AP Physics 2

学生用书

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气体、热量与传热

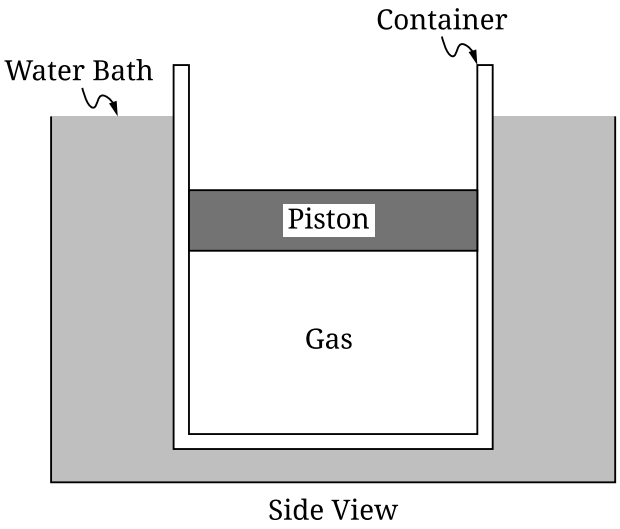
活塞受力：先区分气体、大气与重力

先选清研究对象。对活塞，只画别的物体对它施加的力：下方气体向上顶，上方大气向下压，地球使它受到向下重力。压强是每单位面积上的垂直力，因此均匀压强 P 作用在面积 A 上时，力的大小是 PA；静止时向上与向下的力相抵。

$$F = PA, \qquad \sum F_y = 0.$$

例题 2025 Q2 A

2. A sample of a monatomic ideal gas is sealed in a thermally conducting container by a movable piston of mass M and area A. The container is in a large water bath that is held at a constant temperature T_0. The piston is free to move with negligible friction. At the instant shown, the gas is in thermal equilibrium with the water bath, the piston is at rest, and the gas occupies volume V_0. The pressure of the air above the piston is P_{atm}.



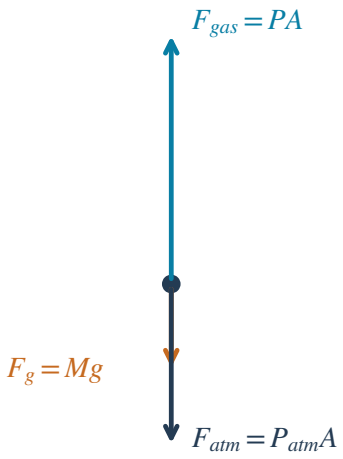
Note: Figure not drawn to scale.

A. On the dot shown, representing the piston, **draw** and **label** the forces that are exerted on the piston. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

解：

只画作用在活塞上的力，所有箭头都从代表活塞的点出发。气体对活塞的压力向上；大气压力和活塞重力向下。此时尚未放方块，不能画方块的作用力。

$$F_{gas} = P_i A \quad \text{向上}, \qquad F_{atm} = P_{atm} A \quad \text{向下}, \qquad F_g = Mg \quad \text{向下}.$$



由活塞平衡求气体压强，再求内能

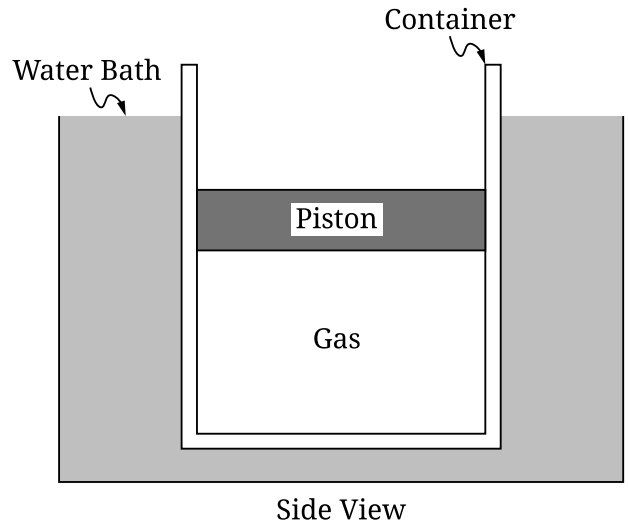
单原子理想气体的内能来自原子随机运动。内能 U 与绝对温度成正比；n 表示气体摩尔数，R 是气体常量。先由活塞平衡找绝对压强，再用气体方程把温度和摩尔数一起消去。大气也压在活塞上，所以气体压强不是仅等于重力除面积。

$$U = \frac{3}{2} nRT, \qquad PV = nRT, \qquad U = \frac{3}{2} PV.$$

例题 2025 Q2 B

本问承接 2025 Q2 A。

2. A sample of a monatomic ideal gas is sealed in a thermally conducting container by a movable piston of mass M and area A. The container is in a large water bath that is held at a constant temperature T_0. The piston is free to move with negligible friction. At the instant shown, the gas is in thermal equilibrium with the water bath, the piston is at rest, and the gas occupies volume V_0. The pressure of the air above the piston is P_{atm}.



Note: Figure not drawn to scale.

B. **Derive** an expression for the internal energy of the gas in terms of M, A, V_0, P_{atm}, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

活塞静止，向上和向下的力平衡。先由受力确定气体绝对压强，再由单原子理想气体内能关系消去温度和摩尔数。

$$P_iA - P_{atm}A - Mg = 0, \qquad P_i = P_{atm} + \frac{Mg}{A}.$$

$$U_i = \frac{3}{2}nRT_0, \qquad P_iV_0 = nRT_0, \qquad U_i = \frac{3}{2}\left(P_{atm} + \frac{Mg}{A}\right)V_0.$$

大气也向下压活塞，所以不能只用 Mg/A 作为气体压强。“单原子”决定内能系数 3/2；“活塞静止”决定压强。

恒温压缩：画 P–V 曲线与过程箭头

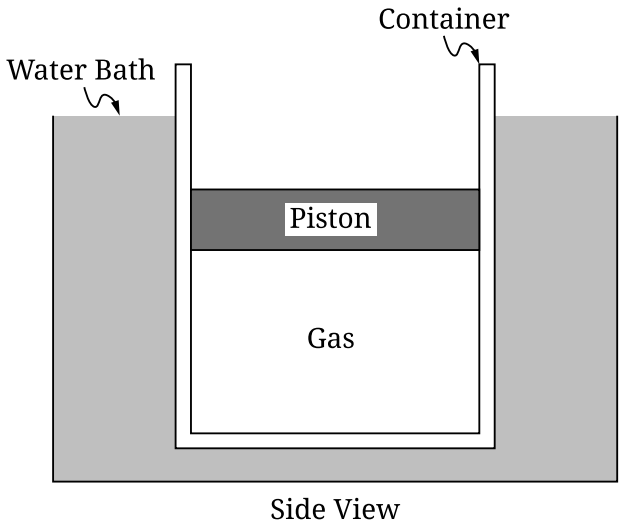
慢慢压缩浸在恒温大水浴中的导热容器，气体不断与水浴交换热量，温度保持水浴温度。封闭使气体量固定，因而 P 与 V 的乘积不变。体积减小时压强增加；先画反比曲线，再按实际过程加箭头。

$$PV = nRT_0 = \text{常量}, \qquad P = \frac{nRT_0}{V}.$$

例题 2025 Q2 C

本问承接 2025 Q2 B。

2. A sample of a monatomic ideal gas is sealed in a thermally conducting container by a movable piston of mass M and area A. The container is in a large water bath that is held at a constant temperature T_0. The piston is free to move with negligible friction. At the instant shown, the gas is in thermal equilibrium with the water bath, the piston is at rest, and the gas occupies volume V_0. The pressure of the air above the piston is P_{atm}.

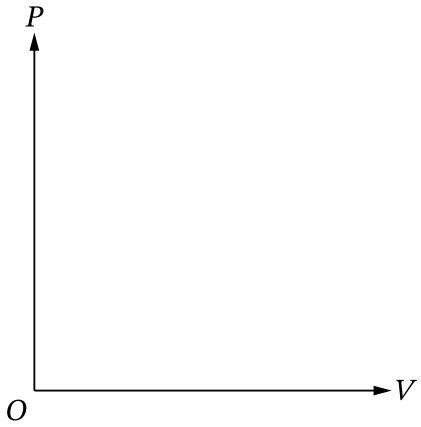


Side View

Note: Figure not drawn to scale.

C. A block, also of mass M, is placed on the piston at time $t = t_0$ and is slowly lowered. The piston comes to rest at $t = t_f$ when the block is completely released.

On the axes provided, **sketch** the expected relationship between the pressure P and volume V of the gas for the thermodynamic process that the gas undergoes during time interval $t_0 \leq t \leq t_f$. **Draw** an arrow on your sketch to represent the direction of the thermodynamic process.



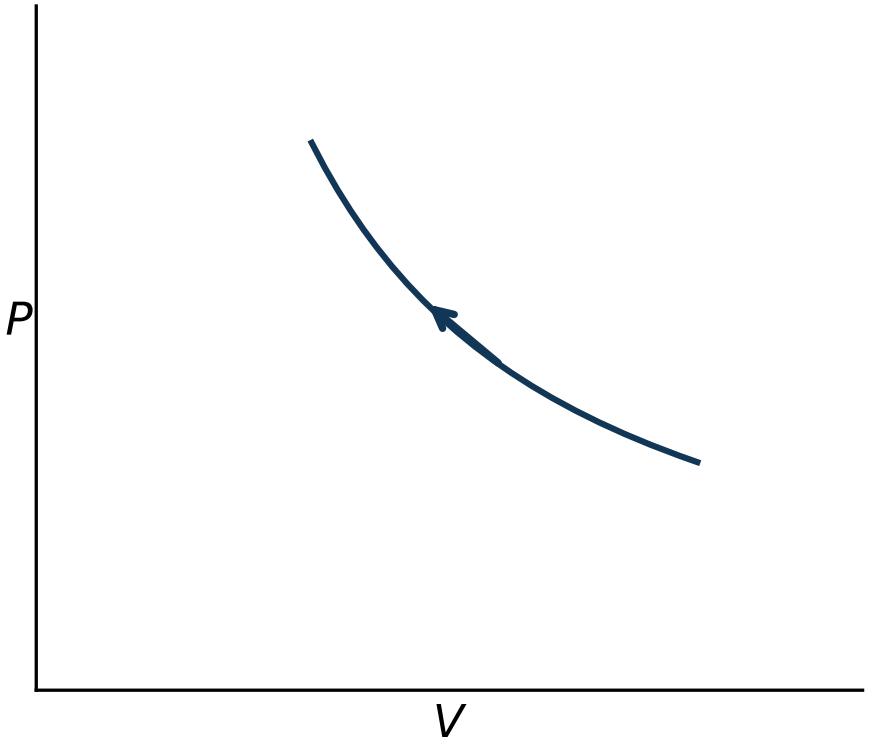
解：

导热容器浸在恒温大水浴中，并且缓慢压缩，所以气体温度保持 T0。封闭气体的量不变，于是：

$$PV = nRT_0 = P_iV_0, \qquad P = \frac{P_iV_0}{V}.$$

在 P 对 V 图上画向上弯曲的反比曲线。方块增加向下负载，压强增大、体积减小；箭头从右下初态指向左上末态。末态承担活塞和方块两个重力：

$$P_f = P_{atm} + \frac{2Mg}{A}, \qquad V_f = \frac{P_i}{P_f}V_0 < V_0.$$



同温压缩：从右下初态沿反比曲线移向左上末态。

同份气体与相同体积：由压强找温度

同一份气体装在固定体积内，压强变化反映绝对温度变化。温度必须用开尔文。先为两个状态分别写 PV=nRT，再相减求温度变化；若目标只比较初末温度，也可比较压强。恢复原体积但活塞负载增大时，先沿用活塞平衡确定压强的变化。

$$T_f - T_i = \frac{(P_f - P_i)V}{nR}.$$

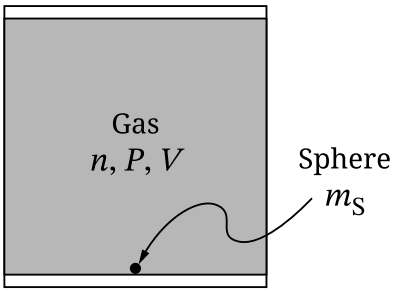
例题 2026 Q1 A(ii)

1. Part A

A sample of n moles of a monatomic, ideal gas is in a large, sealed, thermally conducting container with fixed volume. A small sphere of mass m_S is in the container. The volume of the sphere is much less than the volume of the container.

The gas is initially in State X with pressure P and volume V, as shown in Figure 1.

Figure 1



State X

Note: Figure not drawn to scale.

The sphere is initially in thermal equilibrium with the gas. The gas undergoes a heating process until the gas is in State Y with pressure $3P$ and the sphere is again in thermal equilibrium with the gas. The total energy transferred to the sphere during the heating process is Q_S.

ii. **Derive** an expression for the change ΔT in temperature of the gas for the heating process that the gas undergoes as the gas changes from State X to State Y. Express your answer in terms of n, P, V, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

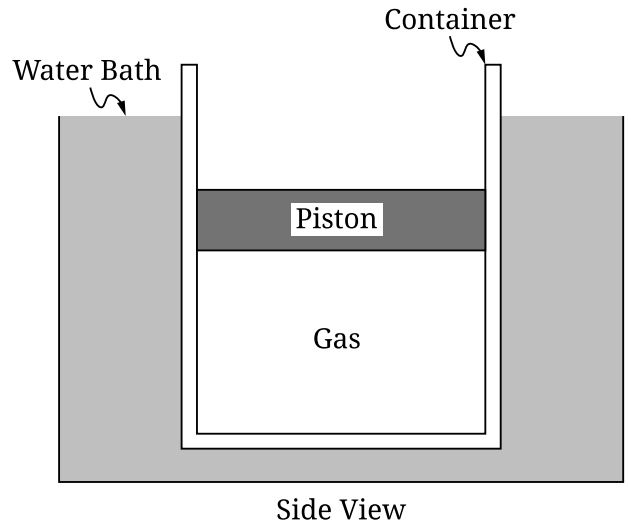
要求的是温度变化，不是末温。分别写初末状态，再相减：

$$T_X = \frac{PV}{nR}, \qquad T_Y = \frac{3PV}{nR}, \qquad \Delta T = T_Y - T_X = \frac{2PV}{nR}.$$

练习 2025 Q2 D

本问承接 2025 Q2 B、2025 Q2 C。

2. A sample of a monatomic ideal gas is sealed in a thermally conducting container by a movable piston of mass M and area A. The container is in a large water bath that is held at a constant temperature T_0. The piston is free to move with negligible friction. At the instant shown, the gas is in thermal equilibrium with the water bath, the piston is at rest, and the gas occupies volume V_0. The pressure of the air above the piston is P_{atm}.



Note: Figure not drawn to scale.

C. A block, also of mass M, is placed on the piston at time $t = t_0$ and is slowly lowered. The piston comes to rest at $t = t_f$ when the block is completely released.

D. With the block still on the piston, the temperature of the water bath is changed to a new constant temperature T_{new}. The gas occupies the original volume V_0 when the sample of gas and the water bath come to thermal equilibrium.

Indicate whether T_{new} is greater than, less than, or equal to T_0.

_____ $T_{\text{new}} > T_0$

_____ $T_{\text{new}} < T_0$

_____ $T_{\text{new}} = T_0$

Briefly **justify** your answer by referencing at least one feature of your answers to parts A, B, or C.

升温后的粒子速率分布

图中横轴是单个原子的速率，纵轴表示相应速率范围内的原子数量。温度上升表示平均随机动能上升，不是每个原子都取得同一速率。较热气体分布向较高速率延伸，峰向右且变低、曲线变宽。同份气体的原子数不变，所以曲线下的总面积不变。

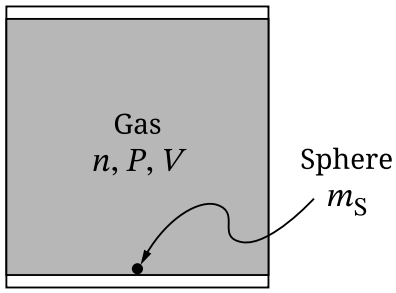
例题 2026 Q1 A(i)

1. Part A

A sample of n moles of a monatomic, ideal gas is in a large, sealed, thermally conducting container with fixed volume. A small sphere of mass m_S is in the container. The volume of the sphere is much less than the volume of the container.

The gas is initially in State X with pressure P and volume V, as shown in Figure 1.

Figure 1



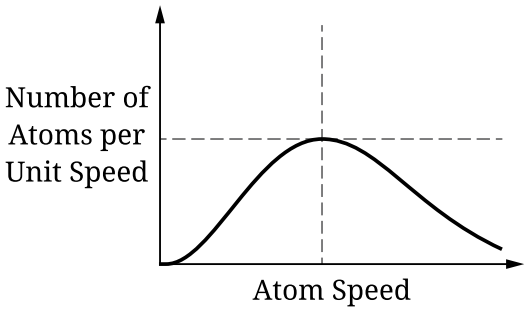
State X

Note: Figure not drawn to scale.

The sphere is initially in thermal equilibrium with the gas. The gas undergoes a heating process until the gas is in State Y with pressure $3P$ and the sphere is again in thermal equilibrium with the gas. The total energy transferred to the sphere during the heating process is Q_S.

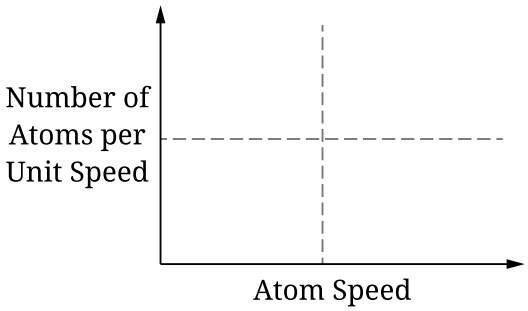
i. The graph in Figure 2 represents the number of atoms per unit speed as a function of atom speed for the gas when the gas is in State X. On Figure 3, **sketch** a curve that could represent the number of atoms per unit speed as a function of atom speed for the gas when the gas is in State Y.

Figure 2



State X

Figure 3



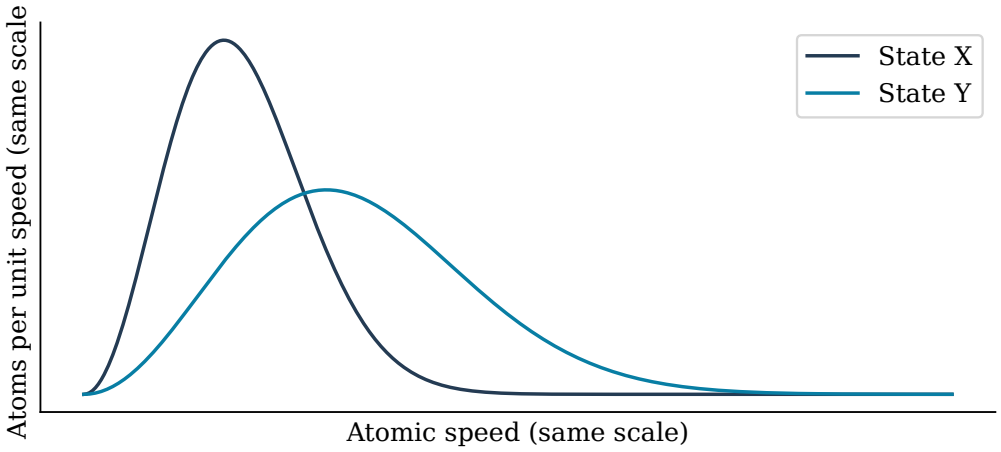
State Y

解：

容器密闭且刚性，气体量和体积均不变。压强从 P 增至 3P，绝对温度因此变成原来的三倍。温度提高表示粒子平均随机动能提高，不表示所有粒子都获得相同速率。

$$PV = nRT_X, \qquad 3PV = nRT_Y, \qquad T_Y = 3T_X.$$

在原图同样的坐标尺度下，末态曲线的峰向右移动、峰降低、分布变宽。气体粒子数没有改变，物理上新旧曲线下的面积相同。



同一粒子数的速率分布：升温使峰右移并降低；横纵轴使用相同尺度。

热平衡下由传入能量求比热容

比热容 c 表示每千克物质温度提高1 K需要多少能量。质量 m、温度变化 ΔT 与传入该物体的热量 Q 相联系；热平衡表示球和气体最终温度相同，若初态也平衡，球经历的温度变化就是前一问求出的气体温度变化。只把传给球的能量代入球的公式。

$$Q = mc\Delta T, \qquad c = \frac{Q}{m\Delta T}.$$

例题 2026 Q1 A(iii)

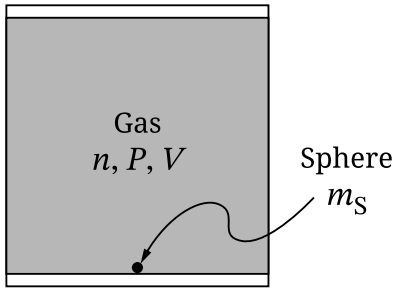
本问承接 2026 Q1 A(ii)。

1. **Part A**

A sample of n moles of a monatomic, ideal gas is in a large, sealed, thermally conducting container with fixed volume. A small sphere of mass m_{S} is in the container. The volume of the sphere is much less than the volume of the container.

The gas is initially in State X with pressure P and volume V, as shown in Figure 1.

Figure 1



State X

Note: Figure not drawn to scale.

The sphere is initially in thermal equilibrium with the gas. The gas undergoes a heating process until the gas is in State Y with pressure $3P$ and the sphere is again in thermal equilibrium with the gas. The total energy transferred to the sphere during the heating process is Q_{S}.

iii. **Derive** an expression for the specific heat c_{S} of the sphere. Express your answer in terms of n, m_{S}, P, V, Q_{S}, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

球在初末两态都与气体达到热平衡，所以球也经历刚求出的温度变化。QS 是传给球的能量；不能把气体的吸热量代到球的公式中。

$$Q_S = m_S c_S \Delta T, \qquad c_S = \frac{Q_S}{m_S \Delta T} = \frac{nRQ_S}{2PVm_S}.$$

球的体积远小于容器体积，使前面使用气体体积 V 的处理成立。

等量能量转移：比较两物体温度变化

隔热容器里球冷却、液体升温；整体没有向外失去能量，所以球失去的能量大小等于液体获得的能量。总热容 mc 是整个物体每升温1 K需要的能量。相同能量对应 ΔT=Q/(mc)：总热容越小，温度改变越多。本问的温差符号取绝对值。

$$m_S c_S |\Delta T_S| = m_L c_L |\Delta T_L|.$$

例题 2026 Q1 B

Part B

An insulated container is filled with a liquid of mass m_{L} and specific heat c_{L}. The original sphere is submerged in the liquid. The sphere has mass m_{S}, where $m_{\mathrm{S}} < m_{\mathrm{L}}$, and specific heat c_{S}, where $c_{\mathrm{S}} < c_{\mathrm{L}}$. The initial temperature of the sphere is greater than the initial temperature of the liquid. Later, the liquid and the sphere reach thermal equilibrium. The absolute values of the changes in the temperatures of the liquid and the sphere are $|\Delta T_{\mathrm{L}}|$ and $|\Delta T_{\mathrm{S}}|$, respectively.

Indicate whether $|\Delta T_{\mathrm{S}}|$ is greater than, less than, or equal to $|\Delta T_{\mathrm{L}}|$. Include one of the following relationships in your response.

- $|\Delta T_{\mathrm{S}}| > |\Delta T_{\mathrm{L}}|$
- $|\Delta T_{\mathrm{S}}| < |\Delta T_{\mathrm{L}}|$
- $|\Delta T_{\mathrm{S}}| = |\Delta T_{\mathrm{L}}|$

Justify your answer. Your justification may include expressions/equations but must include conceptual reasoning beyond algebraic solutions.

解：

取球和液体整体为系统。容器隔热，球失去的能量等于液体获得的能量；这里两个温度变化符号均表示绝对值。

$$m_S c_S \Delta T_S = m_L c_L \Delta T_L.$$

球的质量较小，比热容也较小，因此它的总热容 mScS 较小。转移相同能量时，较小总热容需要较大温度变化，所以：

$$\Delta T_S > \Delta T_L.$$

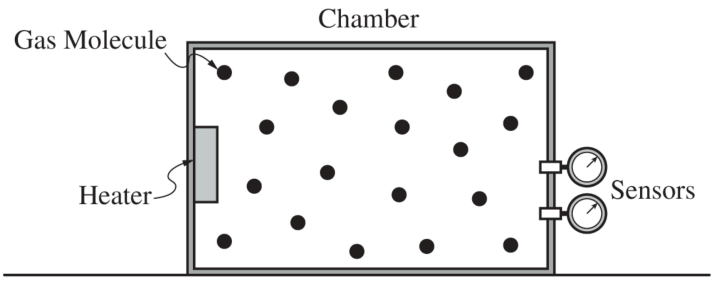
判断依据是能量转移相等、总热容不同；仅说“球原来较热”不能说明温差大小。

用固定体积的压强和温度测分子数

目标是分子个数 N，不是摩尔数。气体实际占据的体积 V 必须测量；温度用开尔文，压强用绝对压强。通过加热取得多组稳定且成对的 P、T 读数，重复测量减少随机误差。玻尔兹曼常量 kB 把单个粒子尺度与温度连接起来。

$$PV = Nk_BT, \qquad P = \frac{Nk_B}{V}T.$$

例题 2024 Q2 a



Note: Figure not drawn to scale.

Figure 1

2. (12 points, suggested time 25 minutes)

In Experiment 1, shown in Figure 1, a sample of an ideal gas is contained in an insulated, sealed chamber with thin, rigid walls. The chamber contains a heater and sensors that measure the temperature and pressure of the gas. A student is asked to design an experiment to determine the number N of molecules of the gas contained in the chamber.

(a) **Describe** a procedure for collecting data that would allow the student to determine an experimental value for N. Provide enough detail so that a student could replicate the experiment, including any steps necessary to reduce experimental uncertainty.

