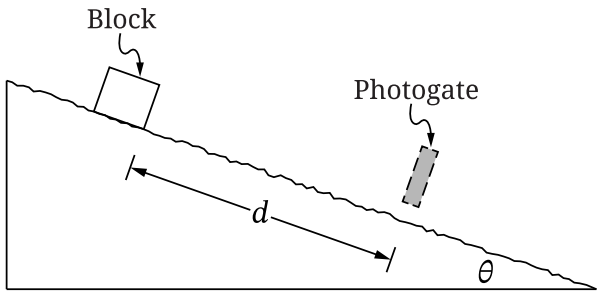
例题 2026 Q3 (d)

本问承接 2026 Q3 (c)(iii)。

The following information applies to parts C and D.

In a different experiment, the students release a block from rest on a rough ramp. The block is released a distance d from a photogate. The ramp is inclined at an angle θ above the horizontal, as shown in Figure 2. The block slides down the ramp and passes through the photogate, which is positioned near the bottom of the ramp. The photogate determines the speed v of the block as it passes through the photogate. The unknown coefficient of kinetic friction between the block and the ramp is μ_k.

Figure 2



The experiment is repeated several times with different release distances d for the same block. Table 1 shows the measured values of d and v.

Table 1

d (m)	v (m/s)
0.20	0.29
0.30	0.38
0.40	0.41
0.50	0.49
0.60	0.52

The students correctly determine that the relationship between d and v is given by

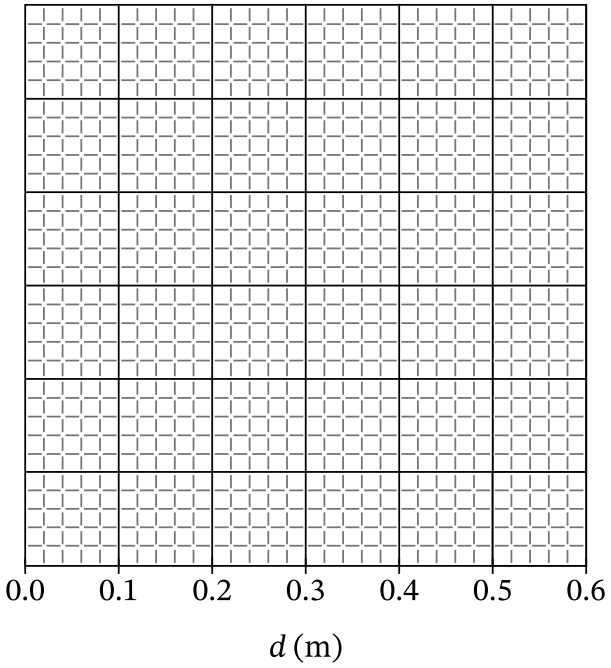
$$v^2 = [2g(\sin\theta - \mu_k \cos\theta)]d,$$

where $\theta = 30^\circ$. The students want to determine μ_k. The students create a graph with d plotted on the horizontal axis.

Figure 3

Quantity (units, if appropriate)

_____ _____



Part D

Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for μ_k.

解：

斜率 s 对应 d 的系数，先由这一对应式解出 μ_k，再代入坡角 30°、重力加速度 9.8 m/s^2：

$$s = 2g(\sin\theta - \mu_k\cos\theta), \qquad \mu_k = \frac{\sin\theta - s/(2g)}{\cos\theta},$$

$$\mu_k = \frac{0.5 - 0.4683/(2\times 9.8)}{\cos 30^\circ} \approx 0.550.$$

μ_k 无单位。合理手绘线的数值可略有差异，但必须使用自己拟合线的斜率，而不是任意挑两个测量点。

用下降距离和时间求末动能：末速率不是平均速率

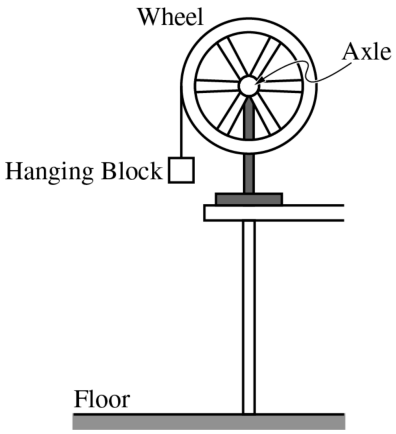
由静止、近似恒加速度下降时，平均速率是末速率的一半。因此先用距离除时间求平均速率，再乘二得到落地前速率，最后代入物块动能。

$$\frac{d}{t} = \frac{0 + v_f}{2}, \qquad v_f = \frac{2d}{t}.$$

d 与 t 是同一次释放的距离和时间；若直接把 d/t 放进动能，会算成平均速率对应的动能。

例题 2022 Q3 (b)

本问承接 2022 Q3 (a)。



3. (12 points, suggested time 25 minutes)

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

(b) Explain how the students could determine the kinetic energy of the block immediately before it reaches the floor using the quantities you indicated in the table in part (a).