解：

气室密封且刚性，气体分子数 N 和体积 V 均固定。先测气室内部长、宽、高，多次测量取平均，求内部容积；加热器等占据体积不可忽略时应扣除。这个量应是气体实际占据的体积。

记录初始稳定状态的温度和绝对压强。打开加热器，在多个不同温度状态记录相互对应的 P、T；每次读数给传感器响应和气体温度均匀留出时间。同一状态重复读数取平均，测量范围应使数据覆盖较大的温度区间，整个过程保持同一密闭气体。温度用开尔文。

$$PV = Nk_BT, \qquad P = \frac{Nk_B}{V}T.$$

以 T 为横轴、P 为纵轴画最佳拟合直线。若斜率为 sP，则：

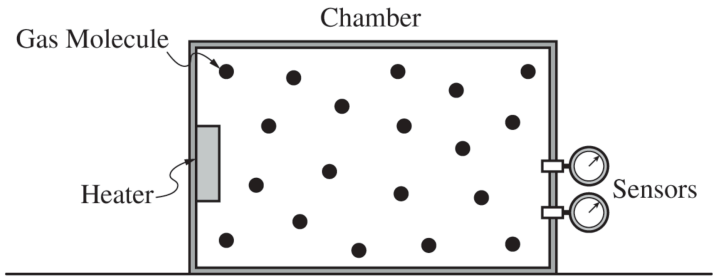
$$s_P = \frac{Nk_B}{V}, \qquad N = \frac{s_PV}{k_B}.$$

只测温度和压力却不测体积，无法确定 N。N 是分子个数，不是摩尔数。

刚性容器加热：在状态图上保持横坐标

坐标图先问哪个量固定。刚性容器的体积不变，横轴为体积时所有状态位于同一个正的横坐标。加热使压强增大，也使内能增大，所以 P–V 与 U–V 都画向上的竖直过程。把竖线画在纵轴上会表示零体积，不能这样画。

例题 2024 Q2 b(i)



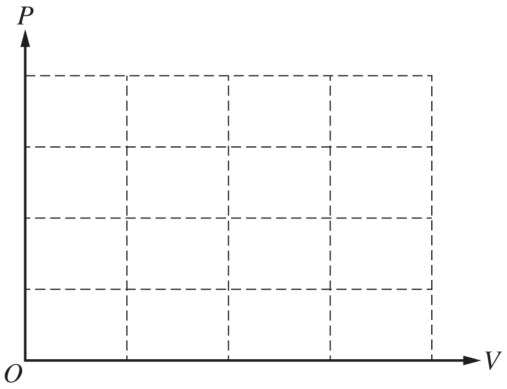
Note: Figure not drawn to scale.

Figure 1

2. (12 points, suggested time 25 minutes)

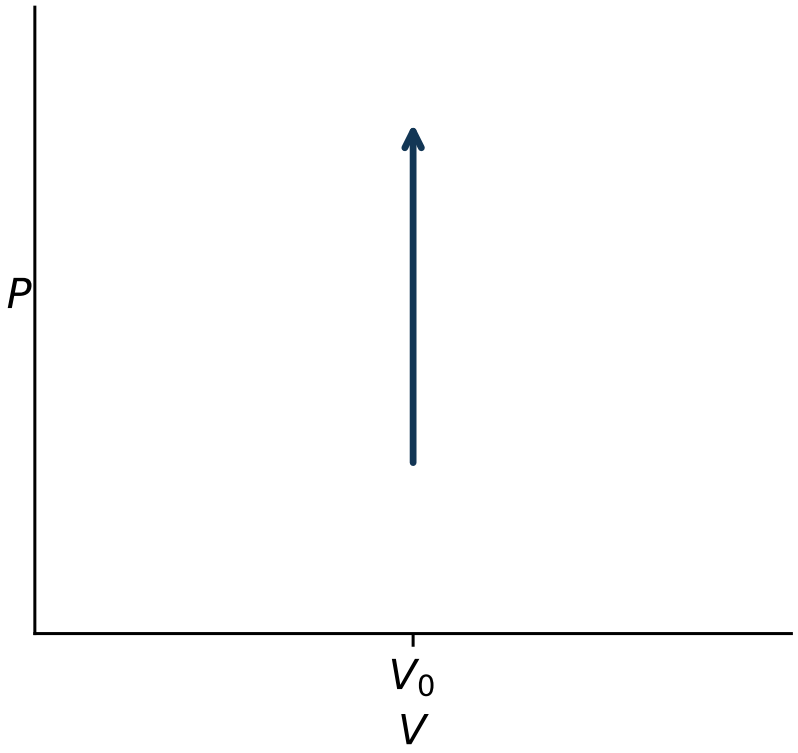
In Experiment 1, shown in Figure 1, a sample of an ideal gas is contained in an insulated, sealed chamber with thin, rigid walls. The chamber contains a heater and sensors that measure the temperature and pressure of the gas. A student is asked to design an experiment to determine the number N of molecules of the gas contained in the chamber.

i. On the following axes, **sketch** a curve or line to represent the expected relationship between the pressure P and the volume V of the gas while the heater is on. **Draw** an arrow on the curve or line to represent the direction of the resulting thermal process.



解：

横轴为 V，纵轴为 P。刚性气室使全过程横坐标固定在一个正的 V0；加热使压强提高。画离开两条坐标轴的竖直线段，箭头向上。



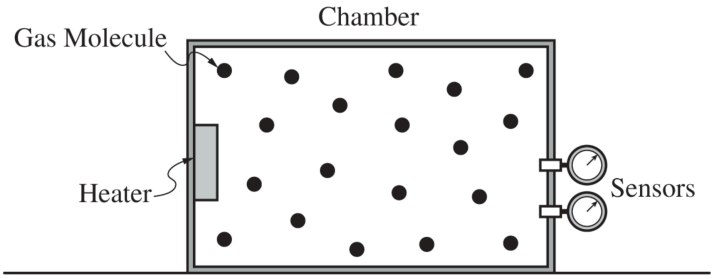
刚性气室中体积不变，加热使压强上升。

定容加热的 U–V 图：体积固定，内能增大

纵轴换成内能U后，仍先保留固定体积V0这个横坐标。刚性边界不移动，气体不以膨胀做功；加热器传入的能量增加气体内能，所以起终点均在正的V0处，过程箭头向上。

$$\Delta U = Q_{\text{in}} > 0, \qquad \Delta V = 0.$$

例题 2024 Q2 b(ii)



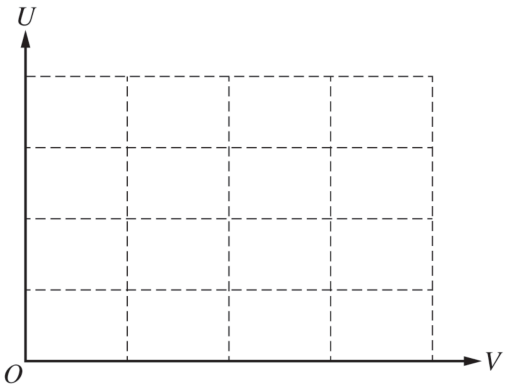
Note: Figure not drawn to scale.

Figure 1

2. (12 points, suggested time 25 minutes)

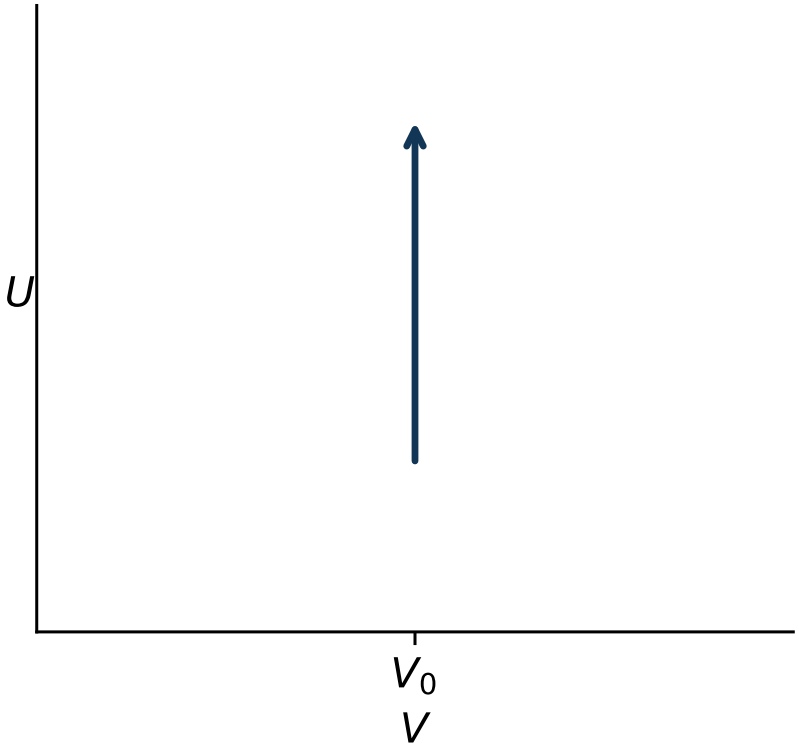
In Experiment 1, shown in Figure 1, a sample of an ideal gas is contained in an insulated, sealed chamber with thin, rigid walls. The chamber contains a heater and sensors that measure the temperature and pressure of the gas. A student is asked to design an experiment to determine the number N of molecules of the gas contained in the chamber.

ii. On the following axes, **sketch** a curve or line to represent the expected relationship between the internal energy U and the volume V of the gas while the heater is on. **Draw** an arrow on the curve or line to represent the direction of the resulting thermal process.



解：

U 对 V 图同样在正的 V0 处竖直向上；起点、终点的内能均为正。



刚性气室中体积不变，内能增加。

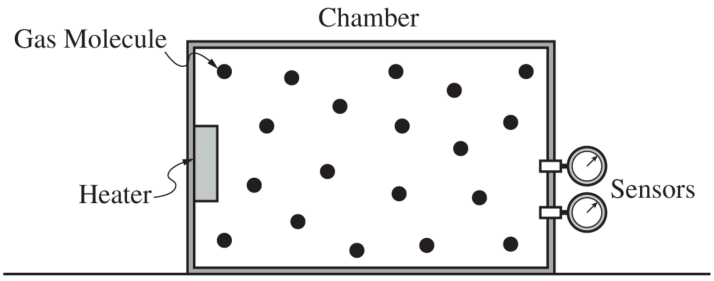
解释定容加热为何内能增加

热力学第一定律是能量账本。Qin 是传入气体的能量，Won 是外界对气体做的功。容器不变形，没有边界体积改变的功；加热器向气体输入能量，所以内能增加。外壁隔热只阻止向外散热，不否认内部加热器传能。

$$\Delta U = Q_{\text{in}} + W_{\text{on}}, \qquad W_{\text{on}} = 0.$$

例题 2024 Q2 b(iii)

本问承接 2024 Q2 b(ii)。



Note: Figure not drawn to scale.

Figure 1

2. (12 points, suggested time 25 minutes)

In Experiment 1, shown in Figure 1, a sample of an ideal gas is contained in an insulated, sealed chamber with thin, rigid walls. The chamber contains a heater and sensors that measure the temperature and pressure of the gas. A student is asked to design an experiment to determine the number N of molecules of the gas contained in the chamber.

iii. Briefly **justify** why the curve or line drawn in part (b)(ii) has the shape that you sketched.

解：

固定体积解释线为什么竖直，加热器输入能量解释箭头为什么向上。气体不对刚性边界做膨胀功：

$$\Delta U = Q_{in} + W_{on}, \qquad W_{on} = 0, \qquad \Delta U = Q_{in} > 0.$$

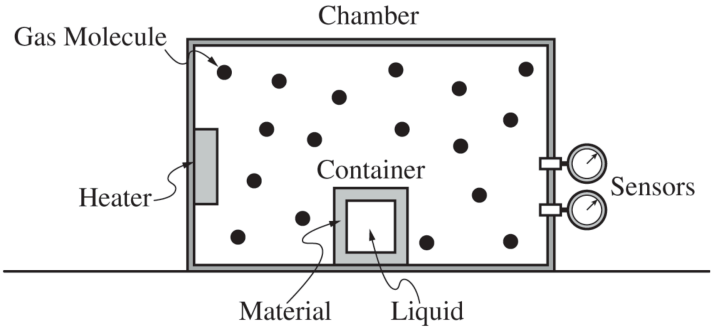
气室外壁隔热表示不向外散热，不能据此否认内部加热器向气体传能。

两侧温差驱动导热：选线性坐标

材料两侧温度不同，能量从高温一侧传向低温一侧。传热率 Q/Δt 是每秒传过的能量；面积 A 越大、厚度 L 越小，传热越快。k 是材料热导率。固定材料尺寸时，传热率与两侧温差成正比，因此应使用两侧温差而非只用其中一侧温度。

$$\frac{Q}{\Delta t} = \frac{kA}{L}(T_G - T_L).$$

例题 2024 Q2 c(i)



Note: Figure not drawn to scale.

Figure 2

In Experiment 2, shown in Figure 2, a liquid-filled container that is completely wrapped with a material of uniform thickness 0.01 m is inside the sealed chamber that is filled with an ideal gas. The material has a total area of 0.06 m^2 in contact with the gas. The heater is turned on. As the temperature T_G of the gas increases, the following data for the temperature T_L of the liquid and the rate $\frac{Q}{\Delta t}$ of energy transfer are collected.

	T_G (K)	T_L (K)	$\frac{Q}{\Delta t}\left(\frac{J}{s}\right)$	
	295	295	0.0	
	371	303	26.3	
	425	308	43.1	
	475	313	60.0	
	528	323	75.0	

(c) The student is asked to determine an experimental value of the thermal conductivity k of the material used to wrap the container inside the sealed chamber.

i. **Indicate** what measured and/or calculated quantities could be graphed to yield a straight line that could be used to calculate an experimental value for the thermal conductivity k of the material. Use the blank columns in the table to list any calculated quantities you graph in addition to the data provided.

Vertical Axis: _____ Horizontal Axis: _____

解：

材料两侧分别是气体和液体，真正驱动导热的是两侧温差 TG−TL。传热率 Q/Δt 已是每秒传过的能量，不能再除一次时间。

$$\frac{Q}{\Delta t} = \frac{kA}{L}(T_G - T_L).$$

横轴选 TG−TL，单位 K；纵轴选 Q/Δt，单位 W。五个温差依次为 0、68、117、162、205 K。

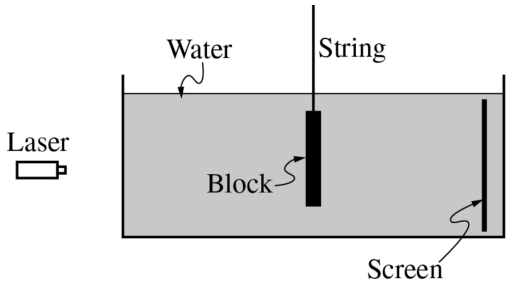
静止液体与流动

浸没物块的受力与拉力

浮力是周围液体压力的合力，方向向上。完全浸没物块排开的液体体积等于物块体积；浮力等于这部分液体的重量。先把重力、悬线拉力和浮力分别画出来，再用静止时合力为零求拉力。ρw 表示液体密度。

$$F_B = \rho_w V_b g, \qquad F_T + F_B - m_b g = 0.$$

例题 2022 Q1 a(i)



(10 points, suggested time 20 minutes)

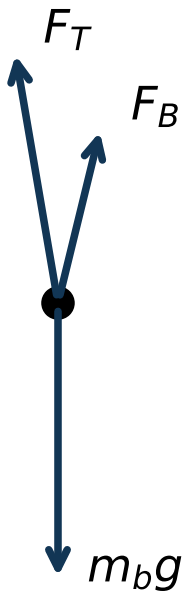
Students are investigating electromagnetic wave phenomena in transparent media. They use a string to support a stationary thin, rectangular block of mass m_b, volume V_b, and density ρ_b. The block has two narrow slits in its center and is submerged in a glass tank containing water with density ρ_w, as shown above.

i. On the dot below, which represents the block, draw and label the forces that are exerted on the block. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



解：

作用在物块上的重力向下；悬线拉力和排开液体产生的浮力向上。所有箭头从原图的点出发。浮力已是液体压力的合力，不额外再画另一项水压力。



只画作用在物块上的三个力。

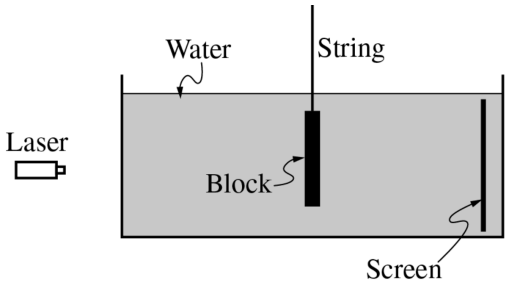
静止浸没物块：由受力平衡求悬线拉力

刚才已画出三种力。现在要求拉力的表达式，取向上为正写合力为零；完全浸没使排水体积等于物块体积，再把浮力写成排开水的重量。最后把已知重力和浮力移到等式另一侧。

$$F_T + F_B - m_b g = 0, \qquad F_B = \rho_w V_b g.$$

例题 2022 Q1 a(ii)

本问承接 2022 Q1 a(i)。



(10 points, suggested time 20 minutes)

Students are investigating electromagnetic wave phenomena in transparent media. They use a string to support a stationary thin, rectangular block of mass m_b, volume V_b, and density ρ_b. The block has two narrow slits in its center and is submerged in a glass tank containing water with density ρ_w, as shown above.

ii. Derive an expression for the force exerted on the block by the string in terms of the given quantities and physical constants, as appropriate.

解：

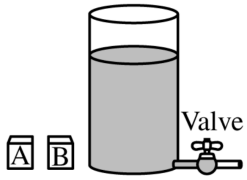
取竖直向上为正，静止使合力为零。完全浸没时排水体积等于物块体积：

$$F_T + F_B - m_b g = 0, \qquad F_B = \rho_w V_b g, \qquad F_T = m_b g - \rho_w V_b g.$$

用粒子碰撞解释向上浮力

压力源于粒子碰撞表面时交换动量。浮块底部受到水粒子向上的作用，上表面受到空气粒子向下的作用。比较这些作用的合效果，而不是只说水有浮力。水的向上作用较大，压力合力向上。

例题 2023 Q3 a(i)



Note: Figure not drawn to scale.

Figure 1

(12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

解：

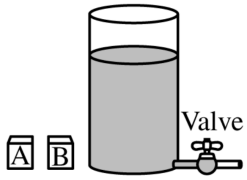
水和空气粒子不断随机运动，碰到浮块表面会交换动量。空气撞击上表面，给浮块向下的压力作用；水撞击底部，给浮块向上的作用。水的向上作用大于空气的向下作用，两者的净效果是向上浮力。不能把空气的压力忽略为不存在。

漂浮深度与物块密度

漂浮且静止时，浮力大小等于重力。两块体积相同时，浸得更深的一块排水更多、浮力更大，也必须更重。密度是质量除总体积，因此可由浸没比例比较密度。

$$\rho_w V_{\text{sub}} g = \rho_b V g, \qquad \frac{\rho_b}{\rho_w} = \frac{V_{\text{sub}}}{V}.$$

例题 2023 Q3 a(ii)



Note: Figure not drawn to scale.

Figure 1

(12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

解：

两块尺寸相同，A 先触底，说明此前静止漂浮时 A 浸得更深、排水体积更大，浮力也更大。各块静止时浮力等于重量，故 A 更重；总体积相同，A 密度更大。

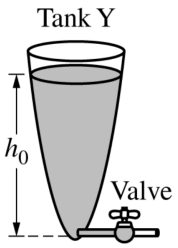
$$\rho_w V_{sub} g = \rho_b V g, \qquad \frac{\rho_b}{\rho_w} = \frac{V_{sub}}{V}, \qquad \rho_A > \rho_B.$$

液面到出口：用能量关系求速度

沿同一理想流动的水，从液面到出口比较压强、单位体积动能和重力势能。两处都接触大气，压强项相同；出口高度取零。液面也在下降，首次写精确关系时保留它的速度，不能先把它设成零。

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2.$$

例题 2023 Q3 b(i)



Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

解：

取液面处为1、出口处为2，出口高度为零。两处均接大气，因此压强相同。按题目理想流动模型使用伯努利关系，保留液面下降速率 vs：

$$P_{atm} + \rho g h + \frac{1}{2}\rho v_s^2 = P_{atm} + \frac{1}{2}\rho v_p^2,$$

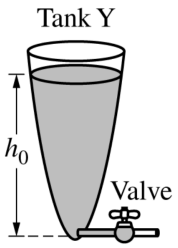
$$v_p^2 = v_s^2 + 2gh, \qquad v_p = \sqrt{v_s^2 + 2gh}.$$

截面积与流速：相同时间流过相同体积

水不可压缩。每秒从水箱减少的体积，等于每秒从管口流出的体积。截面积乘该处速度就是每秒流过的体积；圆形截面积用半径平方乘π。R 是当时液面半径，r 是固定管口半径。

$$A_s v_s = A_p v_p, \qquad (\pi R^2) v_s = (\pi r^2) v_p.$$

例题 2023 Q3 b(ii)



Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \dfrac{R^2}{r^2} v_s$.

解：

水不可压缩，单位时间液面减少的体积等于管口流出的体积。液面半径 R 随时刻改变，管道半径 r 固定：

$$(\pi R^2)v_s = (\pi r^2)v_p, \qquad v_p = \frac{R^2}{r^2}v_s.$$

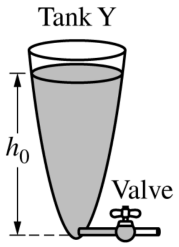
先比较速度，再作大水箱近似

“水箱宽”本身还不是代入公式的理由。用连续性求液面速度与出口速度的比：面积比很小时，液面速度才远小于出口速度，液面动能项才可忽略。

$$\frac{v_s}{v_p} = \frac{r^2}{R^2}, \qquad R \gg r \Longrightarrow v_s^2 \ll v_p^2.$$

例题 2023 Q3 b(iii)

本问承接 2023 Q3 b(i)、2023 Q3 b(ii)。



Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

解：

液面半径远大于管半径，使液面截面积远大于管口面积。由连续性，液面速率与出口速率之比等于管口面积与液面面积之比，因此前者远小于后者。因此液面动能项相对出口动能项可忽略：

$$v_p \approx \sqrt{2gh}.$$

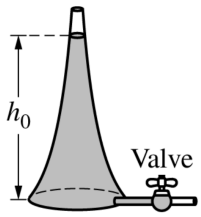
这是有条件的近似。不能在写精确关系时先把 vs 设为零，又声称保留了液面运动。

水位与截面同时改变：联立判断液面速度

水流出时高度降低，若水箱向下变宽，当时的液面面积也改变。不能把上一水箱的“恒定液面速度”移到新水箱。把连续性中的出口速度代入能量关系，写出液面速度只依赖当前高度和半径的表达式，再分别看分子、分母的变化。

例题 2023 Q3 c

本问承接 2023 Q3 b(i)、2023 Q3 b(ii)。



Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

解：

Tank Z 向下越来越宽，水流出时 h 减小、当前液面半径 R 增大。先联立两条关系，避免把未知变化的 vs 当常量：

$$v_p = \frac{R^2}{r^2} v_s, \qquad \frac{R^4}{r^4} v_s^2 = v_s^2 + 2gh, \qquad v_s = \sqrt{\frac{2gh}{R^4/r^4 - 1}}.$$

水箱比管口宽，分母为正。随着 h 减小，分子减小；随着 R 增大，分母增大，因此液面下降速率 vs 减小。该结论同时使用了能量关系与连续性，符合题目要求。

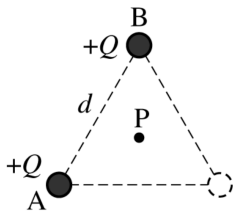
电场、电势与能量

零电场与零电势：向量和标量分开

电场描述正试验电荷的受力方向，是向量；电势描述每单位正电荷的电势能，是带正负号的标量。求零电场必须抵消方向；求零电势则直接相加带符号的贡献。等距不意味着电场大小能直接相加。先按原几何把两电场分解，再与第三电荷比较。

$$\boldsymbol{E}_{\text{net}} = \boldsymbol{E}_1 + \boldsymbol{E}_2 + \boldsymbol{E}_3, \qquad V_{\text{net}} = \frac{kq_1}{a} + \frac{kq_2}{a} + \frac{kq_3}{a}.$$

例题 2023 Q4 a



(10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

Students Y and Z discuss the electric field and the electric potential at Point P after a third charged particle is placed at the bottom-right vertex. The students make the following statements.

Student Y: "If a particle with positive charge $+2Q$ is placed at the bottom-right vertex, the magnitude of the electric field will be zero at Point P."

Student Z: "To make the value of the electric potential zero at Point P, a particle with negative charge $-Q$ should be placed at the bottom-right vertex."

(a) In a coherent, paragraph-length response, evaluate the accuracy of each student's statement. If any aspect of either student's statement is inaccurate, explain how to correct the student's statement. Support your evaluations using appropriate physics principles.

解：

Y 错误。P 到三个顶点距离相等，但电场是有方向的向量。A、B 各自的电场大小相等，夹角 120°；合场大小只等于一个电荷产生的场，并非二者大小相加。

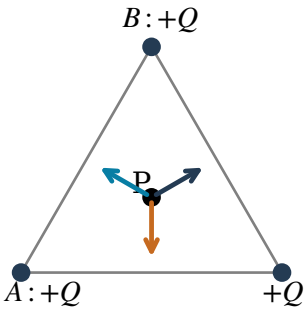
$$E_0 = \frac{kQ}{a^2}, \qquad E_{AB} = 2E_0 \cos 60^\circ = E_0.$$

这里 a 是中心到顶点的距离。合场指向右下顶点；要抵消它，右下顶点应放 +Q，产生等大的左上电场。+2Q 会抵消过多，留下向左上的非零场。

Z 也错误。电势是标量，要把三个带正负号的贡献直接相加。距 P 都为 a，所以零电势要求总电荷为零：

$$V_P = \frac{kQ}{a} + \frac{kQ}{a} + \frac{kq_3}{a} = \frac{k}{a}(2Q + q_3), \qquad q_3 = -2Q.$$

只放 −Q 时还剩 kQ/a 的正电势。零电场所需第三电荷与零电势所需第三电荷不同，不能用同一个“正负抵消”规则处理。

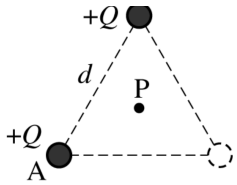


电荷系统能量：逐对计数与外界功

两电荷的相互作用电势能是 kq1q2/r，带电荷的正负号。三电荷系统共有三对，每对只计一次。第三电荷最初在无穷远时，它与原两电荷的相互作用为零，但原两电荷之间的能量仍在。若起止都静止，外界功等于系统电势能增量。

$$U = \sum_{\text{不同电荷对}} \frac{kq_iq_j}{r_{ij}}, \qquad W_{\text{ext}} = U_f - U_i.$$

例题 2023 Q4 b(i)



(10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

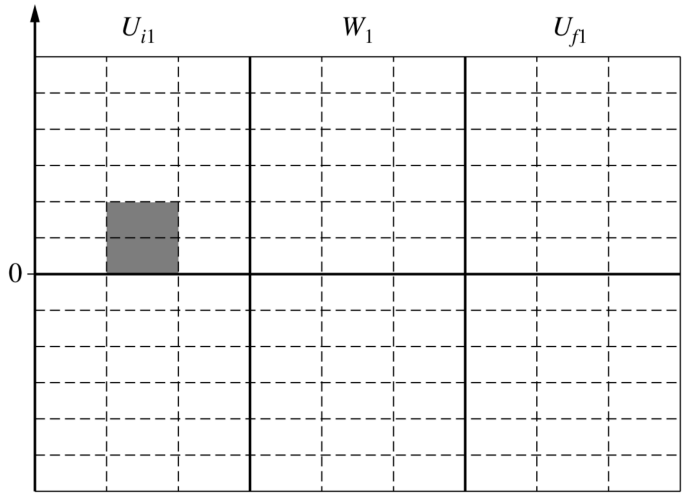
(b) Particles A and B are once again held in place at two vertices of the equilateral triangle. The students want to represent the electric potential energy of a system of particles when a third charged particle is brought from very far away to the bottom-right vertex. Scenarios 1 and 2 are considered.

i. In Scenario 1, a third particle with positive charge $+Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i1} of the system consisting of all three particles when the third particle with positive charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_1 required to move the third particle with positive charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f1} of the system consisting of all three particles when the third particle with positive charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 1

解：

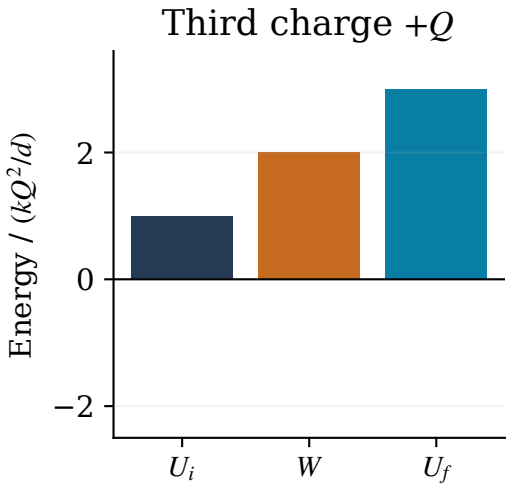
第三电荷开始在无穷远，但 A、B 早已存在并相距 d，因此初态电势能不为零。移入 +Q 后共有 AB、AC、BC 三对，每对只计一次。

$$U_{i1} = \frac{kQ^2}{d}, \qquad U_{f1} = \frac{kQ^2}{d} + \frac{kQ^2}{d} + \frac{kQ^2}{d} = 3U_{i1}.$$

起止粒子都静止，外界输入的功等于电势能变化：

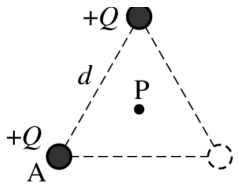
$$W_1 = U_{f1} - U_{i1} = 2U_{i1}.$$

用已有初态柱作 1 单位：W1 向上 2 单位，Uf1 向上 3 单位。原图初态柱是 2 小格，对应功柱 4 小格、末态柱 6 小格。



练习 2023 Q4 b(ii)

本问承接 2023 Q4 b(i)。



(10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

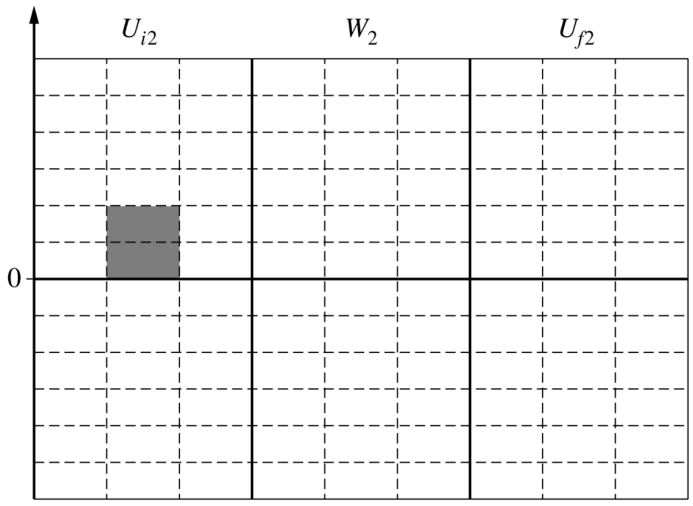
(b) Particles A and B are once again held in place at two vertices of the equilateral triangle. The students want to represent the electric potential energy of a system of particles when a third charged particle is brought from very far away to the bottom-right vertex. Scenarios 1 and 2 are considered.

ii. In Scenario 2, a particle with negative charge $-Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i2} of the system consisting of all three particles when the particle with negative charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_2 required to move the particle with negative charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f2} of the system consisting of all three particles when the particle with negative charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 2

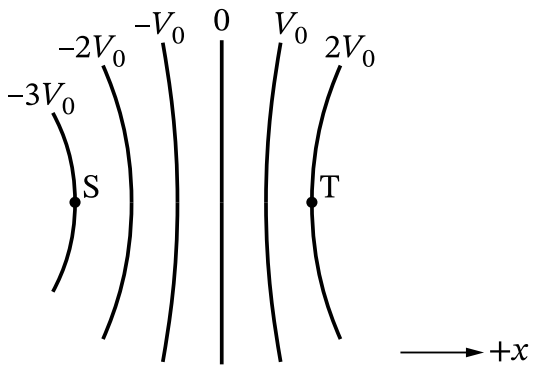
读等势线：判断电荷加速或减速

等势线标签给出位置的电势。电场指向电势降低的方向。正电荷受力与场同向，负电荷反向；再比较力与运动方向决定电场做正功还是负功。只有电场力时，电势能增加对应动能减少。

$$\Delta U = q(V_f - V_i), \qquad \Delta K = -\Delta U.$$

例题 2026 Q4 A

4. Equipotential lines for a region with an electric field are shown.



A small sphere (not shown) with positive charge $+Q$ has initial speed v_S in the +x-direction and kinetic energy K_S when the sphere is at Point S. The sphere moves to Point T. The force from the electric field is the only force that is exerted on the sphere as the sphere moves between Points S and T.

The sphere has final speed v_T and kinetic energy K_T when the sphere is at Point T.

Part A

Indicate whether v_T is greater than, less than, or equal to v_S. Include one of the following relationships in your response.

- $v_T > v_S$
- $v_T < v_S$
- $v_T = v_S$

Justify your answer. Your justification may include expressions/equations but must include conceptual reasoning beyond algebraic solutions.

解：

原图 S 点的电势为 −3V0，T 点为 2V0。沿 S 到 T，电势增加，电场指向电势降低的方向，即 −x。正电荷受力沿电场方向，与运动方向相反，电场做负功，动能和速率减小。

$$V_T - V_S = 2V_0 - (-3V_0) = 5V_0, \qquad v_T < v_S.$$

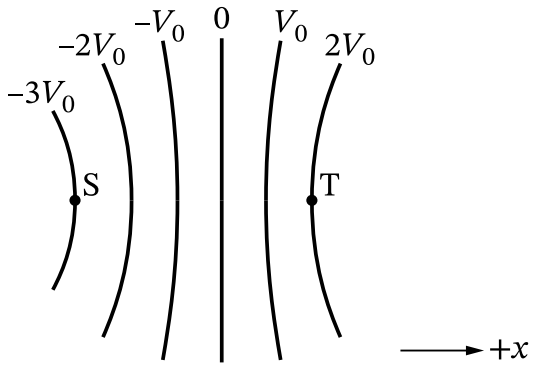
保留电荷正负号：求到达另一点的动能

先写初末总能量相等，再整理所求动能。位置电势差是末电势减初电势；q 是实际带符号电荷。换电荷符号时，位置电势差不变，电势能变化才反号。

$$K_i + qV_i = K_f + qV_f, \qquad K_f = K_i - q(V_f - V_i).$$

例题 2026 Q4 B

4. Equipotential lines for a region with an electric field are shown.



A small sphere (not shown) with positive charge $+Q$ has initial speed v_S in the +x-direction and kinetic energy K_S when the sphere is at Point S. The sphere moves to Point T. The force from the electric field is the only force that is exerted on the sphere as the sphere moves between Points S and T.

The sphere has final speed v_T and kinetic energy K_T when the sphere is at Point T.

Part B

Derive an expression for K_T in terms of V_0, Q, K_S, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

从能量守恒起手。只有电场力做功，动能的减少等于电势能的增加。

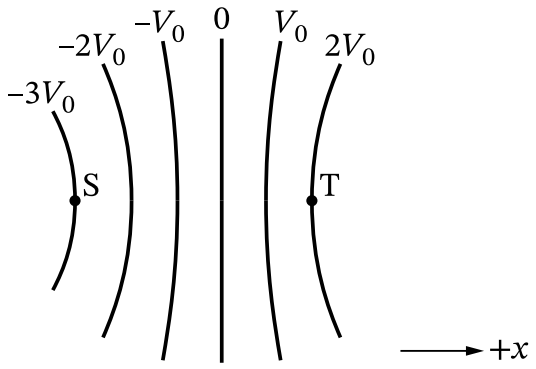
$$K_S + QV_S = K_T + QV_T, \qquad K_T = K_S - Q(V_T - V_S) = K_S - 5QV_0.$$

题干已说明粒子到达 T；初动能必须足够，不能算出负动能后仍把它当成到达 T 的物理结果。

练习 2026 Q4 C

本问承接 2026 Q4 B。

4. Equipotential lines for a region with an electric field are shown.



A small sphere (not shown) with positive charge $+Q$ has initial speed v_S in the $+x$-direction and kinetic energy K_S when the sphere is at Point S. The sphere moves to Point T. The force from the electric field is the only force that is exerted on the sphere as the sphere moves between Points S and T.

The sphere has final speed v_T and kinetic energy K_T when the sphere is at Point T.

Part C

A new, small sphere of the same mass as the original sphere but with negative charge $-Q$ has the same initial speed v_S as the original sphere in the $+x$-direction when the sphere is at Point S. The force from the electric field is the only force that is exerted on the new sphere as the new sphere moves between Points S and T. The new sphere has kinetic energy K_{new} when the new sphere is at Point T.

Indicate whether K_{new} is greater than, less than, or equal to K_T. Include one of the following relationships in your response.

- $K_{new} > K_T$
- $K_{new} < K_T$
- $K_{new} = K_T$

Briefly **justify** your answer by referencing your derivation in part B.

从静止加速：比较电荷大小与动能

装置极性已调整到两种电荷都被加速，所以每个粒子得到正的动能。从静止出发时，动能增量等于电荷大小乘加速电势差大小。比较相同电势差加速的粒子，质量决定速度，但不改变获得的电动能。

$$K = |q|\,|\Delta V|.$$

例题 2024 Q4 a

4. (10 points, suggested time 20 minutes)

Two particles, 1 and 2, have different mass and charge as described by the following.

- Particle 1 has mass M and negative charge $-Q$.
- Particle 2 has mass $\frac{M}{2}$ and positive charge $+2Q$.

In separate trials, a device is used to accelerate each particle in the $-y$-direction from rest through a potential difference of absolute value $|\Delta V|$. The polarity of the potential difference can be adjusted so that a particle with either positive charge or negative charge can be accelerated in the $-y$-direction by the device. Gravitational effects are negligible.

After moving through the potential difference, particles 1 and 2 exit the device with kinetic energies K_1 and K_2, respectively.

(a) **Calculate** the ratio $\frac{K_2}{K_1}$.

解：

两粒子从静止出发，装置调整极性使其都加速。动能增量为电荷大小乘加速电势差大小；实际电荷符号不使获得的动能变成负值。

$$K_1 = Q|\Delta V|, \qquad K_2 = 2Q|\Delta V|, \qquad \frac{K_2}{K_1} = 2.$$

电路、电容与测量

先简化串并联，再求主干电流

串联支路中各电阻流过同一电流，阻值相加；并联支路连接同两节点，电势差相同，电阻倒数相加。先从没有分岔的最内侧支路开始，再逐层合并。主干电流是电池电流，用电池电势差除总阻值。

$$R_s = R_1 + R_2, \qquad \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}, \qquad I = \frac{\Delta V}{R_{\text{total}}}.$$

例题 2024 Q3 a(i)

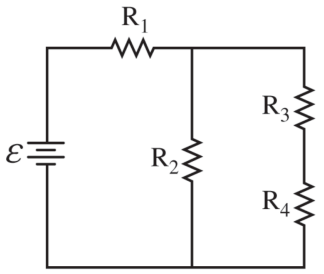


Figure 1

3. (12 points, suggested time 25 minutes)

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors R_1, R_2, R_3, and R_4, each of resistance R, as shown in Figure 1.

(a) For parts (a)(i) and (a)(ii), express your answers in terms of numerical values, $\mathcal{E}$, and R only.

i. **Derive** an expression for the current I_1 in Resistor R_1.

解：

先读导线连接：R3、R4 串联；该支路与 R2 并联；并联整体再与 R1 串联。R1 是分流前的主干，所以它流过电池总电流。

$$R_{34} = 2R, \qquad \frac{1}{R_p} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}, \qquad R_{total} = R + \frac{2R}{3} = \frac{5R}{3}.$$

$$I_1 = \frac{\mathcal{E}}{R_{total}} = \frac{3\mathcal{E}}{5R}.$$

已知主干电流：先求并联支路电压

并联支路各自只分到主干串联电阻之后剩下的电势差。先由主干电流求前段电阻的电势差，再用电池电势差减去它；最后把支路电势差除以该支路的总阻值。不能把整节电池电势差直接代给其中一个支路电阻。

例题 2024 Q3 a(ii)

本问承接 2024 Q3 a(i)。

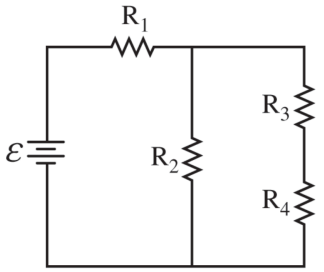


Figure 1

3. (12 points, suggested time 25 minutes)

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors R_1, R_2, R_3, and R_4, each of resistance R, as shown in Figure 1.

(a) For parts (a)(i) and (a)(ii), express your answers in terms of numerical values, $\mathcal{E}$, and R only.

ii. **Derive** an expression for the current I_3 in Resistor R_3.

解：

R1 消耗的电势差为 I1R，剩余部分落在两条并联支路上。R3 所在支路总阻值是 2R，因此：

$$\Delta V_1 = I_1 R = \frac{3}{5}\mathcal{E}, \qquad \Delta V_p = \mathcal{E} - \Delta V_1 = \frac{2}{5}\mathcal{E}, \qquad I_3 = \frac{\Delta V_p}{2R} = \frac{\mathcal{E}}{5R}.$$

串联分压与并联同压：画柱或排序

并联的元件连接相同两节点，所以电势差相等。串联同电流，ΔV=IR，阻值越大，分到的电势差越大。画柱形图时先定相对电池的各电势差，再按相同比例画高度；排序使用同样规则。

$$\Delta V = IR, \qquad \Delta V_{\text{series}} = \Delta V_1 + \Delta V_2.$$

例题 2024 Q3 b

本问承接 2024 Q3 a(i)、2024 Q3 a(ii)。

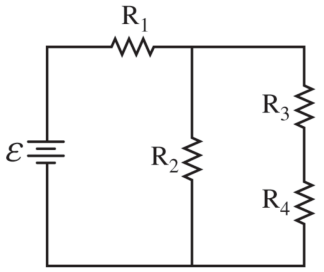


Figure 1

3. (12 points, suggested time 25 minutes)

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors R_1, R_2, R_3, and R_4, each of resistance R, as shown in Figure 1.

(b) The partially completed bar chart in Figure 2 shows a bar that represents the absolute value $|\Delta V|$ of the potential difference across the ideal battery.

- In Figure 2, **draw** a bar to represent $|\Delta V|$ across each resistor, relative to the emf $\mathcal{E}$ of the ideal battery.
- The height of each bar should be proportional to the value of $|\Delta V|$ represented by that bar. If $|\Delta V|$ is zero, write a "0" in that column.

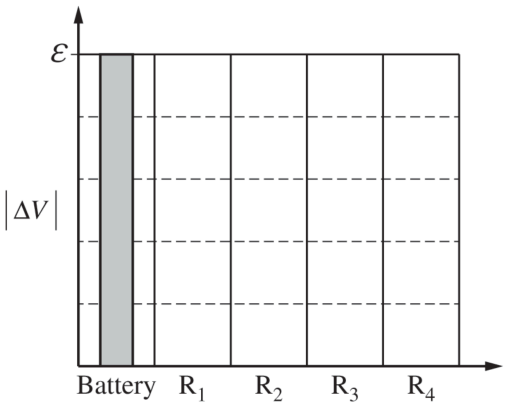
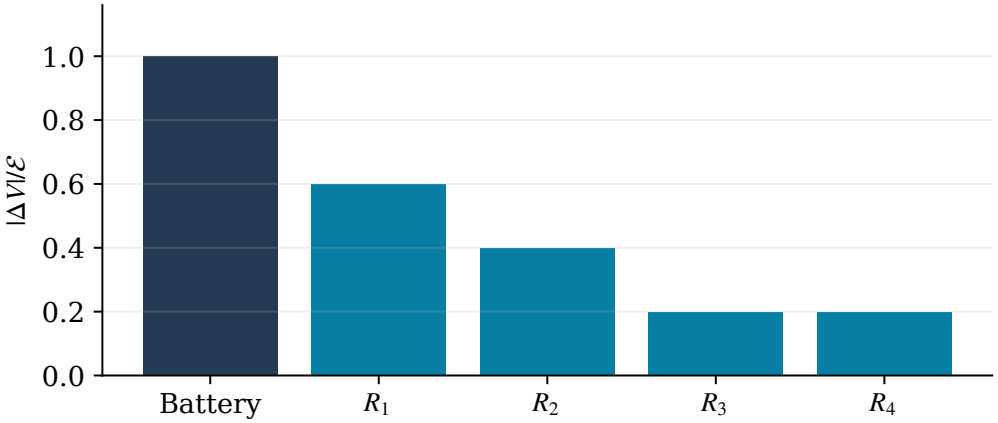


Figure 2

解：

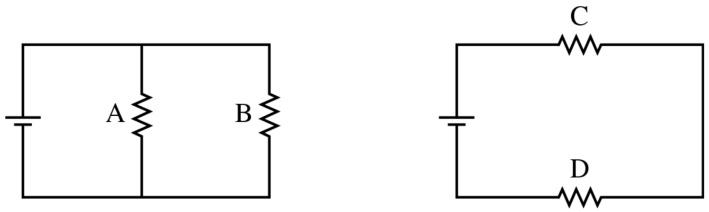
R2 与整个 R3、R4 串联支路连接同两节点，所以电势差相同。R3、R4 阻值相同且串联，把支路电势差平分。

$$|\Delta V_1| = \frac{3}{5}\mathcal{E}, \quad |\Delta V_2| = \frac{2}{5}\mathcal{E}, \quad |\Delta V_3| = |\Delta V_4| = \frac{1}{5}\mathcal{E}.$$



每个柱高以电池电动势为单位。R2 的柱高等于 R3 与 R4 柱高之和。

练习 2022 Q2 a



2. (12 points, suggested time 25 minutes)

Students perform an experiment with a battery and four resistors, A, B, C, and D. The resistance of resistors A and C is $R_A = R_C = R$. The resistance of resistors B and D is $R_B = R_D = 2R$. The students create the two circuits shown above and measure the potential differences ΔV_A, ΔV_B, ΔV_C, and ΔV_D across resistors A, B, C, and D, respectively.

(a) From greatest to least, rank the magnitudes of the potential differences across the resistors. Use "1" for the greatest magnitude, "2" for the next greatest magnitude, and so on. If any potential differences have the same magnitude, use the same number for their ranking.

____ΔV_A ____ΔV_B ____ΔV_C ____ ΔV_D

Justify your answer.

用电池输出核对电路总功率

功率是每秒转移的能量。理想电池每单位电荷提供电动势$\mathcal{E}$，电流I表示每秒通过的电荷量，所以输出功率为$\mathcal{E}$I；理想电路中这等于各电阻耗散功率的总和。把已求出的主干电流代入，逐项核对待评价表达式。

$$P = \mathcal{E}I = \frac{\mathcal{E}^2}{R_{\text{total}}}.$$

例题 2024 Q3 c

本问承接 2024 Q3 a(i)。

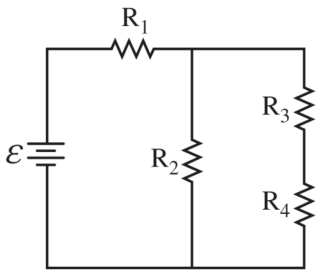


Figure 1

3. (12 points, suggested time 25 minutes)

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors R_1, R_2, R_3, and R_4, each of resistance R, as shown in Figure 1.

A student claims that the rate at which energy is dissipated (power) by the circuit can be expressed as $P = \frac{3\mathcal{E}^2}{5R}$.

(c) **State** whether the expression for P is correct or incorrect. **Justify** your answer by referring to the derivations from part (a) or the bar chart from part (b).

解：

表达式正确。理想电池的全部输出功率在电阻中耗散；引用 (a) 的总电流：

$$P = \mathcal{E}I_1 = \mathcal{E}\frac{3\mathcal{E}}{5R} = \frac{3\mathcal{E}^2}{5R}.$$

内阻加入后：比较外电阻功率

真实电池的内阻 r 与外电路串联。电动势相同，加入正内阻使总阻值增大、主干电流减小；外电路两端的电势差不再等于电动势。对于同一个外电阻，P=I^2R，电流减小便使耗散功率减小。

$$I = \frac{\mathcal{E}}{r + R_{\text{ext}}}, \qquad P_R = I^2 R.$$

例题 2024 Q3 d

本问承接 2024 Q3 a(i)。

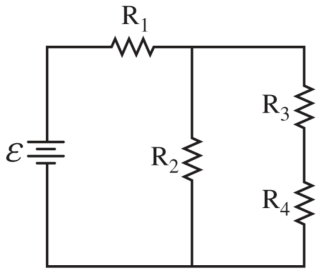


Figure 1

3. (12 points, suggested time 25 minutes)

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors R_1, R_2, R_3, and R_4, each of resistance R, as shown in Figure 1.

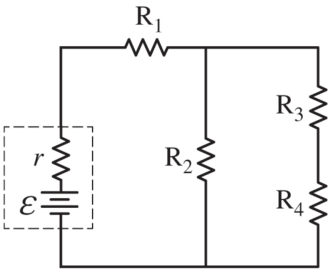


Figure 3

When the ideal battery is connected in the original circuit, the rate at which energy is dissipated by Resistor R_1 is P_{original}. The ideal battery is now replaced with a nonideal battery of emf $\mathcal{E}$ and internal resistance r to form the new circuit shown in Figure 3. The rate at which energy is dissipated by Resistor R_1 in the new circuit is P_{new}.

(d) **Indicate** whether P_{new} is greater than, less than, or equal to P_{original}.

_____ $P_{\text{new}} > P_{\text{original}}$ _____ $P_{\text{new}} < P_{\text{original}}$ _____ $P_{\text{new}} = P_{\text{original}}$

Briefly **justify** your answer.

解：

新电池电动势相同，但正的内阻 r 与外电路串联，总电阻增加，R1 电流下降。R1 阻值不变，功率随电流平方减小：

$$I_{new} = \frac{\mathcal{E}}{r + 5R/3} < \frac{\mathcal{E}}{5R/3}, \qquad P_{new} = I_{new}^2 R < P_{original}.$$

不能把电动势相同理解成外电路端电压仍相同；一部分电势差落在内阻上。

电流表串联，电压表并联

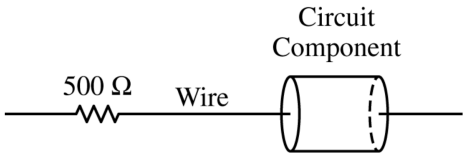
电流表要测“流过”的电荷量，必须成为被测支路的一部分；电压表要比较两个点的电势，接在这两个点之间。接线前先标被测元件的两端，画完整闭合回路。把电流表直接跨接电源或把电压表串进主干，都不能完成原测量目标。

例题 2023 Q2 a(i)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance $500\,\Omega$.

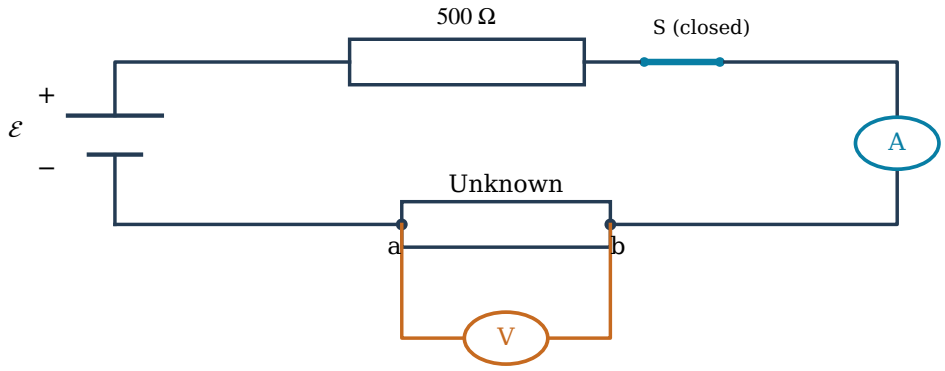
(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.