解：

由静止开始的恒加速度运动，平均速度是末速度的一半：

$$v_f = \frac{2d}{t}, \qquad K_{\text{block},f} = \frac{1}{2}m\left(\frac{2d}{t}\right)^2.$$

若只把 d/t 代入动能，就把平均速度误当成末速度。

实验加速度反求 g：补测坡角或等价几何量

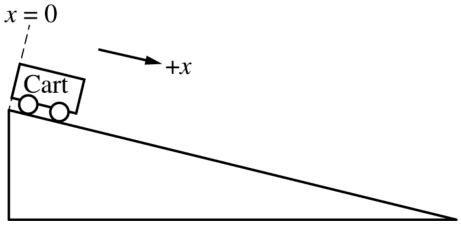
已求到沿坡加速度 a，要反求 g，还需知道向下重力的哪一部分沿坡。这个比例由坡角决定；也可测竖直高度差 h 与沿坡长度 L_r，由两长度的比得到坡角的正弦。

先分清沿坡长度与水平投影长度；两者不能互换。

$$\sin\theta = \frac{h}{L_r}.$$

例题 2023 Q2 (b)(i)

本问承接 2023 Q2 (a)(iii)。



2. (12 points, suggested time 25 minutes)

(b) The students are asked to determine an experimental value for the acceleration due to gravity g_{exp} using their data.

i. What additional quantities do the students need to measure in order to calculate g_{exp} from a ?

解：

还要测坡角 θ；或测坡两端竖直高度差 h 和沿坡长度 L_r，由竖直高度差除以沿坡长度得到坡角的正弦。

写 g 的反求式：重力的沿坡分量进入第二定律

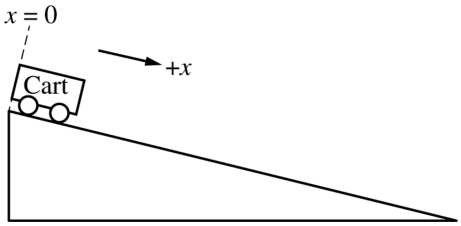
在题目使用的质点近似中，沿坡重力为 mg sinθ，质量乘加速度为 ma。让二者相等，约掉质量，再解出 g。

$$mg\sin\theta = ma, \qquad g = \frac{a}{\sin\theta}.$$

代入几何测量时，坡角正弦用竖直高度差除以沿坡长度；把自己测得的 a 与坡角连起来，不能把 a 直接当 g。

例题 2023 Q2 (b)(ii)

本问承接 2023 Q2 (b)(i)。



2. (12 points, suggested time 25 minutes)

(b) The students are asked to determine an experimental value for the acceleration due to gravity g_{exp} using their data.

ii. Write an expression for the value of g_{exp} in terms of a.

解：

在暂时忽略轮转动的质点模型中，沿坡重力分量提供加速度：

$$mg_{\text{exp}}\sin\theta = ma \;\Rightarrow\; g_{\text{exp}} = \frac{a}{\sin\theta} = \frac{aL_r}{h}.$$

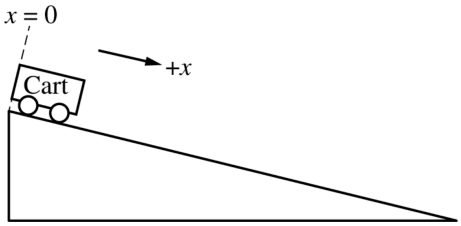
指出物理模型遗漏：具体到会改变运动的因素

实验值与接受值差得明显时，单写“人为误差”没有给出物理原因。原题排除了摩擦和空气阻力，还能检查模型忽略了哪些实际运动：例如车轮除平动外还转动。

指出具体对象与遗漏，先满足本问的识别任务；下一问再按这个遗漏追踪为什么偏大或偏小。

例题 2023 Q2 (c)(i)

本问承接 2023 Q2 (b)(ii)。



2. (12 points, suggested time 25 minutes)

(c) The students calculate the value of g_{exp} to be significantly lower than the accepted value of 9.8 m/s^2.

i. What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of g_{exp} ?

解：

可指出轮有不可忽略的转动惯量。这是具体的模型遗漏，不能只写“人为误差”。

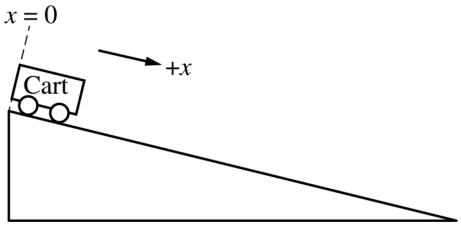
解释误差方向：遗漏→加速度变化→反求值变化

不能只说“轮转动所以不准”。须连成因果：势能分给平动与轮转动，平动加速比只算平动的模型慢；在坡角相同的重力加速度反求关系中，g 与 a 成正比；加速度偏小会使实验 g 偏小。

轮转动可以无耗散，转动动能不是自动的摩擦损失。解释方向时沿用上一问自己指出的具体因素，不换一种原因。

例题 2023 Q2 (c)(ii)

本问承接 2023 Q2 (c)(i)。



2. (12 points, suggested time 25 minutes)

(c) The students calculate the value of g_{exp} to be significantly lower than the accepted value of 9.8 m/s^2.

ii. Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of g_{exp} .

解：

真实下坡时，重力势能分给平动和轮转动，平动加速比把全部能量给平动的模型慢。因此测到的加速度小于质点模型预期值，将这个偏小的加速度代入重力加速度关系，就得到偏小的实验重力加速度；轮可无耗散地滚动，不能把转动能一概说成摩擦损耗。