解：

将电池、开关、电流表、500 Ω 电阻和未知元件串联成闭合回路。电流表必须在主干内，测流过元件的电流；若使用电压表，应并联在未知元件两端。



A: series in the main loop V: across nodes a and b

电流表串联；电压表并联在待测元件两端。

闭合后持续观察：区分电阻与电容

未充电电容起初两端电势差为零，随后电荷堆积，使电容电势差增加。电池总电势差不变，电阻分到的电势差与电流就逐渐减小。电阻则维持稳定电流。实验必须既测刚闭合时，也测较长时间后；每次重复前让电容放电，保证初始状态相同。

例题 2023 Q2 a(ii)

本问承接 2023 Q2 a(i)。

Students are given an unknown circuit component that is connected in series to a resistor with known resistance $500\ \Omega$.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

解：

先让元件初始未充电，闭合开关，记录刚接通时的电流，并在较长时间后再次读电流。每次重复前断开并使电容放电，保证相同初始状态。电阻接入后产生持续稳定电流；未充电电容则产生逐渐衰减到零的电流。

电容充电结果：用电荷和电势差解释电流变化

记录到电流变化后，还要解释原因。未充电时电容q=0，电容两端电势差为零。电荷逐渐堆积使ΔVC=q/C增大，固定电池电势差里留给电阻的份额就减少；IR变小，电流也变小，最终趋于零。

$$\mathcal{E} = IR + \Delta V_C, \qquad \Delta V_C = \frac{q}{C}.$$

例题 2023 Q2 a(iii)

本问承接 2023 Q2 a(i)、2023 Q2 a(ii)。

Students are given an unknown circuit component that is connected in series to a resistor with known resistance $500\,\Omega$.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

解：

初始电容两端电势差为零，电荷经回路移到两板，电容电荷增大，其电势差随之增加。电池总电势差固定，电容占的份额增加使电阻电势差和电流减小；长时间后电容电势差等于电池电势差，电阻两端电势差为零，电流趋于零。

$$\mathcal{E} = IR + \Delta V_C, \qquad q = C\Delta V_C.$$

已知电容和自身电压：计算电荷

电容 C 是储存电荷量大小与自身两端电势差的比。用电容自己两端的电势差，不要把串联整个回路的电池电压给每个元件。微法拉μF要换成10^{-6} F，才能得到以库仑为单位的电荷。

$$q = C\Delta V.$$

例题 2022 Q2 b(i)

In another experiment, the students have a capacitor with unknown capacitance C_{U}. They want to determine C_{U} by using a battery of potential difference 4.5 V and several other capacitors of known capacitance. They create circuits with the battery, the unknown capacitor, and one of the capacitors of known capacitance. The students wait until the capacitors are fully charged and then record the potential difference $\Delta V_{\mathrm{known}}$ across the known capacitor and the potential difference ΔV_{U} across the unknown capacitor. Their data are shown in the table on the following page.

Known Capacitance of Capacitors (μF)	$\Delta V_{\mathrm{known}}$ (V)	ΔV_{U} (V)		
200	0.91	3.53		
300	0.65	3.74		
400	0.51	3.95		
500	0.42	4.06		
600	0.36	4.17		

i. Calculate the amount of charge on the capacitor of known capacitance of 200 μF in the students' experiment.

解：

用已知电容自身的电势差求电荷，而不是整节电池的 4.5 V：

$$Q = C_{known}\Delta V_{known} = (200 \times 10^{-6}\ \mathrm{F})(0.91\ \mathrm{V}) = 1.82 \times 10^{-4}\ \mathrm{C}.$$

用电压数据判断串联，说明并联测量的不足

串联电容中间节点不能从电源取得净电荷，两个电容两板储存的电荷大小相等；总电势差分配到两个电容。并联电容则各自直接连接电池两端，电势差相同，但电荷可能不同。只有电压读数时，并联并不能告诉未知电容储了多少电荷，因而无法由q/ΔV确定它。

例题 2022 Q2 b(ii)

In another experiment, the students have a capacitor with unknown capacitance C_U. They want to determine C_U by using a battery of potential difference 4.5 V and several other capacitors of known capacitance. They create circuits with the battery, the unknown capacitor, and one of the capacitors of known capacitance. The students wait until the capacitors are fully charged and then record the potential difference ΔV_{known} across the known capacitor and the potential difference ΔV_U across the unknown capacitor. Their data are shown in the table on the following page.

Known Capacitance of Capacitors (μF)	ΔV_{known} (V)	ΔV_U (V)		
200	0.91	3.53		
300	0.65	3.74		
400	0.51	3.95		
500	0.42	4.06		
600	0.36	4.17		

ii. Briefly explain why the data in the table provide evidence that the capacitors are connected in series.

解：

同一行的两个电容电势差不同，而二者之和接近电池的 4.5 V，符合串联分压；并联连接相同两节点，电势差应相同。

练习 2022 Q2 b(iii)

In another experiment, the students have a capacitor with unknown capacitance C_U. They want to determine C_U by using a battery of potential difference 4.5 V and several other capacitors of known capacitance. They create circuits with the battery, the unknown capacitor, and one of the capacitors of known capacitance. The students wait until the capacitors are fully charged and then record the potential difference ΔV_{known} across the known capacitor and the potential difference ΔV_U across the unknown capacitor. Their data are shown in the table on the following page.

Known Capacitance of Capacitors (μF)	ΔV_{known} (V)	ΔV_{U} (V)		
200	0.91	3.53		
300	0.65	3.74		
400	0.51	3.95		
500	0.42	4.06		
600	0.36	4.17		

iii. Briefly explain why connecting the capacitors in parallel would not provide enough information to determine the capacitance of the unknown capacitor if the only measuring device available is a voltmeter.

没有计时器时：分别测 R 与 C

时间常数τ=RC表征电容充电快慢，不一定要直接用时间测量。给定电池、电流表与尺时，可用纯电阻回路的电流求R，用极板面积和板间距求空气平行板电容C。测正方形边长后平方得到面积，重复测量取平均；空气介电常数按题设取1。

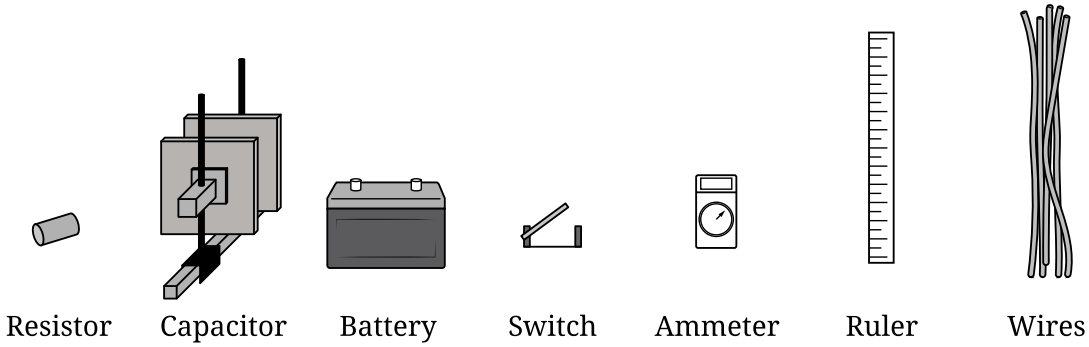
$$\tau = RC, \qquad R = \frac{\mathcal{E}}{I}, \qquad C = \frac{\varepsilon_0 A}{d}.$$

例题 2025 Q3 A

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3. In Experiment 1, a student is given a resistor of unknown resistance and an air-filled parallel-plate capacitor of unknown capacitance. The student is asked to predict the expected time constant τ of a circuit if these two circuit elements were connected in series with a battery. The student has access to a battery of known emf, a switch, an ammeter, a ruler, and wires, as shown in Figure 1. The plates of the capacitor are square, and the separation between the plates is small compared to the dimensions of the plates. The capacitor is initially uncharged. Assume that the dielectric constant of air is 1.

Figure 1



Note: Figure not drawn to scale.

A. Describe a procedure for collecting data that would allow the student to determine the expected time constant τ. In your description, include the measurements to be made. Include any steps necessary to reduce experimental uncertainty.

解：

要预测时间常数，可分别求电阻 R 与电容 C。题目没有计时器，所以采用不直接计时的方法。用尺多次测正方形极板边长 a 和板间距 d，取平均。把电池、开关、电流表和未知电阻串联，暂不接电容；闭合开关测稳定电流 I，重复读数取平均并避免明显升温改变电阻。

由测量结果分别求电阻、电容和时间常数

测完正方形边长a、极板距离d和纯电阻回路电流I后，逐项把测量量换成目标物理量。面积是a^2；电阻用电池电动势除电流，电容用真空介电常量乘面积除板距。得到R、C后相乘才是时间常数τ，所有长度先用米。

$$R=\frac{\mathcal{E}}{I},\qquad C=\frac{\varepsilon_0 a^2}{d},\qquad \tau=RC.$$

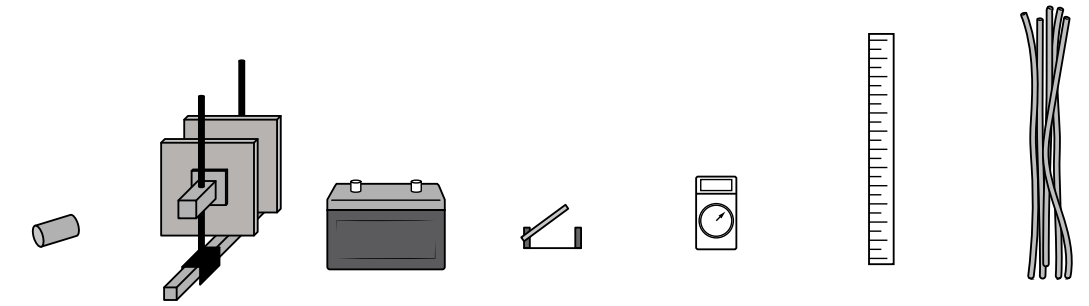
例题 2025 Q3 B

本问承接 2025 Q3 A。

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3. In Experiment 1, a student is given a resistor of unknown resistance and an air-filled parallel-plate capacitor of unknown capacitance. The student is asked to predict the expected time constant τ of a circuit if these two circuit elements were connected in series with a battery. The student has access to a battery of known emf, a switch, an ammeter, a ruler, and wires, as shown in Figure 1. The plates of the capacitor are square, and the separation between the plates is small compared to the dimensions of the plates. The capacitor is initially uncharged. Assume that the dielectric constant of air is 1.

Figure 1



Note: Figure not drawn to scale.

B. **Describe** how the collected data could be analyzed to determine τ. Include references to appropriate equations and to relationships between measured and known quantities.

解：

面积是边长平方，空气介电常数按题设取 1，边缘效应因板距远小于边长可忽略。理想电池电势差全部落在纯电阻上：

$$A = a^2, \qquad C = \varepsilon_0 \frac{a^2}{d}, \qquad R = \frac{\mathcal{E}}{I}, \qquad \tau = RC = \frac{\mathcal{E}}{I}\frac{\varepsilon_0 a^2}{d}.$$

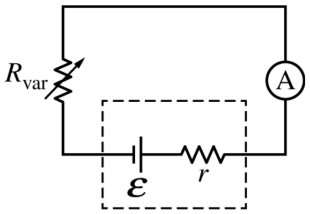
长度用米，电流用安培，结果单位为秒。若接入电容测电阻，应使用刚接通时的初始电流，而不能取任意充电时刻的电流。

含内阻的电路：先列回路电压账本

绕完整闭合回路，电池使电势升高$\mathcal{E}$，内阻与外电阻各使电势降低IR。它们串联并流过同一电流；一圈回到起点，升降总和为零。

$$\mathcal{E} - Ir - IR_{\text{var}} = 0.$$

例题 2023 Q2 b(i)



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\,\Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)		
0.087	200		
0.060	300		
0.042	450		
0.027	700		
0.016	1200		

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

解：

内阻 r 与可变外电阻串联，二者流过同一电流。绕回路电势升高和降低相抵：

$$\mathcal{E} - Ir - IR_{var} = 0, \qquad \mathcal{E} = I(r + R_{var}).$$

从 q=CΔV 选线性坐标

同一电容的C固定，电荷与其电势差成正比。横轴电势差、纵轴电荷时斜率就是C；换轴后斜率代表倒数，不能混用。若是串联未知电容，先用已知电容及其电压计算共同的电荷，再与未知电容的电压配成坐标。

$$q_U = q_{\text{known}} = C_{\text{known}}\Delta V_{\text{known}}, \qquad q_U = C_U \Delta V_U.$$

例题 2025 Q3 C(i)

C. In Experiment 2, the student is asked to determine the capacitance *C* of a new parallel-plate capacitor. For each trial, the absolute value $|\Delta V|$ of the potential difference across the capacitor is varied and the charge *q* stored on one plate of the fully charged capacitor is measured. Table 1 contains the data collected.

Table 1

$\lvert\Delta V\rvert$ (V)	$q\ (\times 10^{-10}\ \text{C})$
3.0	2.4
5.0	4.2
7.2	5.6
8.0	6.6
10.0	8.0

i. **Indicate** two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine *C*.

Vertical axis: _____ Horizontal axis: _____

解：

C(i) 电容的电荷与两端电势差成正比。横轴用 ΔV（V），纵轴用 q（C），斜率就是 C。

练习 2022 Q2 c(i)

本问承接 2022 Q2 b(ii)。

In another experiment, the students have a capacitor with unknown capacitance C_{U}. They want to determine C_{U} by using a battery of potential difference $4.5\ \mathrm{V}$ and several other capacitors of known capacitance. They create circuits with the battery, the unknown capacitor, and one of the capacitors of known capacitance. The students wait until the capacitors are fully charged and then record the potential difference $\Delta V_{\mathrm{known}}$ across the known capacitor and the potential difference ΔV_{U} across the unknown capacitor. Their data are shown in the table on the following page.

Known Capacitance of Capacitors (μF)	$\Delta V_{\mathrm{known}}$ (V)	ΔV_{U} (V)		
200	0.91	3.53		
300	0.65	3.74		
400	0.51	3.95		
500	0.42	4.06		
600	0.36	4.17		

(c) The students want to produce a linear graph of the data so that the capacitance C_{U} of the unknown capacitor can be determined from the slope of the best-fit line for the data.

i. Indicate two quantities that could be plotted to produce the desired graph. Use the empty columns of the data table in part (b) to record any values that you need to calculate.

Vertical axis _____ Horizontal axis _____

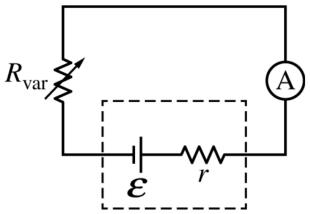
反求电动势：总电阻先加再取倒数

内阻已知时，回路关系I=$\mathcal{E}$/(r+Rvar)。固定的$\mathcal{E}$乘总阻值倒数，得到电流，因此横轴选总阻值倒数、纵轴选电流能得到直线。先加内阻再取倒数；只取外阻倒数会漏掉题设内阻。

$$I = \mathcal{E}\left(\frac{1}{r + R_{\mathrm{var}}}\right).$$

例题 2023 Q2 b(ii)

本问承接 2023 Q2 b(i)。



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\,\Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)		
0.087	200		
0.060	300		
0.042	450		
0.027	700		
0.016	1200		

ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis:_____ Vertical Axis:_____

解：

求电动势可选总电阻的倒数为横轴 x，单位 Ω^{-1}；纵轴 I，单位 A。因为：

$$I = \mathcal{E}\left(\frac{1}{r + R_{var}}\right),$$

直线斜率为电动势。先把 30 Ω 内阻加到外电阻，再取倒数；只用 1/Rvar 不是本题的线性模型。

磁场、带电粒子与电磁感应

长直导线磁场与导线受力方向

先按原坐标确定电流和观察点的位置。求长直导线产生的磁场时，右手拇指沿正电流，四指环绕的方向就是磁场方向；进入纸面用×，离开纸面用点表示。求另一根导线受到的磁力时，右手四指先沿这根导线的正电流方向伸直，再向它所在位置的磁场方向弯曲，伸出的拇指就是磁力方向。两种手势中拇指的作用不同：求磁场时拇指指电流，求磁力时拇指指力。本题先把拇指向右，观察上方导线下侧的环绕方向，得到磁场进入纸面；再把四指向右伸直并向纸内弯曲，拇指指向上方，因此下方导线受力向上。

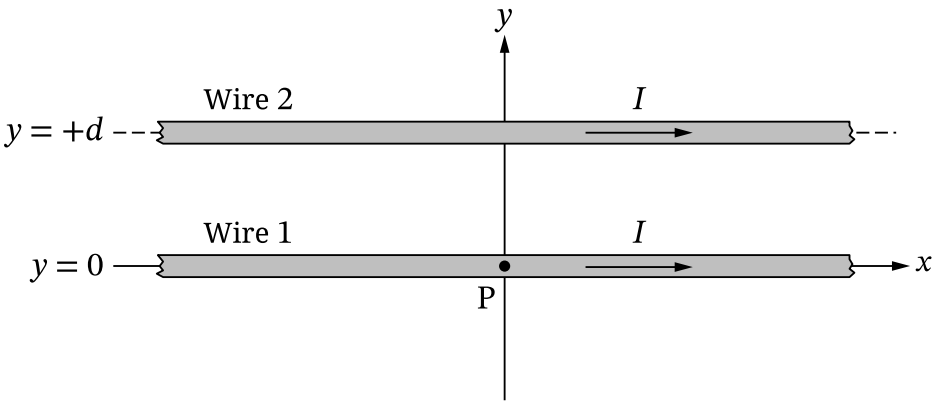
$$\boldsymbol{F} = I\ell \times \boldsymbol{B}.$$

例题 2025 Q1 A(i)

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1. Very long Wire 1 carries current I in the $+x$-direction along the line $y = 0$. Very long Wire 2 carries current I in the $+x$-direction along the line $y = +d$. Point P is located along Wire 1 at the origin, as shown in Figure 1. The diameters of the wires are small compared to the distance between the wires. Both wires are in the xy-plane.

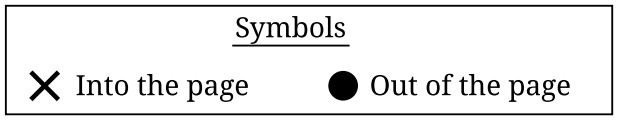
Figure 1



Note: Figure not drawn to scale.

i. Complete the following tasks in figures 2 and 3. Use either arrows or the symbols shown in the box above the figures for your response.

- **Indicate** the direction of the magnetic field from Wire 2 at Point P in Figure 2.
- **Indicate** the direction of the magnetic force that is exerted on Wire 1 by Wire 2 in Figure 3.



Magnetic Field from Wire 2 at Point P

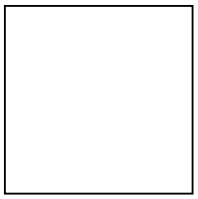


Figure 2

Magnetic Force on Wire 1 by Wire 2

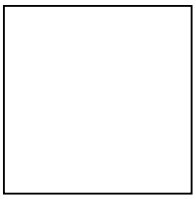


Figure 3

解：

沿原坐标 +x 向右、+y 向上、+z 朝纸外。导线2电流沿 +x，P 在它下方。右手拇指沿电流，四指指向纸内，所以 B2 在 P 朝 −z。导线1电流也沿 +x，$I\ell \times B$ 指向 +y；同向电流相吸，力指向上方导线2。

两导线力抵消：先确定第三根的位置侧

同向电流相吸。要抵消上方导线的向上吸引，第三根同向导线必须位于下方。比较同一段目标导线，I 和长度相同，力大小抵消等价于两外场大小相等。磁场随电流增大，随距离增大而减小；最后把正的距离改回带符号坐标。

$$B = \frac{\mu_0 I}{2\pi r}.$$

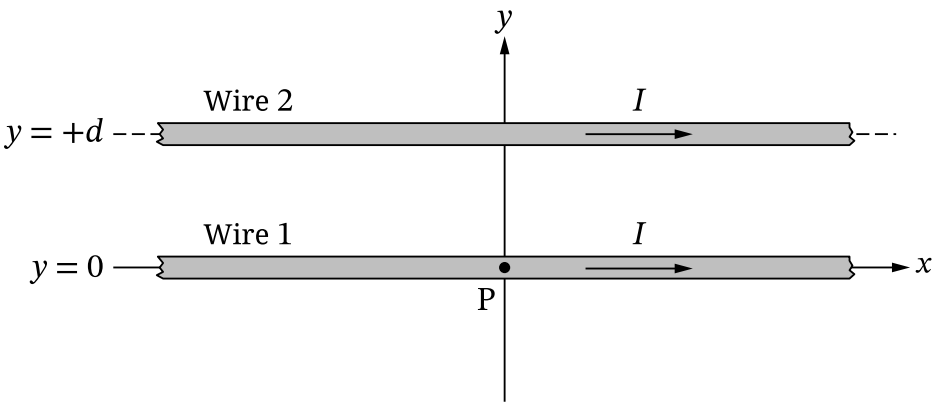
例题 2025 Q1 A(ii)

本问承接 2025 Q1 A(i)。

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1. Very long Wire 1 carries current I in the $+x$-direction along the line $y = 0$. Very long Wire 2 carries current I in the $+x$-direction along the line $y = +d$. Point P is located along Wire 1 at the origin, as shown in Figure 1. The diameters of the wires are small compared to the distance between the wires. Both wires are in the xy-plane.

Figure 1



Note: Figure not drawn to scale.

ii. Very long Wire 3 carrying current $2I$ in the $+x$-direction is placed in the xy-plane along the line $y = y_3$. The net magnetic force exerted on Wire 1 by the currents in wires 2 and 3 is zero.

Derive an expression for y_3 in terms of d. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

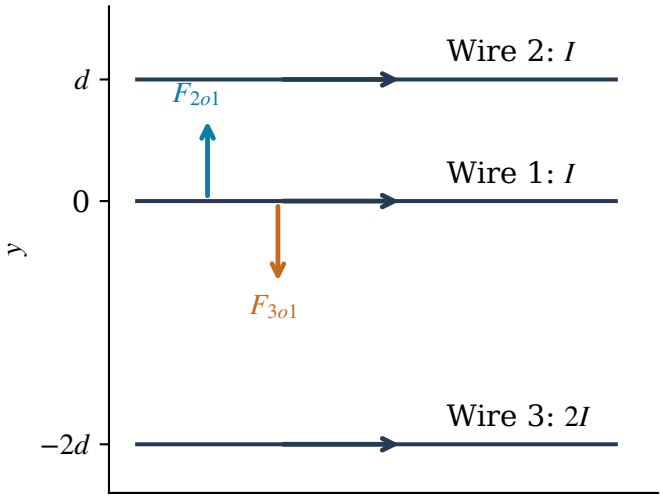
解：

第三根电流同向，也吸引导线1。要抵消导线2向上的力，第三根必须放在原点下方。取导线1同一段长度，其电流和长度相同，抵消力等价于抵消场大小：

$$B_2 = \frac{\mu_0 I}{2\pi d}, \qquad B_3 = \frac{\mu_0 (2I)}{2\pi |y_3|}, \qquad B_2 = B_3.$$

$$\frac{I}{d} = \frac{2I}{|y_3|}, \qquad |y_3| = 2d, \qquad y_3 = -2d.$$

最后一步必须结合方向：2d 是距离，不是带符号坐标。

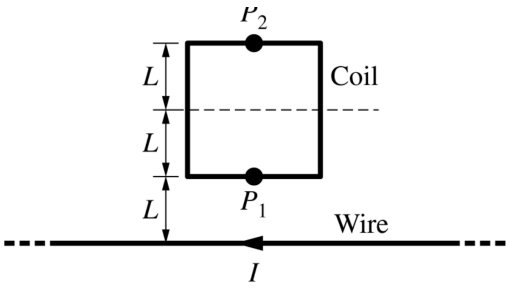


长直导线场强的反比位置

同一根导线电流不变，Br是常量。要找场强为某数值的位置，比较两个状态的B乘距离，再把距离落在原图上；距离从导线量起，不能从线圈边缘量起。

$$B_1 r_1 = B_2 r_2.$$

例题 2022 Q4 b(i)



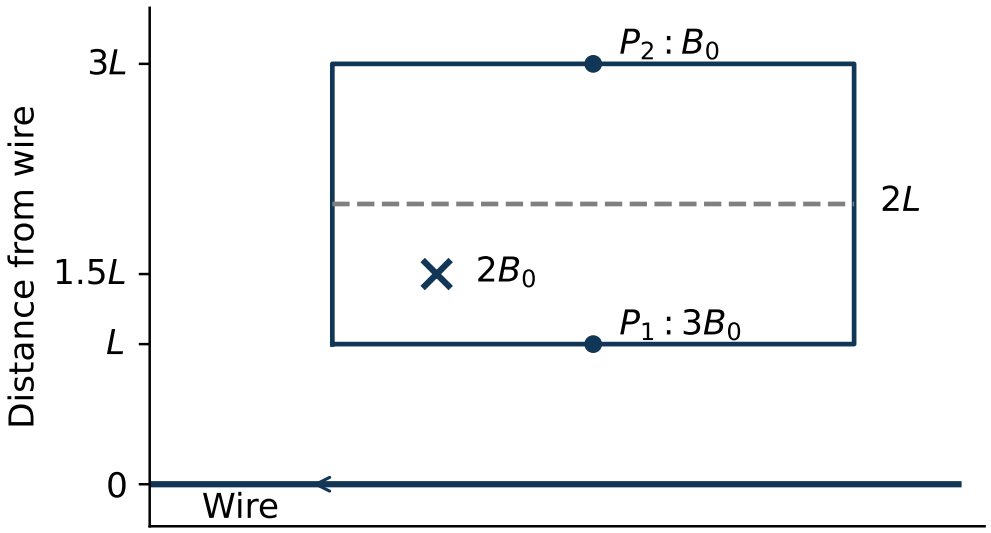
(b) The charged object is removed, and a square coil with side length $2L$ is placed near the long, straight wire, as shown above. The bottom of the coil is a distance L from the wire. The magnitude of the magnetic field due to the current in the wire is $3B_0$ at point P_1 and B_0 at point P_2.

i. Write an "X" at a location on the figure where the magnitude of the magnetic field is $2B_0$. Briefly justify your reasoning.

解：

同一导线 B 与距离成反比。离线 L 处为 3B0；要降到 2B0，距离应增为原来的 3/2。因此在离导线 1.5L 的水平位置标 X，处于 P1 和中间虚线之间。

$$(3B_0)L = (2B_0)r, \qquad r = 1.5L.$$



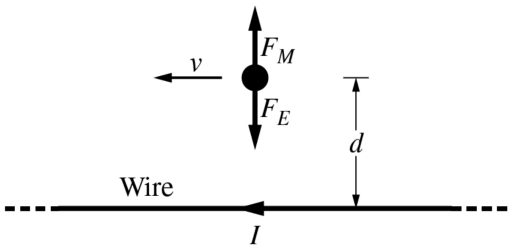
原图中线在2L，场强2B0的位置在1.5L，处于P1与中线之间。

不偏转：电力与磁力平衡求速度

匀速或不偏转并忽略重力时，题示相反的电力和磁力大小相等。速度与磁场垂直，磁力大小为|q|vB，电力大小为|q|E；电荷量能消去，因此无需猜电荷数值。长直导线磁场则要用粒子所在处的距离。

$$|q|E = |q|vB, \qquad v = \frac{E}{B}.$$

例题 2022 Q4 a



(10 points, suggested time 20 minutes)

At the instant shown above, a negatively charged object is moving to the left with constant velocity v near a long, straight wire that has a current I directed to the left. The region contains a uniform electric field of magnitude E, and the charged object is at a distance d from the wire. The figure shows the electric and magnetic forces, F_E and F_M, respectively, exerted on the charged object.

(a) Derive an expression for v in terms of E, d, I, and physical constants, as appropriate.

解：

带电物体匀速，题示两力相反，所以合力为零。以 |q| 表示电荷量大小，速度与磁场垂直：

$$F_E = F_M, \qquad |q|E = |q|vB, \qquad v = \frac{E}{B}.$$

物体距离长直导线为 d，用该处磁场表达式：

$$B = \frac{\mu_0 I}{2\pi d}, \qquad v = \frac{2\pi d E}{\mu_0 I}.$$

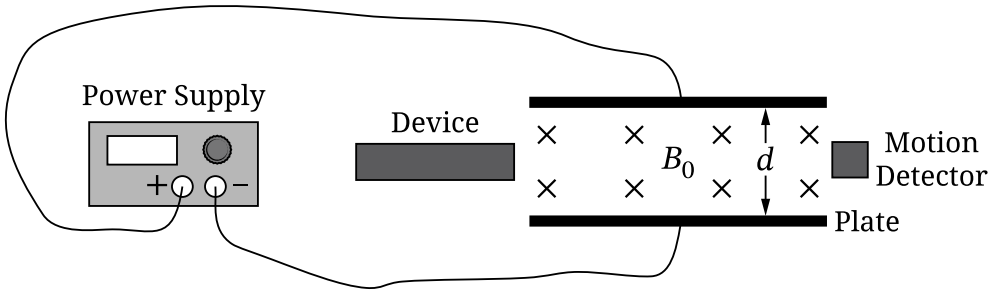
不偏转实验：用已有仪器选成对测量量

原设备已能测速度，另有电压表，板距已知。对每个速度调节板间电压到粒子不偏转，再记录一对速度与板间电压。电压表跨接两板；不能添加题目没有提供的磁场计或其他传感器。同一条件重复读数、换多个条件重复过程，减小个别读数误差。

例题 2026 Q3 A(i)

3. In Experiment 1, scientists want to collect data that can be graphed to determine the magnitude B_0 of an external, uniform magnetic field. The field is directed into Figure 1 in the region between identical, parallel, conducting plates that are connected to a power supply of variable emf $\mathcal{E}$. The plates are a known distance d apart, where d is much smaller than the dimensions of the plates.

Figure 1



Note: Figure not drawn to scale.

In each trial, a device emits small, charged spheres with speed v through the region between the plates. $\mathcal{E}$ is varied until the spheres move undeflected toward a motion detector. The scientists can vary v between trials. Gravitational effects are negligible.

In addition to the equipment shown in Figure 1, the scientists also have access to a voltmeter. The scientists do not have access to other measuring tools, devices, sensors, or equipment.

i. **Indicate** quantities that could be measured using the available equipment that would allow the scientists to determine B_0 by using a linear graph.

解：

实验1用运动探测器测每次不偏转时的速度 v，用电压表并接两板测板间电势差 $\mathcal{E}$。对多个不同速度调节 $\mathcal{E}$ 直到小球不偏转，再记录成对的 (v,$\mathcal{E}$)。板距 d 已知，无须新增题目没有提供的磁场计。

重复不偏转测量，减小随机读数误差

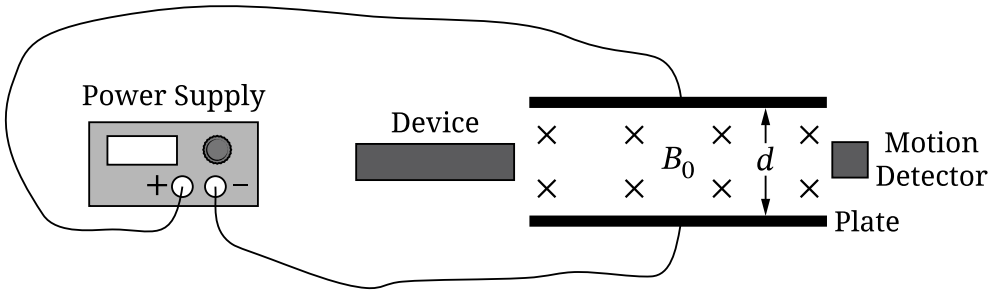
同样的速度条件下多次调到不偏转并读取板间电压，把重复读数取平均，可减小某一次随机偏差的影响。再换多个速度重复成对读数，使随后直线由多组数据决定。不能把改换仪器或增加题目没有的设备当成误差控制。

例题 2026 Q3 A(ii)

本问承接 2026 Q3 A(i)。

3. In Experiment 1, scientists want to collect data that can be graphed to determine the magnitude B_0 of an external, uniform magnetic field. The field is directed into Figure 1 in the region between identical, parallel, conducting plates that are connected to a power supply of variable emf $\mathcal{E}$. The plates are a known distance d apart, where d is much smaller than the dimensions of the plates.

Figure 1



Note: Figure not drawn to scale.

In each trial, a device emits small, charged spheres with speed v through the region between the plates. $\mathcal{E}$ is varied until the spheres move undeflected toward a motion detector. The scientists can vary v between trials. Gravitational effects are negligible.

In addition to the equipment shown in Figure 1, the scientists also have access to a voltmeter. The scientists do not have access to other measuring tools, devices, sensors, or equipment.

ii. Briefly **describe** a method to reduce experimental uncertainty for the measured quantities indicated in part A (i).

解：

同一设定重复测 v、$\mathcal{E}$ 取平均，再换多个速度重复“不偏转—测量”，以多组数据拟合减少个别读数误差。

平行板与速度选择：线性图求磁场

两块很大的平行板间距d远小于板尺寸，中央电场近似匀强，E=$\mathcal{E}$/d。不偏转条件E=vB0把已测的板间电压与速度连接起来，得到$\mathcal{E}$=B0dv。选电压为纵轴、速度为横轴，斜率包含已知板距；求B0时再除以d。

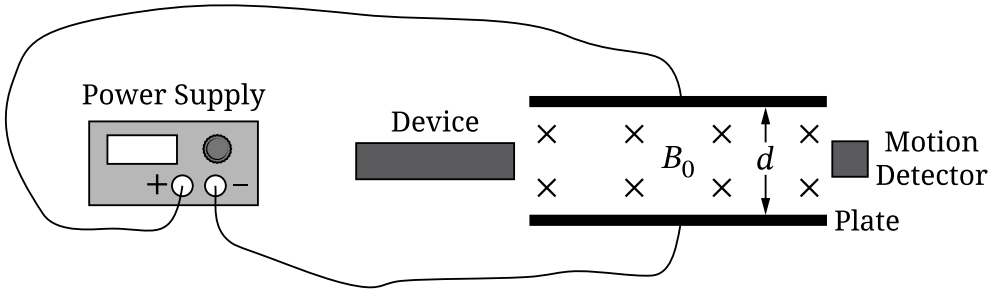
$$\mathcal{E} = B_0 d v, \qquad s = B_0 d, \qquad B_0 = \frac{s}{d}.$$

例题 2026 Q3 B(i)

本问承接 2026 Q3 A(i)。

3. In Experiment 1, scientists want to collect data that can be graphed to determine the magnitude B_0 of an external, uniform magnetic field. The field is directed into Figure 1 in the region between identical, parallel, conducting plates that are connected to a power supply of variable emf $\mathcal{E}$. The plates are a known distance d apart, where d is much smaller than the dimensions of the plates.

Figure 1



Note: Figure not drawn to scale.

In each trial, a device emits small, charged spheres with speed v through the region between the plates. $\mathcal{E}$ is varied until the spheres move undeflected toward a motion detector. The scientists can vary v between trials. Gravitational effects are negligible.

In addition to the equipment shown in Figure 1, the scientists also have access to a voltmeter. The scientists do not have access to other measuring tools, devices, sensors, or equipment.

i. **Indicate** what quantities the scientists could graph on the horizontal and vertical axes to create a linear graph that can be used to determine B_0. Clearly indicate which quantity corresponds to each axis.

解：

B(i) 不偏转且忽略重力，电力和磁力大小相等。板间近似匀强场：

$$|q|E = |q|vB_0, \qquad E = \frac{\mathcal{E}}{d}, \qquad \mathcal{E} = B_0 d v.$$

横轴 v（m/s），纵轴 $\mathcal{E}$（V），是理论上过原点的直线。

本问只选择横纵坐标：先把板间电场代入不偏转条件，再把所选两量排成 y=sx；下一小问才把斜率换成磁场大小。

电压对速度直线：斜率除板距得磁场

刚才选择纵轴板间电压、横轴速度。回到已推导的$\mathcal{E}$=B0dv，固定系数B0d乘速度，所以这张图的斜率s就是B0d。目标是B0而板距d已知，先两边除以d；若换了横纵轴，必须先重写自己的斜率关系。

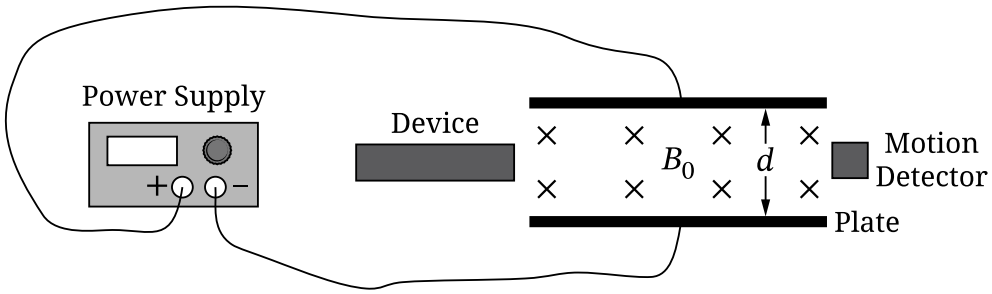
$$s = B_0 d, \qquad B_0 = \frac{s}{d}.$$

例题 2026 Q3 B(ii)

本问承接 2026 Q3 B(i)。

3. In Experiment 1, scientists want to collect data that can be graphed to determine the magnitude B_0 of an external, uniform magnetic field. The field is directed into Figure 1 in the region between identical, parallel, conducting plates that are connected to a power supply of variable emf $\mathcal{E}$. The plates are a known distance d apart, where d is much smaller than the dimensions of the plates.

Figure 1



Note: Figure not drawn to scale.

In each trial, a device emits small, charged spheres with speed v through the region between the plates. $\mathcal{E}$ is varied until the spheres move undeflected toward a motion detector. The scientists can vary v between trials. Gravitational effects are negligible.

In addition to the equipment shown in Figure 1, the scientists also have access to a voltmeter. The scientists do not have access to other measuring tools, devices, sensors, or equipment.

ii. Briefly **describe** the relationship between B_0 and one feature of the graph from part B (i). Your answer may include an equation that relates B_0 and the indicated feature of the graph.

解：

B(ii) 若斜率为 s，比较直线关系可知：

$$s = B_0 d, \qquad B_0 = \frac{s}{d}.$$

已知粒子动能与质量：解速率

动能只取决于质量和速率。先用题目给这一个粒子的实际质量写K=$mv^2/2$，再解正的速率。原题给M/2时，先把质量括起来代入，避免错当成2M。

$$v = \sqrt{\frac{2K}{m}}.$$

例题 2024 Q4 b(i)

4. (10 points, suggested time 20 minutes)

Two particles, 1 and 2, have different mass and charge as described by the following.

- Particle 1 has mass M and negative charge $-Q$.
- Particle 2 has mass $\frac{M}{2}$ and positive charge $+2Q$.

In separate trials, a device is used to accelerate each particle in the $-y$-direction from rest through a potential difference of absolute value $|\Delta V|$. The polarity of the potential difference can be adjusted so that a particle with either positive charge or negative charge can be accelerated in the $-y$-direction by the device. Gravitational effects are negligible.

After moving through the potential difference, particles 1 and 2 exit the device with kinetic energies K_1 and K_2, respectively.

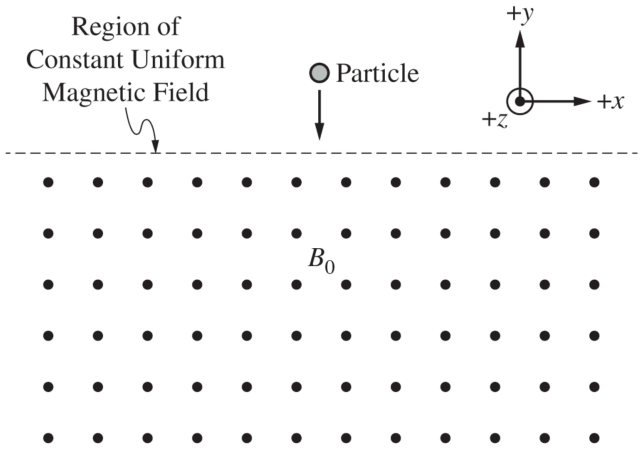


Figure 1

After exiting the device, the particles enter a large region of constant uniform magnetic field of magnitude B_0 that is directed in the $+z$-direction (out of the page), as shown in Figure 1. Each particle is moving in the $-y$-direction when entering the region, and each particle is moving in the $+y$-direction when exiting the region.

i. **Determine** an expression for the speed of Particle 2 in the region. Express your answer in terms of M, K_2, and physical constants, as appropriate.

解：

粒子2质量为 M/2，不能误读成 2M。由它自己的动能式求速度：

$$K_2 = \frac{1}{2}\left(\frac{M}{2}\right)v_2^2, \qquad v_2 = 2\sqrt{\frac{K_2}{M}}.$$

磁力作向心合力：半圆位移是直径

磁力始终垂直速度，不改变速率，只弯曲轨迹。它本身提供圆周运动所需的向心合力，“向心力”不是另加的一种力。先把磁力与mv^2/r相等，再代入这一个粒子的质量、速度和电荷大小。入口向下、出口向上表示走半圆，水平距离是2r。

$$|q|vB = \frac{mv^2}{r}, \qquad r = \frac{mv}{|q|B}.$$

例题 2024 Q4 b(ii)

本问承接 2024 Q4 b(i)。

4. (10 points, suggested time 20 minutes)

Two particles, 1 and 2, have different mass and charge as described by the following.

- Particle 1 has mass M and negative charge $-Q$.
- Particle 2 has mass $\frac{M}{2}$ and positive charge $+2Q$.

In separate trials, a device is used to accelerate each particle in the $-y$-direction from rest through a potential difference of absolute value $|\Delta V|$. The polarity of the potential difference can be adjusted so that a particle with either positive charge or negative charge can be accelerated in the $-y$-direction by the device. Gravitational effects are negligible.

After moving through the potential difference, particles 1 and 2 exit the device with kinetic energies K_1 and K_2, respectively.

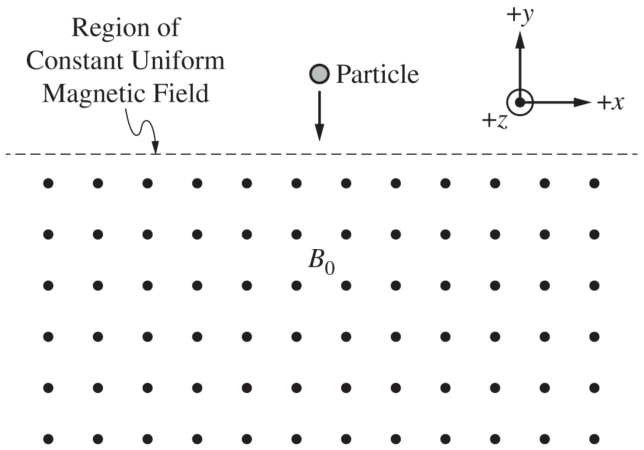


Figure 1

After exiting the device, the particles enter a large region of constant uniform magnetic field of magnitude B_0 that is directed in the $+z$-direction (out of the page), as shown in Figure 1. Each particle is moving in the $-y$-direction when entering the region, and each particle is moving in the $+y$-direction when exiting the region.

ii. **Derive** an expression for the horizontal distance Δx between the locations where Particle 2 enters and leaves the region. Express your answer in terms of M, Q, K_2, B_0, and physical constants, as appropriate.

解：

纯磁场中磁力垂直速度，不改变速率。它提供圆轨道所需的向心合力；“向心力”不是另加的一项力。

$$|q|vB_0 = \frac{mv^2}{r}, \qquad r = \frac{mv}{|q|B_0}.$$

$$r_2 = \frac{(M/2)\,2\sqrt{K_2/M}}{(2Q)B_0} = \frac{\sqrt{MK_2}}{2QB_0}.$$

粒子进入向下、离开向上，在图示区域走半圆；水平位移大小是直径，不是半径：

$$\Delta x = 2r_2 = \frac{\sqrt{MK_2}}{QB_0}.$$

不同符号电荷：先转向，再比较半径

先按正电荷的速度叉乘磁场求力方向，负电荷反向。力决定最初向哪侧弯；质量、速率、电荷大小共同决定弯曲半径。前面已比较加速所得动能，代回各自质量求速率后，再比较mv/(|q|B)，不能只凭电荷正负猜半径。

例题 2024 Q4 c

本问承接 2024 Q4 a、2024 Q4 b(i)、2024 Q4 b(ii)。

4. (10 points, suggested time 20 minutes)

Two particles, 1 and 2, have different mass and charge as described by the following.

- Particle 1 has mass M and negative charge $-Q$.
- Particle 2 has mass $\frac{M}{2}$ and positive charge $+2Q$.

In separate trials, a device is used to accelerate each particle in the $-y$-direction from rest through a potential difference of absolute value $|\Delta V|$. The polarity of the potential difference can be adjusted so that a particle with either positive charge or negative charge can be accelerated in the $-y$-direction by the device. Gravitational effects are negligible.

After moving through the potential difference, particles 1 and 2 exit the device with kinetic energies K_1 and K_2, respectively.

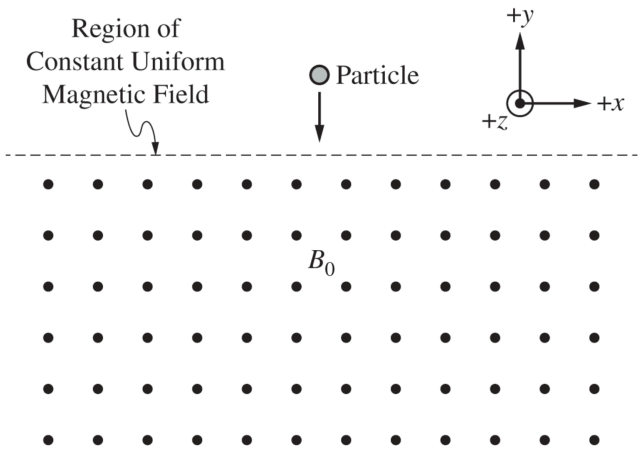
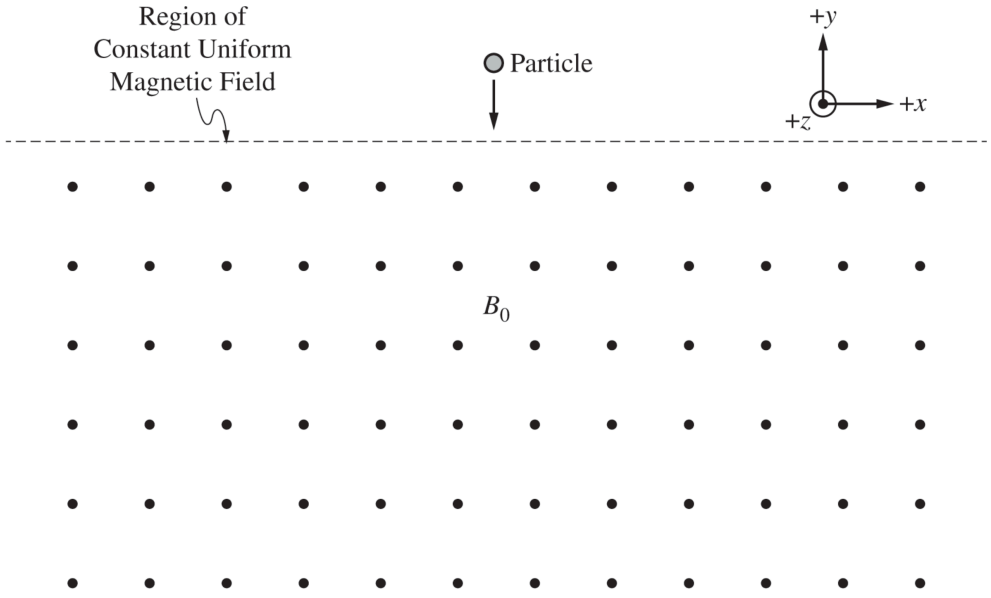


Figure 1

After exiting the device, the particles enter a large region of constant uniform magnetic field of magnitude B_0 that is directed in the $+z$-direction (out of the page), as shown in Figure 1. Each particle is moving in the $-y$-direction when entering the region, and each particle is moving in the $+y$-direction when exiting the region.

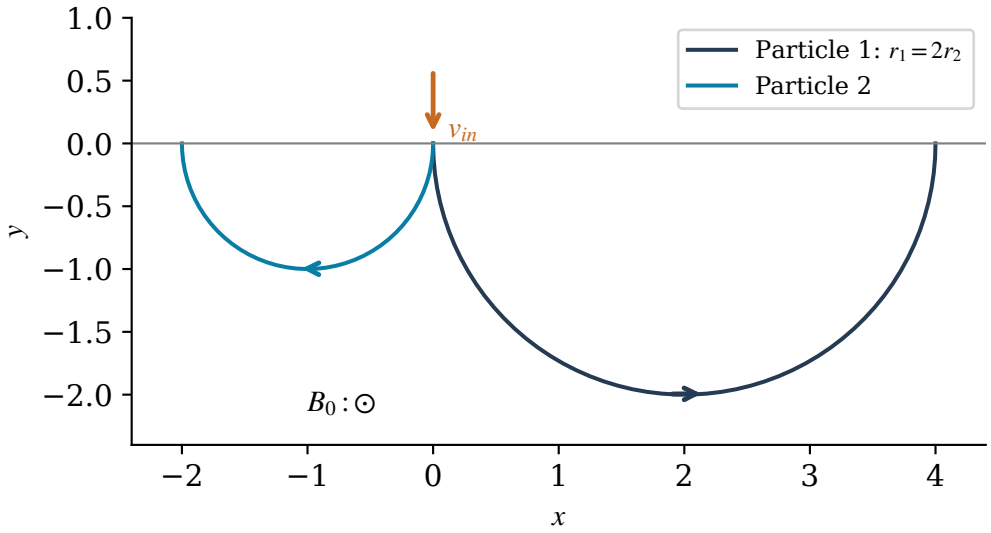
(c) On the following diagram in Figure 2, **sketch** and clearly **label** the paths of both particles 1 and 2 in the region.



解：

磁场朝纸外，初速度向下。正电荷的 v×B 向左，所以粒子2向左转；负电荷粒子1向右转。比较半径还要考虑质量、速率和电荷大小：

$$v_1 = \sqrt{\frac{2K_1}{M}} = \sqrt{\frac{K_2}{M}}, \qquad r_1 = \frac{Mv_1}{QB_0} = 2r_2.$$



粒子1向右走较大半圆；粒子2向左走较小半圆，半径比为 2：1。

磁力已知时：反求平衡电场方向

直线匀速需要合力为零。先由速度和磁场找该粒子的磁力，电力必须反向；最后看电荷符号。负电荷电力与电场反向，因此“所需电力方向”不能直接当成电场方向。

例题 2024 Q4 d

本问承接 2024 Q4 c。

4. (10 points, suggested time 20 minutes)

Two particles, 1 and 2, have different mass and charge as described by the following.

- Particle 1 has mass M and negative charge $-Q$.
- Particle 2 has mass $\frac{M}{2}$ and positive charge $+2Q$.

In separate trials, a device is used to accelerate each particle in the $-y$-direction from rest through a potential difference of absolute value $|\Delta V|$. The polarity of the potential difference can be adjusted so that a particle with either positive charge or negative charge can be accelerated in the $-y$-direction by the device. Gravitational effects are negligible.

After moving through the potential difference, particles 1 and 2 exit the device with kinetic energies K_1 and K_2, respectively.

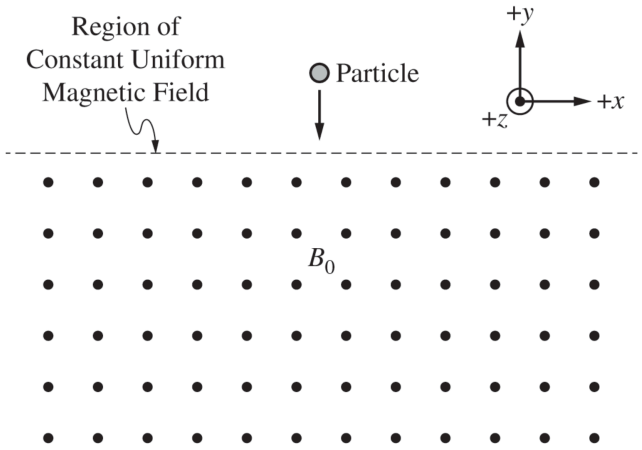


Figure 1

After exiting the device, the particles enter a large region of constant uniform magnetic field of magnitude B_0 that is directed in the $+z$-direction (out of the page), as shown in Figure 1. Each particle is moving in the $-y$-direction when entering the region, and each particle is moving in the $+y$-direction when exiting the region.

(d) A uniform electric field is added to the region such that Particle 1 of negative charge $-Q$ travels with constant speed in a straight line through the region. **Determine** the direction of the electric field.

解：

粒子1的磁力向右。要直线匀速，电力需向左；粒子1负电，电力与电场反向，因此电场应沿 +x 向右。

圆轨道半径与磁场反比：化为直线

同一种粒子电荷和速率固定，圆轨道半径与磁场大小成反比。横轴用1/B才能把反比关系化成直线。质量m隐藏在斜率中；先明确单位和斜率表达式，再在后续图中求质量。

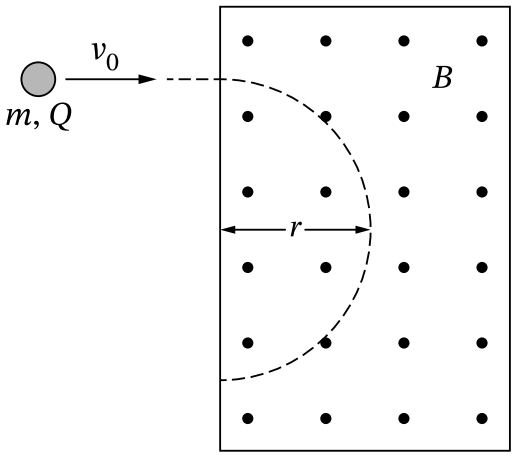
$$Qv_0B = \frac{mv_0^2}{r}, \qquad r = \left(\frac{mv_0}{Q}\right)\frac{1}{B}.$$

例题 2026 Q3 C(i)

In Experiment 2, scientists want to use a graph to determine the mass m of identical, charged particles that are emitted from a device. Each particle has charge $Q = +6.4 \times 10^{-19}$ C.

In each trial, a particle is emitted with speed $v_0 = 3.0 \times 10^6$ m/s toward a region of external, uniform magnetic field of variable magnitude B. The radius r of the path along which the particle moves while in the field, shown in Figure 2, is recorded.

Figure 2



In subsequent trials, B is increased and the resulting r is recorded. Gravitational effects are negligible. Collected data are provided in Table 1.

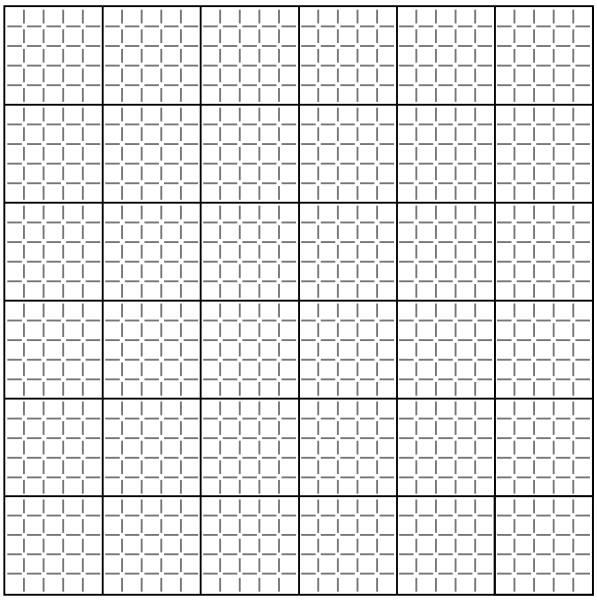
Table 1

B (T)	r (m)
0.04	1.8
0.06	1.2
0.14	0.5
0.16	0.4
0.20	0.3

i. **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine m.

Quantity（units, if appropriate)

_____ _____



Quantity（units, if appropriate)

_____ _____

解：

C(i) 实验2速度 v0 固定、电荷 Q 固定，磁力提供向心合力：

$$Qv_0B = \frac{mv_0^2}{r}, \qquad r = \frac{mv_0}{Q}\left(\frac{1}{B}\right).$$

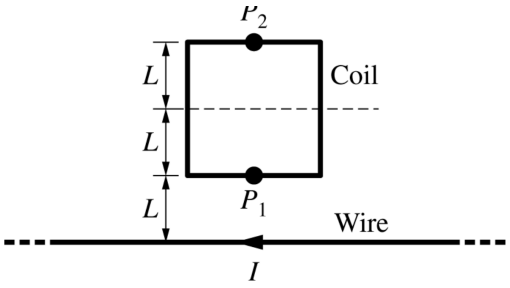
横轴选 1/B（T^{-1}），纵轴选 r（m）。这把原来的反比曲线变成直线，其斜率为 mv0/Q。

磁通变化量与平均感应电流

磁通Φ描述穿过线圈的磁场量。单匝线圈平均感应电动势大小，是磁通改变的大小除时间；再用整个线圈电阻求平均电流。所求大小取绝对值，不能把负的变化当成负的电流大小。

$$|\mathcal{E}_{\text{avg}}| = \frac{|\Delta\Phi|}{\Delta t}, \qquad |I_{\text{avg}}| = \frac{|\mathcal{E}_{\text{avg}}|}{R}.$$

例题 2022 Q4 b(ii)



(b) The charged object is removed, and a square coil with side length $2L$ is placed near the long, straight wire, as shown above. The bottom of the coil is a distance L from the wire. The magnitude of the magnetic field due to the current in the wire is $3B_0$ at point P_1 and B_0 at point P_2.

ii. Over a time interval of 2.0 s, the current in the wire is decreased. The initial magnetic flux through the coil is 5.0×10^{-5} T·m^2 and the final magnetic flux through the coil is 1.0×10^{-5} T· m^2. The coil has a total resistance of 10 Ω. Calculate the magnitude of the average current in the coil during the 2.0 s time interval.

解：

先用磁通变化除以时间求平均感应电动势大小，再用线圈总电阻求平均电流大小。本题单匝。

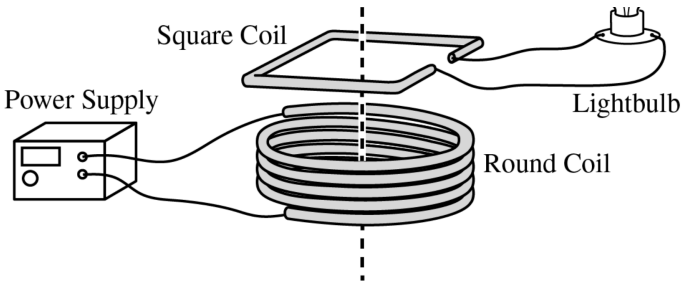
$$|\mathcal{E}_{avg}| = \frac{|\Delta\Phi|}{\Delta t} = \frac{|(1.0-5.0)\times 10^{-5}|}{2.0}\ \mathrm{V} = 2.0\times 10^{-5}\ \mathrm{V},$$

$$|I_{avg}| = \frac{|\mathcal{E}_{avg}|}{R} = \frac{2.0\times 10^{-5}}{10}\ \mathrm{A} = 2.0\times 10^{-6}\ \mathrm{A}.$$

持续改变磁场：解释灯泡一直亮

一只线圈的电流改变，会使穿过附近另一只线圈的磁通改变。只要这种改变持续存在，闭合接收线圈便持续有感应电动势和电流。两线圈不需要导线连接；单纯存在一个恒定磁场并不足以产生持续感应。

例题 2022 Q4 c



The wire is removed and the square coil is positioned so that the coil is directly above and concentric with a round coil of wire connected to a power supply. A part of the square coil is removed and a lightbulb is connected to the coil, as shown above.

(c) During a short time interval, the current in the power supply is constantly increasing. Use physics principles to explain why the lightbulb is lit during the entire time interval.

解：

圆线圈电流产生磁场，磁场穿过上方方线圈形成磁通。圆线圈电流在整个时间段持续增大，因此穿过方线圈的磁通持续改变。法拉第定律使闭合方线圈持续出现感应电动势和电流，电流经过灯泡，所以整个时间段都亮。

两线圈无需导线连接。决定持续感应的是磁通持续变化，不能只说“存在磁场”，也不能说只有刚开始时才亮。

楞次定律：先找原磁通怎样变

先用长直导线右手规则找线圈内原场方向。再看运动使场变强还是变弱。感应场阻止的是磁通的变化：原向内磁通减小时，感应场也向内；向内场增强时，感应场才向外。最后用线圈右手规则把所需感应场变成电流顺逆时针。

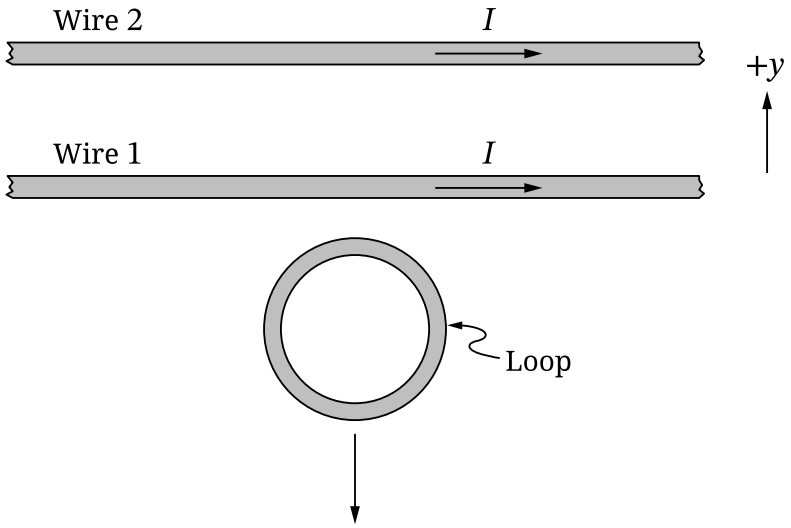
例题 2025 Q1 B

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1. Very long Wire 1 carries current I in the $+x$-direction along the line $y = 0$. Very long Wire 2 carries current I in the $+x$-direction along the line $y = +d$. Point P is located along Wire 1 at the origin, as shown in Figure 1. The diameters of the wires are small compared to the distance between the wires. Both wires are in the xy-plane.

B. Wire 3 is moved very far away from wires 1 and 2. A circular conducting loop in the xy-plane is initially held at rest below Wire 1. The loop is then moved at a constant speed in the $-y$-direction, as shown in Figure 4.

Figure 4



Note: Figure not drawn to scale.

Indicate whether there is a clockwise induced current in the loop, a counterclockwise induced current in the loop, or no induced current in the loop.

_____ Clockwise

_____ Counterclockwise

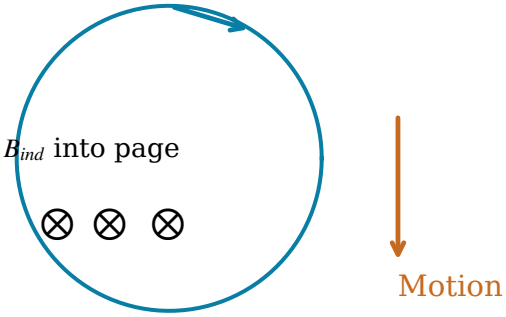
_____ There is no induced current in the loop.

Justify your answer.

解：

两导线下方的磁场都进入纸面。线圈向 –y 远离两根导线，磁场随距离增加而变弱，穿过线圈的向内磁通减少。感应场要阻止这个减少，故也应向内；右手拇指朝纸内，四指顺时针弯曲，所以感应电流顺时针。

楞次定律反抗磁通的变化，并非一律让感应场与原场反向；本题二者同向。


B_{ind} into page
Motion

折射、反射与干涉

空气—液体—镜面：逐个界面画光路

角度从法线量起，法线垂直界面。进入折射率较大的介质，光向法线靠近；镜面反射的入射角等于反射角；从高折射率介质离开到空气时远离法线。每遇一个界面便重新画法线，按这一处关系继续，而不是直接连入口与出口。

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \qquad \theta_{\text{反射}} = \theta_{\text{入射}}.$$

例题 2023 Q1 a

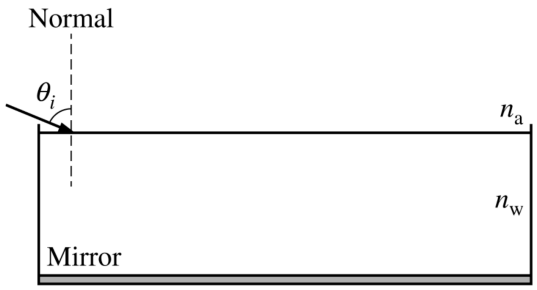
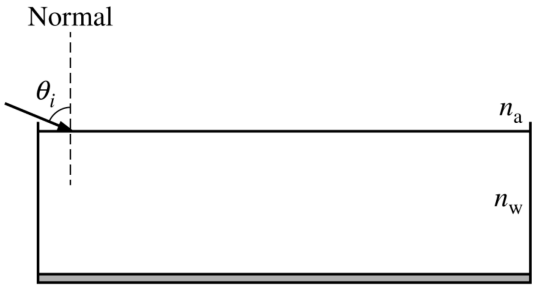


Figure 1

(10 points, suggested time 20 minutes)

A rectangular tank with a mirrored bottom is filled with water (index of refraction n_w). A beam of light passes from air (index of refraction n_a) into the water at angle θ_i from the normal, as shown in Figure 1. Index of refraction n_w is greater than index of refraction n_a.

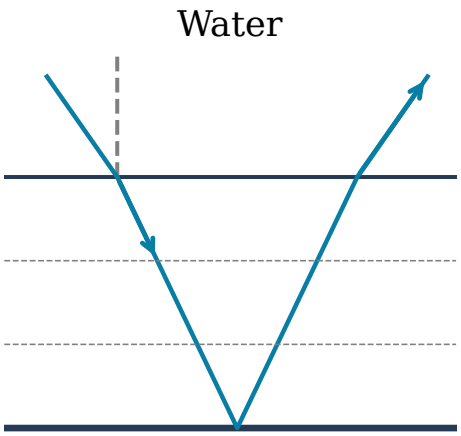
(a) On the following diagram, sketch the entire path of the beam as the beam enters, travels through, and then exits the water.



解：

角度从法线量起。光从空气进入水，水的折射率较大，折射线更靠近竖直法线。镜面底部入射角等于反射角，光改为向右上方传播。再次从水进入空气时远离法线偏折；出射光与竖直法线的夹角大小等于原空气入射角，但传播方向朝右上方。

$$n_a \sin\theta_i = n_w \sin\theta_w, \qquad n_w \sin\theta_w = n_a \sin\theta_o, \qquad \theta_o = \theta_i.$$



进入介质：速度变化，频率保持

同一束光通过静止界面，频率由光源决定并保持。折射率n越大，传播速度c/n越小；波长是速度除频率，所以也越短。这里的波长是介质中光的波长，别把它与电子物质波混在一起。

$$v = \frac{c}{n}, \qquad \lambda = \frac{v}{f} = \frac{c}{nf}.$$

例题 2023 Q1 b

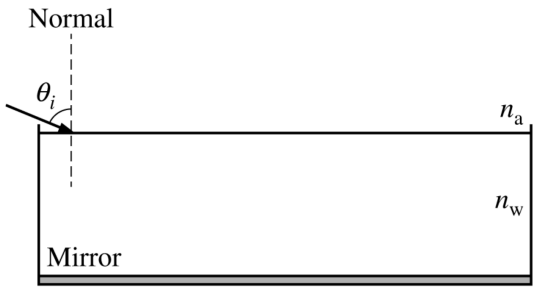


Figure 1

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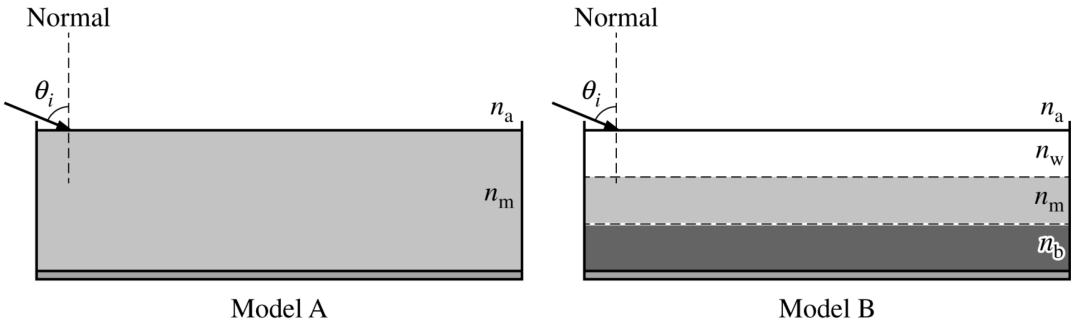


Figure 2

Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with water. The models are shown in Figure 2.

Model A: The sugar is uniformly mixed throughout the water, resulting in a mixture with index of refraction n_m such that $n_m > n_w$.

Model B: Layers are formed of varying concentrations of sugar in the water. There are three distinct layers of equal volume. The top layer is only water (index of refraction n_w). The middle layer has the same concentration of sugar as the mixture in Model A (index of refraction n_m). The bottom layer has the highest concentration of sugar (index of refraction n_b).

(b) Consider Model A. Briefly describe how the observed wavelength of light changes, if at all, as the beam travels from air into the mixture.

解：

同一束光在静止介质界面频率保持不变；混合液折射率大于空气，速度减小。波长等于速度除以频率，所以波长减小。

$$v = \frac{c}{n}, \qquad \lambda = \frac{v}{f} = \frac{c}{nf}.$$

多个平行界面：连续应用折射定律

平行界面的法线平行，所以前一界面的折射角就是后一界面的入射角。连续应用n sinθ保持不变，中间各层相消。先写原入射介质与最终介质的等式，再求最后的角。

$$n_a \sin\theta_i = n_w \sin\theta_2 = n_m \sin\theta_3 = n_b \sin\theta_4.$$

例题 2023 Q1 c(i)

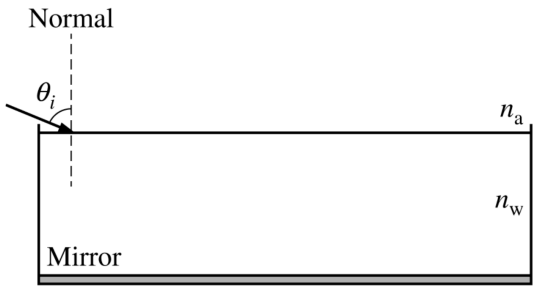


Figure 1

(10 points, suggested time 20 minutes)

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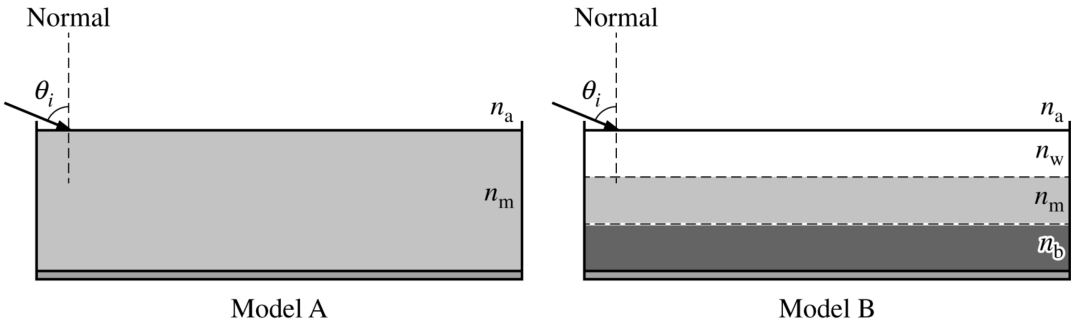


Figure 2

Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with water. The models are shown in Figure 2.

Model A: The sugar is uniformly mixed throughout the water, resulting in a mixture with index of refraction n_{m} such that $n_{\mathrm{m}} > n_{\mathrm{w}}$.

Model B: Layers are formed of varying concentrations of sugar in the water. There are three distinct layers of equal volume. The top layer is only water (index of refraction n_{w}). The middle layer has the same concentration of sugar as the mixture in Model A (index of refraction n_{m}). The bottom layer has the highest concentration of sugar (index of refraction n_{b}).

(c) Relevant angles between the beam and the normal for the various layers present in models A and B are defined in the following table.

Model A		Model B	
θ_i	Incident angle of the beam in air	θ_i	Incident angle of the beam in air
θ_1	Angle the beam makes with the normal in the mixture in Model A	θ_2	Angle the beam makes with the normal in the top layer in Model B
		θ_3	Angle the beam makes with the normal in the middle layer in Model B
		θ_4	Angle the beam makes with the normal in the bottom layer in Model B

i. Determine an expression for θ_4 in terms of θ_i, n_{a}, and n_{b}.

解：

各界面平行，法线均竖直，可连续使用折射定律：

$$n_a \sin\theta_i = n_w \sin\theta_2 = n_m \sin\theta_3 = n_b \sin\theta_4,$$

$$\theta_4 = \arcsin\left(\frac{n_a}{n_b}\sin\theta_i\right).$$

相同 n sinθ：由折射率排序角度

相同光束在平行各层中n sinθ固定。在0到90度间，正弦随角增大；折射率大的一层，角反而小。相同折射率且入射条件相同的角相等；排序要保留并列。

例题 2023 Q1 c(ii)

本问承接 2023 Q1 c(i)。

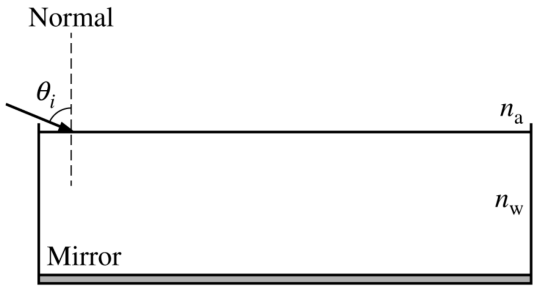


Figure 1

(10 points, suggested time 20 minutes)

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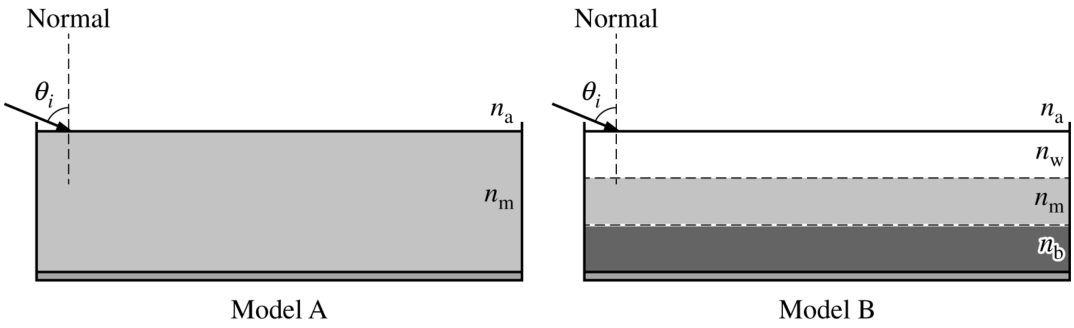


Figure 2

Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with water. The models are shown in Figure 2.

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		θ_3	Angle the beam makes with the normal in the middle layer in Model B
		θ_4	Angle the beam makes with the normal in the bottom layer in Model B

ii. Rank the angles from greatest to least, with 1 being greatest. If two angles are the same value, give them the same ranking.

_____ θ_1 _____ θ_2 _____ θ_3 _____ θ_4

Briefly explain your reasoning using appropriate physics principles and/or mathematical models.

解：

同一空气入射条件下，n sinθ 相同。折射率越大，锐角 θ 越小。水的折射率最低，底层最高；Model A 和 Model B 中层都为 nm：

$$\theta_2 > \theta_1 = \theta_3 > \theta_4.$$

按 θ1、θ2、θ3、θ4 的顺序填 2、1、2、3。

把分层光路的水平距离相加

深度H的一层，光与竖直法线夹角θ，单程水平距离为H tanθ。底部反射后朝相同水平侧传播，回程距离要相加；分层时各层分别算。比较新旧图：角变小便使该层水平距离变小。

例题 2023 Q1 d

本问承接 2023 Q1 c(ii)。

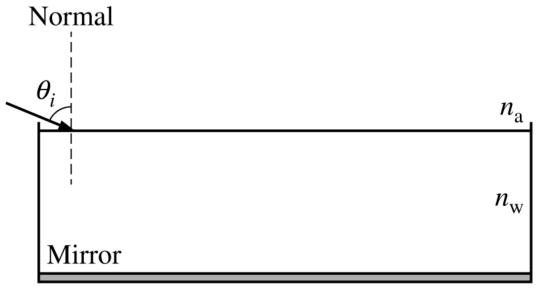


Figure 1

(10 points, suggested time 20 minutes)

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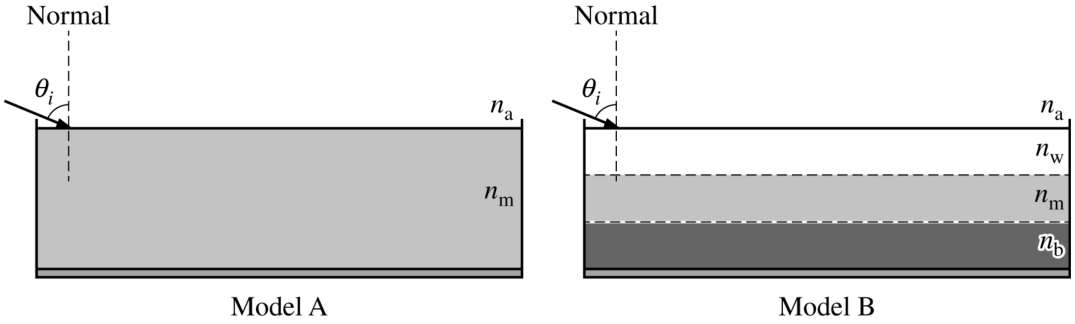
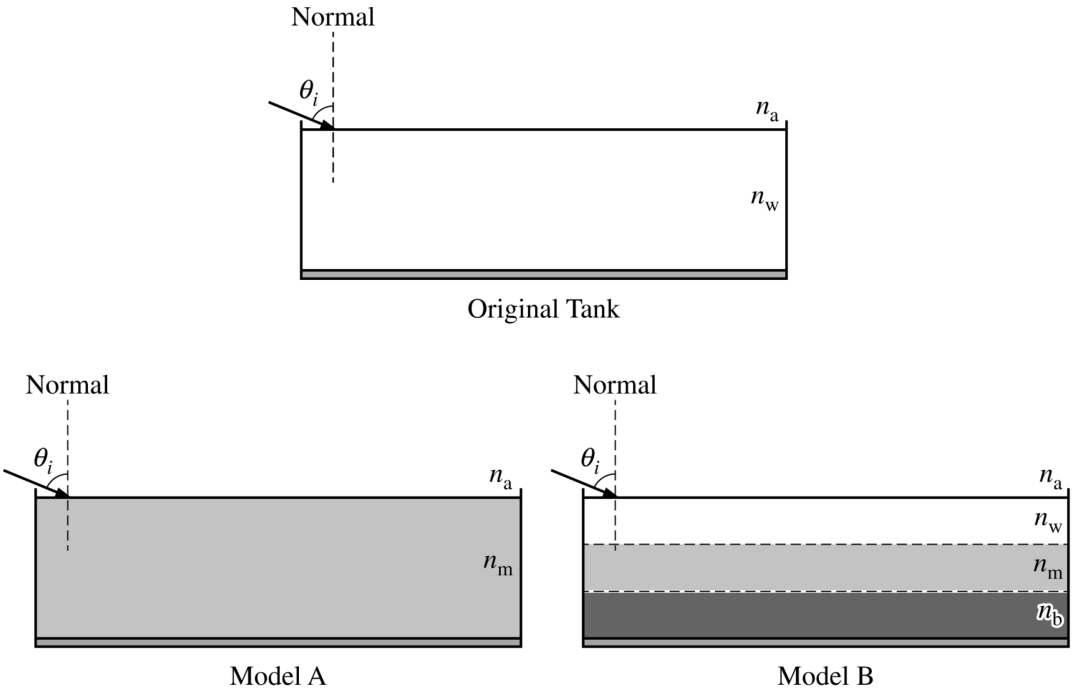


Figure 2

Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with water. The models are shown in Figure 2.

Model A: The sugar is uniformly mixed throughout the water, resulting in a mixture with index of refraction n_m such that $n_m > n_w$.

Model B: Layers are formed of varying concentrations of sugar in the water. There are three distinct layers of equal volume. The top layer is only water (index of refraction n_w). The middle layer has the same concentration of sugar as the mixture in Model A (index of refraction n_m). The bottom layer has the highest concentration of sugar (index of refraction n_b).



For the original tank filled with water, the beam is observed to exit the surface of the water a horizontal distance d_{w} from the entry point. For models A and B, the horizontal distances are d_{A} and d_{B}, respectively.

(d) Determine whether d_{A} and d_{B} are each greater than, less than, or equal to d_{w}. It is NOT necessary to compare d_{A} to d_{B}. Briefly justify your answer.

解：

深度 H、均匀折射角 θ 的单程水平距离为 Htanθ，来回两段相加。原水箱：

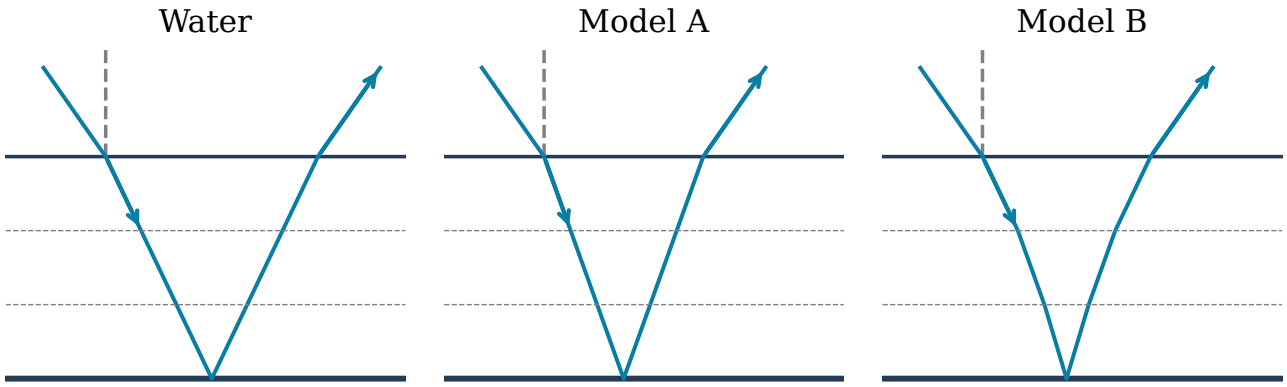
$$d_w = 2H\tan\theta_w.$$

Model A 的角更小，所以水平距离小于原水箱的距离。Model B 三层等体积、等深；上层角与原水箱相同，中、下两层角更小，所以总水平距离也更小：

$$d_B = 2\frac{H}{3}(\tan\theta_2 + \tan\theta_3 + \tan\theta_4) < 2H\tan\theta_w = d_w.$$

$$d_A < d_w, \qquad d_B < d_w.$$

题目不要求比较 dA 与 dB。反射前后光都朝右，所以水平距离相加，不相减。

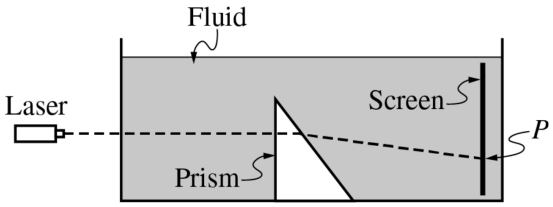


棱镜外介质改变：在斜面重画法线

垂直入射棱镜第一面时方向不改变。决定偏折的是出口斜面；法线垂直斜面，不能用水平或竖直方向代替。棱镜和内部入射角不变，外介质折射率降低使出射角增大，再据原图方向判断屏幕位置。

$$n_{\text{prism}} \sin\theta_{\text{in}} = n_{\text{out}} \sin\theta_{\text{out}}.$$

例题 2022 Q1 c

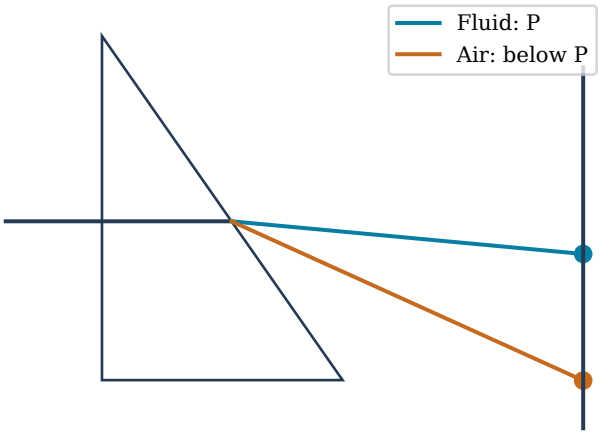


(c) The block is replaced by a triangular prism, as shown above. The path of the beam is indicated by the dotted line, and the beam reaches the screen at point P. The fluid is then removed from the tank, and the prism is surrounded by air. Predict whether the beam will reach the side of the tank above point P, at point P, or below point P when the prism is surrounded by air. Support your answer using physics principles.

解：

入口处光垂直棱镜侧面，入射方向不改变。主要比较斜面出射：棱镜折射率不变，外介质由液体变为空气，折射率减小。同一内侧入射角下，Snell 定律要求外侧折射角增大，光线更远离法线，按原图方向偏得更向下，所以落在 P 下方。

$$n_{prism} \sin\theta_{in} = n_{outside} \sin\theta_{out}.$$

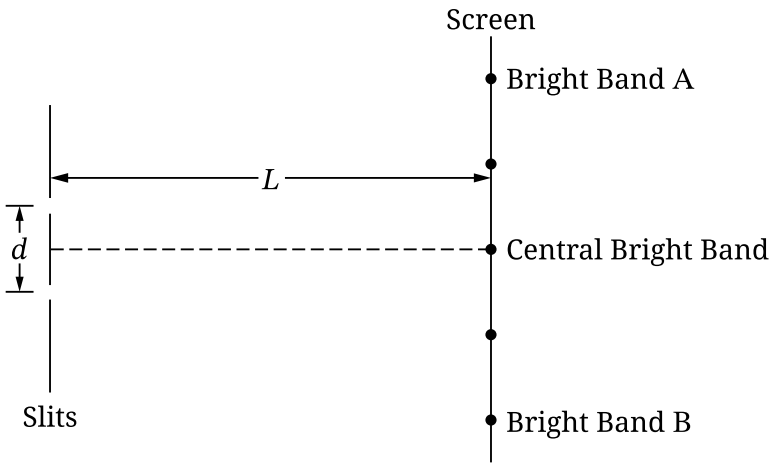


同一级亮纹：用光程差比较间距

亮纹出现于两路光路程相差整数个波长的位置。相同级数下，波长较短便需要较小光程差，观察点更靠近中央。先以这个物理条件解释紫光和红光的差别；介质改变时，还要先说明速度、频率与波长怎样变。

例题 2025 Q4 A

4. Two narrow slits are a distance d apart. A screen is a distance L from the midpoint of the slits, where $L \gg d$. When a laser emits monochromatic light toward the slits, a pattern of narrow bright and dark bands is observed on the screen. The centers of bright bands A and B are indicated. Three additional bright bands, including the central bright band, are observed on the screen between bands A and B, as shown.



Note: Figure not drawn to scale.

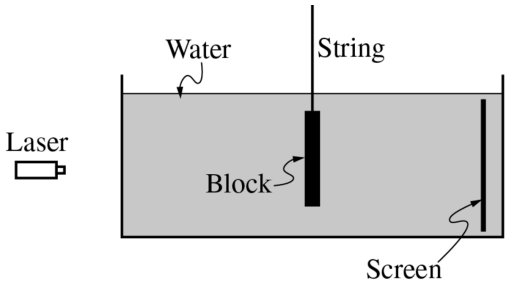
A student claims that the distance between the center of Band A and the center of the central bright band is smaller when using a laser that emits violet light than when using a laser that emits red light.

A. Indicate whether the student's claim is correct or incorrect. Without manipulating equations, **justify** your answer by referencing the difference in path length traveled by the light from each slit to the center of Band A.

解：

学生说法正确。紫光波长比红光短；同一级亮纹要求两缝到观察点的路程相差相同整数个波长，所以紫光需要更小的光程差。观察点越靠近中央，光程差越小，因此 A 移向中央。这个解释按题意从光程差出发，不需要先操作间距公式。

练习 2022 Q1 b



(10 points, suggested time 20 minutes)

Students are investigating electromagnetic wave phenomena in transparent media. They use a string to support a stationary thin, rectangular block of mass m_b, volume V_b, and density ρ_b. The block has two narrow slits in its center and is submerged in a glass tank containing water with density ρ_w, as shown above.

(b) A monochromatic laser beam is incident perpendicular to the wall of the tank. The beam passes through the slits in the block. An interference pattern is formed on the screen inside the tank. The water is then replaced with a clear fluid with a greater index of refraction than that of water. In a coherent, paragraph-length response, describe how the greater index of refraction of the new fluid affects the interference pattern. Explain your reasoning in terms of speed, frequency, and wavelength of the light.

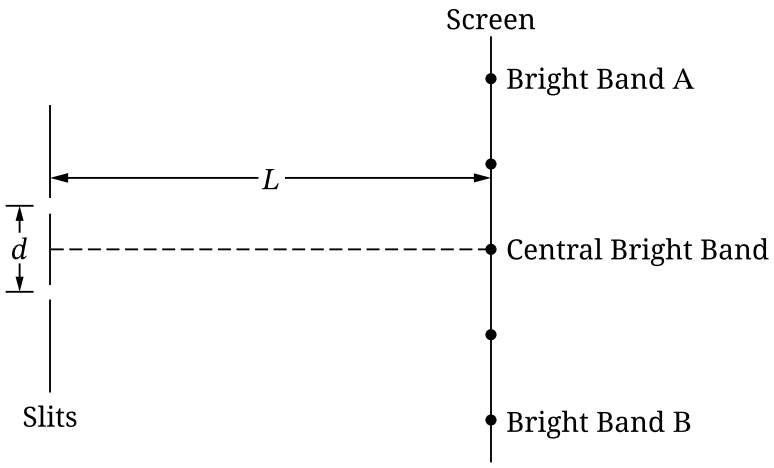
数清亮纹级数，再推导两条纹距离

中央亮纹是0级，相邻亮纹每次级数改变1。题目两亮纹之间出现三条亮纹且包括中央时，先在图上逐条编号，再算两端级数差。缝距d、屏距L且L远大于d，可用小角近似连接屏幕位置y和光程差；频率给定时用λ=c/f。

$$d\sin\theta = m\lambda, \qquad \sin\theta \approx \tan\theta = \frac{y}{L}, \qquad y_m \approx \frac{m\lambda L}{d}.$$

例题 2025 Q4 B

4. Two narrow slits are a distance d apart. A screen is a distance L from the midpoint of the slits, where $L \gg d$. When a laser emits monochromatic light toward the slits, a pattern of narrow bright and dark bands is observed on the screen. The centers of bright bands A and B are indicated. Three additional bright bands, including the central bright band, are observed on the screen between bands A and B, as shown.

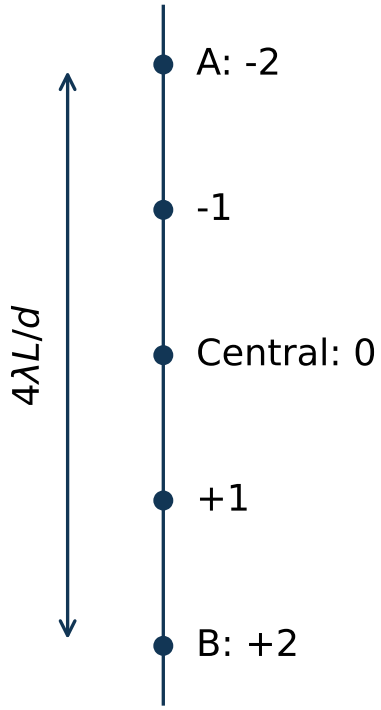


Note: Figure not drawn to scale.

B. **Derive** an expression for the distance between the centers of bands A and B when light of frequency f is emitted toward the slits. Express your answer in terms of d, L, f, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

中央亮纹为零级，两边每经过一条亮纹，级数改变1。A、B中间另有三条亮纹，包括中央，所以A和B分别在中央两侧的第二级位置，共相隔四个条纹间距。取沿屏幕向下为正，A的坐标是负二级，B是正二级。



沿原屏幕方向编号：A为−2级，B为+2级；正方向取向下。

从亮纹条件起手。小角度近似把方向角与屏幕坐标连接：

$$d\sin\theta = m\lambda, \qquad \sin\theta \approx \tan\theta = \frac{y_m}{L}, \qquad y_m \approx \frac{m\lambda L}{d}.$$

波长用光速除以频率，再取两侧二级亮纹坐标之差：

$$\Delta y = y_{+2} - y_{-2} = \frac{4\lambda L}{d} = \frac{4cL}{fd}.$$

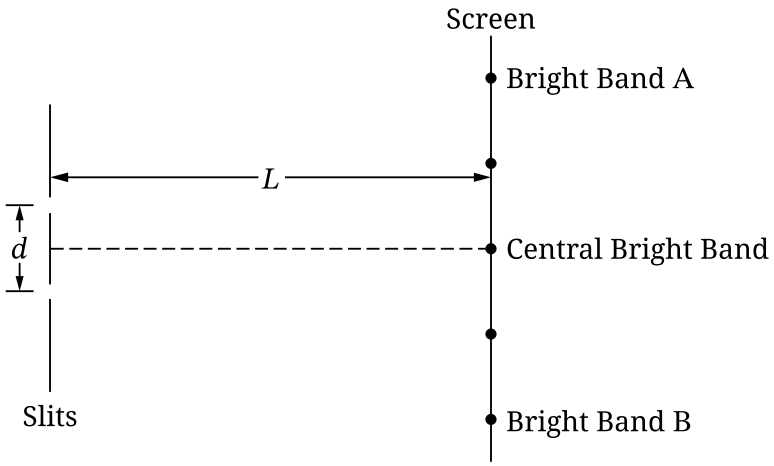
从反比关系核对定性判断

核对一致性不是把刚才的结论再抄一遍。指出表达式中变化的是哪一个量、其余量为何相同，再解释所求距离随该量增大还是减小；与先前光程差判断相对照。

例题 2025 Q4 C

本问承接 2025 Q4 A、2025 Q4 B。

4. Two narrow slits are a distance d apart. A screen is a distance L from the midpoint of the slits, where $L \gg d$. When a laser emits monochromatic light toward the slits, a pattern of narrow bright and dark bands is observed on the screen. The centers of bright bands A and B are indicated. Three additional bright bands, including the central bright band, are observed on the screen between bands A and B, as shown.



Note: Figure not drawn to scale.

A student claims that the distance between the center of Band A and the center of the central bright band is smaller when using a laser that emits violet light than when using a laser that emits red light.

C. **Indicate** whether the expression you derived in part B is or is not consistent with your answer from part A. Briefly **justify** your answer.

解：

一致。紫光频率较大，式中其他量固定，亮纹间距与频率成反比，因此距离缩小，符合 A 中“紫光条纹靠近中央”的判断。

光子、电子与原子

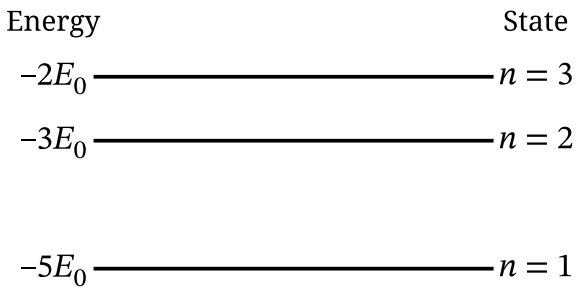
原子发光：枚举向下能级跃迁

能级线给出允许的原子能量。原子发光时失去能量，终态能级低于初态，所以箭头向下。按不同初末能级对列举，每对一条；不能把连续能量曲线当作原子允许的所有能级。

例题 2026 Q2 A

2. Figure 1 represents the energy levels and their corresponding states for a hypothetical atom.

Figure 1

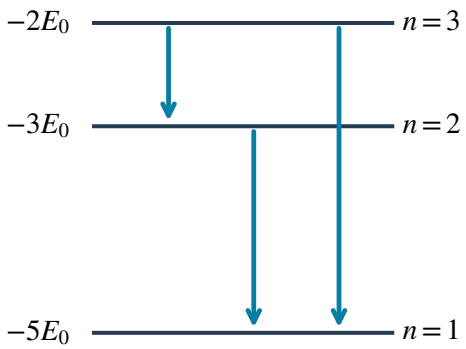


Part A

On Figure 1, **draw** arrows to represent all possible atomic transitions that could result in the emission of a photon.

解：

原图三个能级分别为 −5E0、−3E0、−2E0。发光表示原子能量降低，所以只画向下箭头：3→2、2→1、3→1，三种不同的能级对各一条。



光子频率或能级差：求能量和波长

h是普朗克常量，光子能量与频率成正比。真空光速c等于频率乘波长。原子发出光子时，光子能量取初末能级的正差；能量最大的光子来自最大的向下能级差，并有最短波长。

$$E_\gamma = hf, \qquad c = f\lambda, \qquad E_\gamma = \frac{hc}{\lambda}.$$

例题 2022 Q3 d(i)

(d) Experiments show that a hydrogen atom can absorb a photon of frequency 3.2×10^{15} Hz.

i. Calculate the energy of a photon with this frequency.

解：

光子频率已知，能量等于普朗克常量乘频率：

$$E_\gamma = (6.63 \times 10^{-34}\ \mathrm{J\,s})(3.2 \times 10^{15}\ \mathrm{s^{-1}}) = 2.12 \times 10^{-18}\ \mathrm{J}.$$

最大向下能级差：求最高能光子的波长

先从原能级图选择初态最高、终态最低的允许跃迁，用初能量减末能量求光子能量。再用E=hf和c=fλ消去频率。能量越大对应波长越短；注意减去负能级时，不能漏掉括号里的负号。

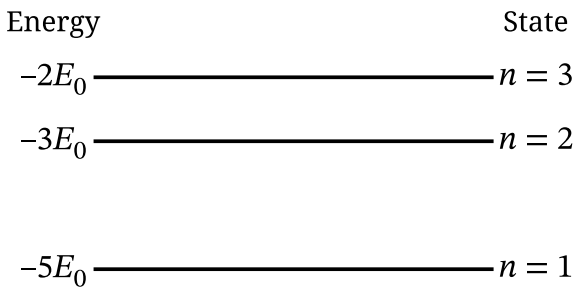
$$E_\gamma = E_{\text{initial}} - E_{\text{final}}, \qquad \lambda = \frac{hc}{E_\gamma}.$$

例题 2026 Q2 B

本问承接 2026 Q2 A。

2. Figure 1 represents the energy levels and their corresponding states for a hypothetical atom.

Figure 1



Part B

Derive an expression for the wavelength of the highest-energy photon that can be emitted from the atom. Express your answer in terms of E_0 and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

光子能量等于原子失去的能量。最大差从 −2E0 降到 −5E0：

$$E_{max} = (-2E_0) - (-5E_0) = 3E_0.$$

光子能量与频率成正比、波长与频率成反比，联立并解出波长：

$$E = hf, \qquad c = f\lambda, \qquad E = \frac{hc}{\lambda}, \qquad \lambda_{min} = \frac{hc}{3E_0}.$$

连续光源：用已知点定反比曲线

连续光源可发出连续范围的波长；E=hc/λ表明波长加倍，能量变成一半。先用给定的λ0和4E0确定乘积hc，再在规定区域画下降且越向右越平的曲线；检查原题端点范围和阴影禁画区。

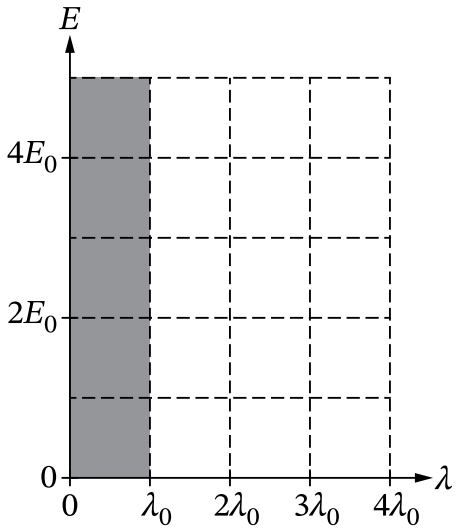
例题 2026 Q2 C

The following information applies to parts C and D.

A device can emit monochromatic electromagnetic radiation. The wavelength λ of the radiation can be varied over a continuous range of wavelengths.

Part C

On Figure 2, **sketch** the energy E of emitted photons from the device as a function of λ for $\lambda_0 < \lambda < 4\lambda_0$. Photons with wavelength λ_0 have energy $4E_0$. Sketches made in the shaded region will not be scored.

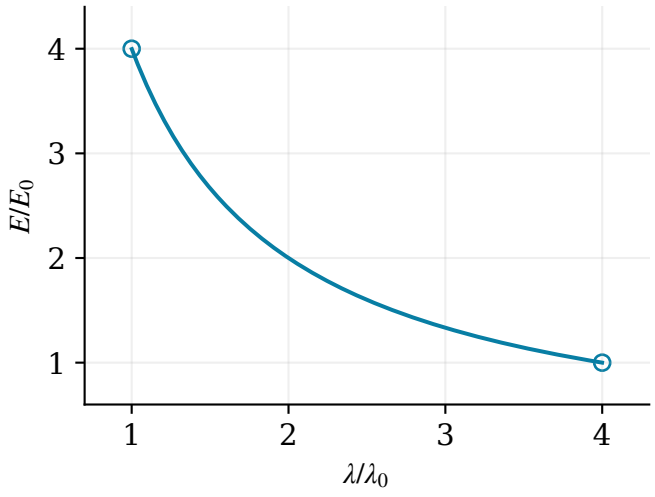


解：

连续设备的波长能在范围内任意改变。λ0 时能量为 4E0，因此光速与普朗克常量的乘积等于这组能量与波长的乘积，图线为：

$$\frac{E}{E_0} = \frac{4}{\lambda/\lambda_0}, \qquad \lambda_0 < \lambda < 4\lambda_0.$$

曲线始终下降，并且越往右越平，向上弯曲；靠近 λ0 时能量靠近 4E0，靠近 4λ0 时靠近 E0，只画题定非阴影区域。



有限能级：检查给定光子是否可能

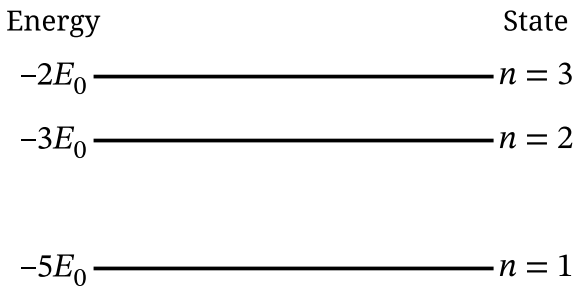
光子能量必须等于某两个允许能级的差。把所有向下跃迁的正能量差列出来，再检查目标光子能量在不在这个列表。设备能发出某光子，并不使原子的离散能级多出这一能量差。

例题 2026 Q2 D

本问承接 2026 Q2 A。

2. Figure 1 represents the energy levels and their corresponding states for a hypothetical atom.

Figure 1



The following information applies to parts C and D.

A device can emit monochromatic electromagnetic radiation. The wavelength λ of the radiation can be varied over a continuous range of wavelengths.

Part D

A student claims that, based on the energy states shown in Figure 1, the atom can emit a photon of wavelength λ_0 and energy $4E_0$ like the device described in part C.

Indicate whether the claim is correct or incorrect. Briefly **justify** your answer by referencing the representation in part A.

解：

说法错误。三条允许跃迁的能量差分别是 E0、2E0、3E0；没有 4E0 的能级差。连续设备可以产生该光子，不代表这三个离散能级之间可以发出它。

电子德布罗意波长：先求动量再求动能

电子是有质量粒子，德布罗意波长λe与动量p满足λe=h/p。题目允许经典动量p=mv时，由K=mv^2/2得K=p^2/(2m)。先把电子波长换成动量，再算动能；不能把电子波长直接代进光子能量hc/λ。

$$p = \frac{h}{\lambda_e}, \qquad K = \frac{p^2}{2m_e} = \frac{h^2}{2m_e\lambda_e^2}.$$

例题 2024 Q1 b

In each trial of a photoelectric experiment, a scientist uses a device to shine light of a single frequency on two different metals, 1 and 2. The device can emit light with frequency f_{A}, f_{B}, or f_{C}. Each frequency of light is used to test both metals.

The scientist determines the minimum de Broglie wavelength λ_e of the electrons ejected from the metal in each trial of the experiment. The following table summarizes the results of the experiment. For each trial, the scientist analyzes only the electrons with the minimum de Broglie wavelength.

Trial	Frequency of Light	Metal Tested	λ_e ($\times 10^{-10}$ m)
1	f_{A}	Metal 1	6.9
2	f_{A}	Metal 2	9.4
3	f_{B}	Metal 1	No electrons ejected
4	f_{B}	Metal 2	No electrons ejected
5	f_{C}	Metal 1	5.3
6	f_{C}	Metal 2	6.3

(b) **Calculate** the maximum kinetic energy of the electrons ejected from Metal 1 in Trial 1. Assume that the momentum p of an ejected electron can be described by the classical definition $p = mv$.

解：

Trial 1 的电子波长是 6.9×10^{-10} m。题目指定动量等于质量乘速率，所以可由电子波长先求动量，再求动能：

$$p = \frac{h}{\lambda_e}, \qquad K_{max} = \frac{h^2}{2m_e\lambda_e^2} = \frac{(6.63\times10^{-34})^2}{2(9.11\times10^{-31})(6.9\times10^{-10})^2}\ \mathrm{J} \approx 5.07\times10^{-19}\ \mathrm{J}.$$

换为电子伏特约 3.2 eV。这里是有质量电子，不能使用光子能量 hc/λe。

同金属比较：由逸出与电子波长判断频率

光子使电子离开金属，先付出逸出功φ，其余能量成为最大动能。比较同金属实验，φ固定；没有电子逸出表示低于阈值。逸出电子最短波长越小，最大动量和动能越大，因此入射光频率越大。表里电子波长不是入射光波长。

$$hf = \phi + K_{\text{max}}.$$

例题 2024 Q1 a

In each trial of a photoelectric experiment, a scientist uses a device to shine light of a single frequency on two different metals, 1 and 2. The device can emit light with frequency f_{A}, f_{B}, or f_{C}. Each frequency of light is used to test both metals.

The scientist determines the minimum de Broglie wavelength λ_e of the electrons ejected from the metal in each trial of the experiment. The following table summarizes the results of the experiment. For each trial, the scientist analyzes only the electrons with the minimum de Broglie wavelength.

Trial	Frequency of Light	Metal Tested	λ_e ($\times 10^{-10}$ m)
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2	f_{A}	Metal 2	9.4
3	f_{B}	Metal 1	No electrons ejected
4	f_{B}	Metal 2	No electrons ejected
5	f_{C}	Metal 1	5.3
6	f_{C}	Metal 2	6.3

(a) In a coherent, paragraph-length response, **indicate** which frequency, f_{A}, f_{B}, or f_{C}, is greatest and which frequency is least. **Justify** your answer using physics principles.

解：

fB 最小、fC 最大。入射光子的能量 hf 随频率增加，电子需先付出金属逸出功 φ，剩下成为最大动能。fB 对两金属都不能使电子逸出，说明低于相应阈值；fA、fC 都有电子逸出。

$$hf = \phi + K_{max}, \qquad \lambda_e = \frac{h}{p}, \qquad K_{max} = \frac{p^2}{2m_e} = \frac{h^2}{2m_e\lambda_e^2}.$$

在同一金属中，fC 对应电子波长较短，表示最大动量、最大动能较大，因此 fC 大于 fA。不能把“电子波长较短”直接当作入射光波长的观测。

$$f_C > f_A > f_B.$$

同频光比较两金属：反求逸出功

选用相同入射频率的两行，才能保证光子能量相同。φ=hf−Kmax；电子最大动能较大的一种金属，把较少能量用于逸出，因此逸出功较小。先由电子波长比较动能，再完成能量比较。

例题 2024 Q1 c

本问承接 2024 Q1 a。

In each trial of a photoelectric experiment, a scientist uses a device to shine light of a single frequency on two different metals, 1 and 2. The device can emit light with frequency f_{A}, f_{B}, or f_{C}. Each frequency of light is used to test both metals.

The scientist determines the minimum de Broglie wavelength λ_e of the electrons ejected from the metal in each trial of the experiment. The following table summarizes the results of the experiment. For each trial, the scientist analyzes only the electrons with the minimum de Broglie wavelength.

Trial	Frequency of Light	Metal Tested	λ_e $(\times 10^{-10}\,\mathrm{m})$
1	f_{A}	Metal 1	6.9
2	f_{A}	Metal 2	9.4
3	f_{B}	Metal 1	No electrons ejected
4	f_{B}	Metal 2	No electrons ejected
5	f_{C}	Metal 1	5.3
6	f_{C}	Metal 2	6.3

(c) **Indicate** whether the work function of Metal 1 is greater than, less than, or equal to the work function of Metal 2. **Justify** your answer by referring to the table of results.

解：

比较同频 fA 的 Trial 1 与 Trial 2：入射光子能量相同，Metal 1 的电子波长更短，最大动能更大。逸出功等于入射光子能量减最大电子动能，所以在相同总光子能量中用于逸出的部分较小，所以：

$$\phi_1 < \phi_2.$$

同频 fC 的两行也支持这个结论；不能拿不同频率下的两行直接比较逸出功。

光子能量能否造出电子：比较静能

质量m的静能为mc^2。要否定题目“光子能量足够增加一个电子”的说法，只需把单光子能量与一个电子的静能用同单位比较。焦耳对焦耳，保留科学计数法指数；光子能量小于所需静能时，能量已不足。

$$E_{\text{rest}} = m_e c^2.$$

例题 2022 Q3 d(ii)

本问承接 2022 Q3 d(i)。

(d) Experiments show that a hydrogen atom can absorb a photon of frequency 3.2×10^{15} Hz.

ii. A student claims that when a hydrogen atom absorbs a photon at this frequency, the energy could be converted into mass, adding an electron to the atom. Calculate the amount of energy needed to create a particle with the mass of an electron and determine whether or not there is sufficient energy gained by the atom to add another electron.

解：

单个电子的静能用 mec^2，仍以焦耳比较：

$$E_{rest} = m_e c^2 = (9.11 \times 10^{-31})(3.00 \times 10^8)^2 \text{ J} = 8.20 \times 10^{-14} \text{ J},$$

$$\frac{E_\gamma}{E_{rest}} \approx 2.59 \times 10^{-5} \ll 1.$$

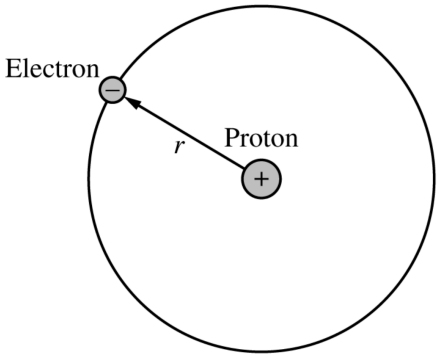
该光子连一个电子的静能都远未达到，因此题中“有足够能量添加一个电子”的判断错误。此能量不足已足以否定该判断，无需再假设额外能量来源。

库仑吸引作圆周合力：求电子速度

使用原题的圆轨道模型，质子静止、引力忽略。e表示电荷量大小；质子与电子异号相吸，库仑力是唯一径向合力。把它直接与mev^2/r相等，不额外再加一个向心力。

$$\frac{ke^2}{r^2} = \frac{m_e v^2}{r}.$$

例题 2022 Q3 a



Note: Figure not drawn to scale.

(12 points, suggested time 25 minutes)

A hydrogen atom can be modeled as an electron in a circular orbit of radius r about a stationary proton, as shown above. The gravitational force between the proton and electron is negligible compared to the electrostatic force between them.

(a) Derive an equation for the speed v of the electron in terms of r and physical constants, as appropriate.

解：

按题定模型，质子静止、电子沿半径 r 做圆周运动，忽略引力。e 表示电荷大小，电子与质子异号相吸。库仑力就是径向合力，不另加一种“向心力”：

$$\frac{ke^2}{r^2}=\frac{m_e v^2}{r},\qquad v^2=\frac{ke^2}{m_e r},\qquad v=\sqrt{\frac{ke^2}{m_e r}}.$$

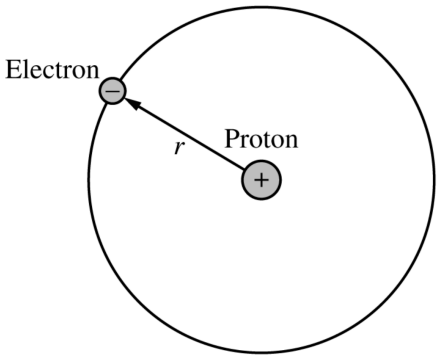
束缚原子总能量：正动能加负电势能

把无限远的相互作用电势能定为零，异号电荷相互作用为负。总能量等于动能加电势能；圆轨道速度已经从受力求出，代入动能式消去速度，再与负的电势能相加。负总能量表示束缚态，不表示动能为负。

$$U_E=-\frac{ke^2}{r},\qquad K=\frac{1}{2}m_e v^2,\qquad E=U_E+K.$$

例题 2022 Q3 b

本问承接 2022 Q3 a。



Note: Figure not drawn to scale.

(12 points, suggested time 25 minutes)

A hydrogen atom can be modeled as an electron in a circular orbit of radius r about a stationary proton, as shown above. The gravitational force between the proton and electron is negligible compared to the electrostatic force between them.

(b) Derive an equation for the total energy of the atom in terms of r and physical constants, as appropriate.

解：

以无限远处相互作用电势能为零，异号电荷的电势能为负。电子动能用 (a) 消去速度：

$$U_E = \frac{k(+e)(-e)}{r} = -\frac{ke^2}{r}, \qquad K = \frac{1}{2}m_e v^2 = \frac{ke^2}{2r}.$$

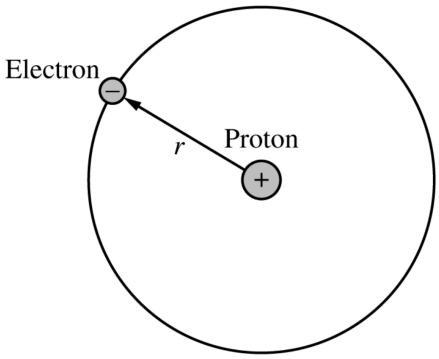
$$E = U_E + K = -\frac{ke^2}{r} + \frac{ke^2}{2r} = -\frac{ke^2}{2r}.$$

半径增大：负能量向零靠近

先看符号，再看分母。负数−C/r中C是正量，r增大使绝对值减小，数值反而增大、向零靠近。这与原子吸收能量相符。解释一致性时要把“半径变大”和“能量增加”用负号明确连接。

例题 2022 Q3 c

本问承接 2022 Q3 b。



Note: Figure not drawn to scale.

(12 points, suggested time 25 minutes)

A hydrogen atom can be modeled as an electron in a circular orbit of radius r about a stationary proton, as shown above. The gravitational force between the proton and electron is negligible compared to the electrostatic force between them.

(c) When the hydrogen atom absorbs a photon, the electron moves to an orbit with a larger radius and the total energy of the atom increases. Is your equation for the energy derived in part (b) consistent with this description of the model of a hydrogen atom absorbing a photon? Explain why the equation is or is not consistent.

解：

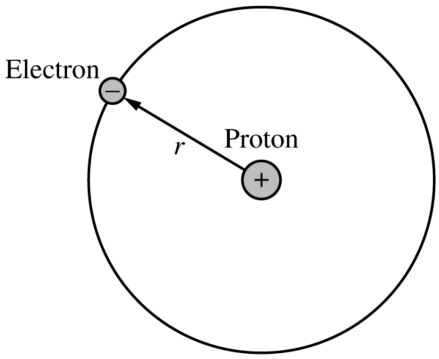
一致。r 增大使这个负数的绝对值减小，能量从更负变成较不负，是能量增加，符合吸收光子。不能把“分母增大”机械判断成能量减小，要先看负号。

较大半径束缚轨道：同步改变两能量柱

从圆轨道受力与能量结果可知，动能正且与1/r成正比，电势能负且等于动能的负两倍。吸收后半径变大，两柱的绝对高度都缩短，保持U=−2K。末态须仍是物理上可行的一组柱，不只随意画一条更高总能量。

例题 2022 Q3 d(iii)

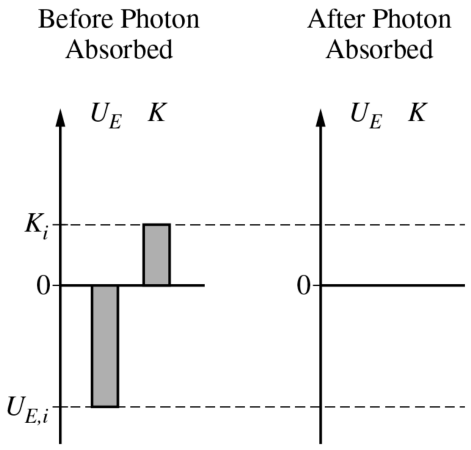
本问承接 2022 Q3 a、2022 Q3 b、2022 Q3 c。



Note: Figure not drawn to scale.

(12 points, suggested time 25 minutes)

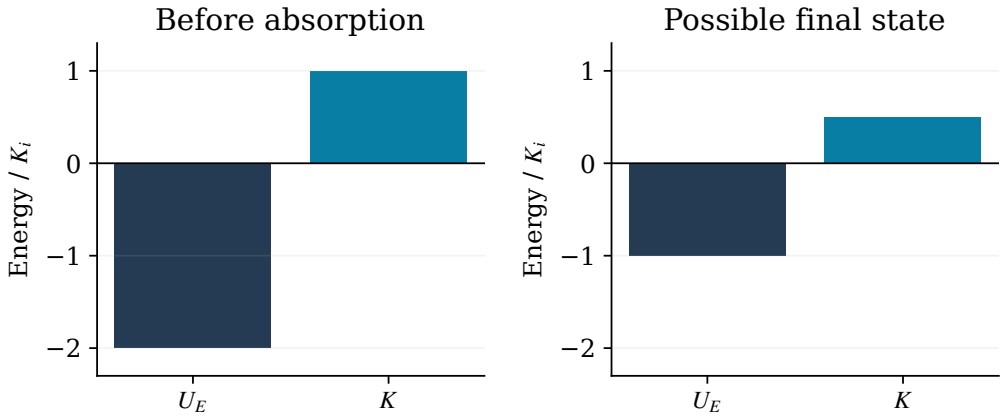
A hydrogen atom can be modeled as an electron in a circular orbit of radius r about a stationary proton, as shown above. The gravitational force between the proton and electron is negligible compared to the electrostatic force between them.



iii. The left bar chart in the figure above is complete and represents the initial electric potential energy $U_{E,\,i}$ of the atom and the initial kinetic energy K_i of the electron before the photon is absorbed. In the space provided on the right, draw a bar chart to represent a possible final electric potential energy of the atom and final kinetic energy of the electron.

解：

吸收后仍在较大半径的束缚圆轨道。UE 仍负，但柱的向下长度较短；K 仍正，但向上长度也较短。同一圆轨道的电势能等于动能的负二倍，因此可画一种保持这一比例的末态。



一种允许的末态示意：两柱绝对值都变小，UE 仍为负、K 仍为正；并非指定唯一末半径。

实验数据、作图与参数反求

先把数据换成坐标，再定刻度与描点

用前一问已选择的横纵轴量，把每行原数据对应成一个坐标。量纲或数量级写在轴旁；等距离刻度必须代表相同增量。用覆盖全部数据的尺度，让点占图的大部分区域，按坐标放点；这个阶段只描点，最佳拟合线在下一动作才画。

例题 2025 Q3 C(ii)

本问承接 2025 Q3 C(i)。

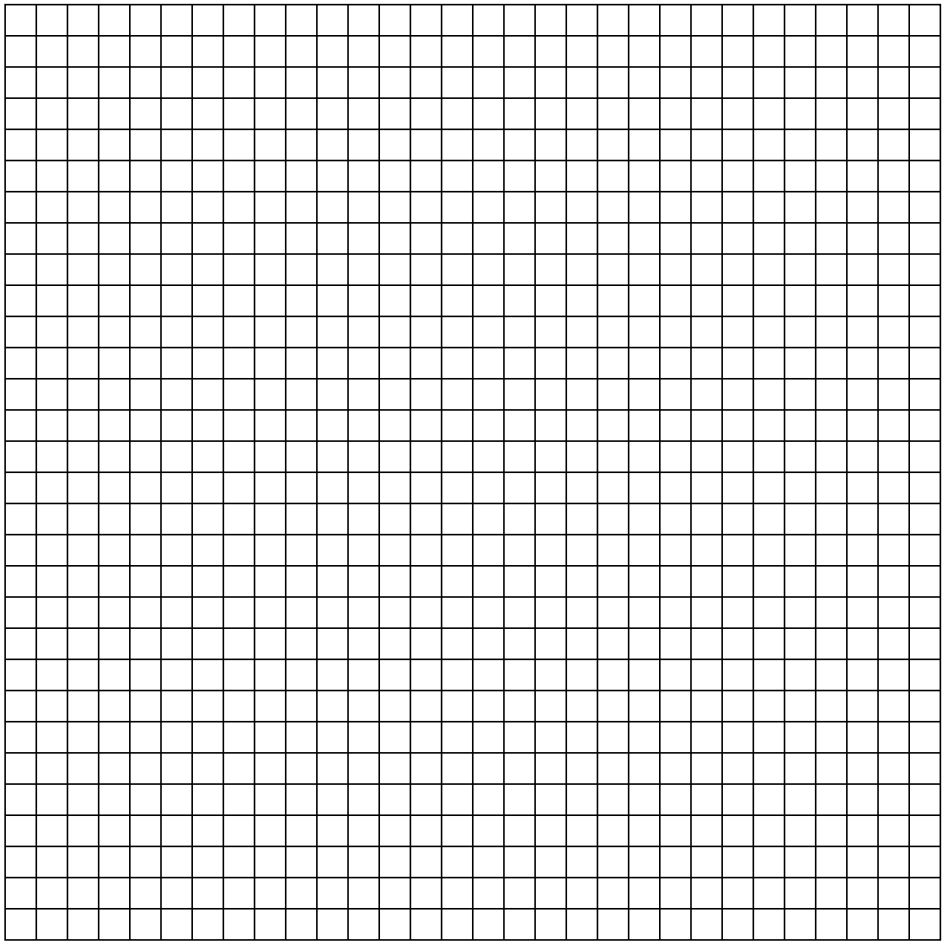
C. In Experiment 2, the student is asked to determine the capacitance C of a new parallel-plate capacitor. For each trial, the absolute value $|\Delta V|$ of the potential difference across the capacitor is varied and the charge q stored on one plate of the fully charged capacitor is measured. Table 1 contains the data collected.

Table 1

$\lvert\Delta V\rvert$ (V)	$q\ (\times 10^{-10}\ \mathrm{C})$
3.0	2.4
5.0	4.2
7.2	5.6
8.0	6.6
10.0	8.0

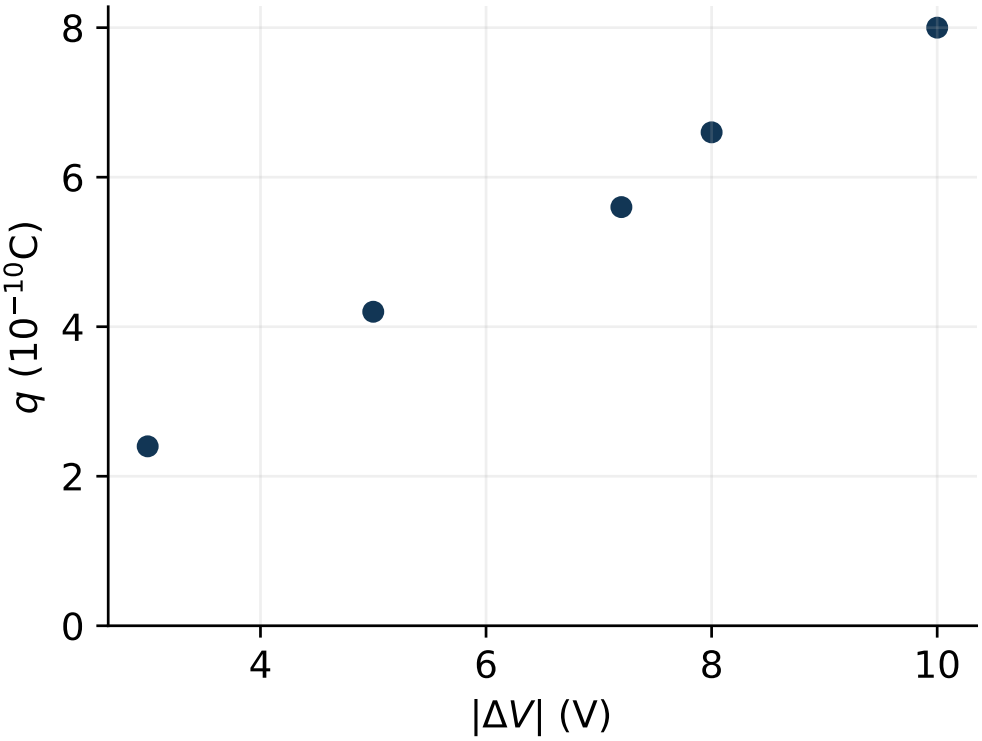
ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.
- **Plot** the points you recorded in Table 2.



解：

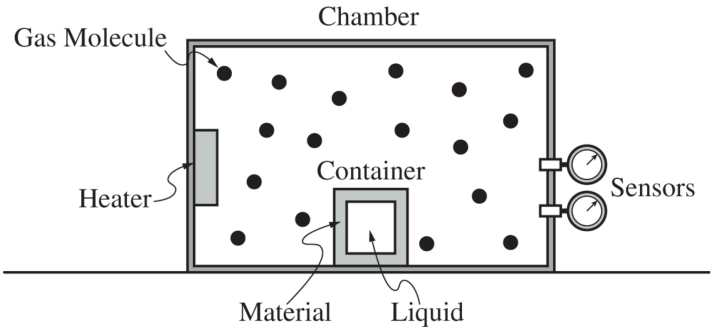
C(ii) 按原表画五点；若纵轴按 10^{-10} C 标度，五点为 (3.0,2.4)、(5.0,4.2)、(7.2,5.6)、(8.0,6.6)、(10.0,8.0)。两轴使用均匀数值刻度，单位写在轴旁。



只描五个原始实验坐标，尚未画拟合线。

练习 2024 Q2 c(ii)

本问承接 2024 Q2 c(i)。



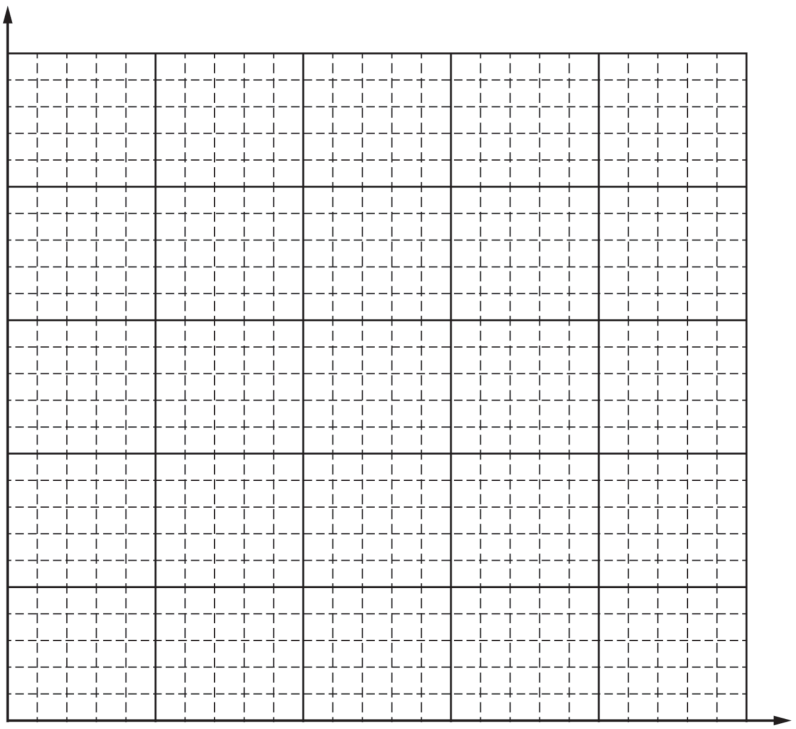
Note: Figure not drawn to scale.

Figure 2

In Experiment 2, shown in Figure 2, a liquid-filled container that is completely wrapped with a material of uniform thickness 0.01 m is inside the sealed chamber that is filled with an ideal gas. The material has a total area of 0.06 m^2 in contact with the gas. The heater is turned on. As the temperature T_G of the gas increases, the following data for the temperature T_L of the liquid and the rate $\frac{Q}{\Delta t}$ of energy transfer are collected.

	T_G (K)	T_L (K)	$\frac{Q}{\Delta t}\left(\frac{J}{s}\right)$	
	295	295	0.0	
	371	303	26.3	
	425	308	43.1	
	475	313	60.0	
	528	323	75.0	

ii. **Plot** the data points for the quantities indicated in part (c)(i) on the graph provided. Clearly scale and **label** all axes, including units, as appropriate.



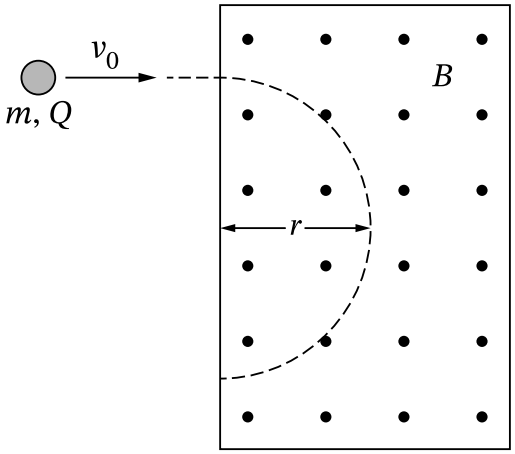
练习 2026 Q3 C(ii)

本问承接 2026 Q3 C(i)。

In Experiment 2, scientists want to use a graph to determine the mass m of identical, charged particles that are emitted from a device. Each particle has charge $Q = +6.4 \times 10^{-19}$ C.

In each trial, a particle is emitted with speed $v_0 = 3.0 \times 10^{6}$ m/s toward a region of external, uniform magnetic field of variable magnitude B. The radius r of the path along which the particle moves while in the field, shown in Figure 2, is recorded.

Figure 2



In subsequent trials, B is increased and the resulting r is recorded. Gravitational effects are negligible. Collected data are provided in Table 1.

Table 1

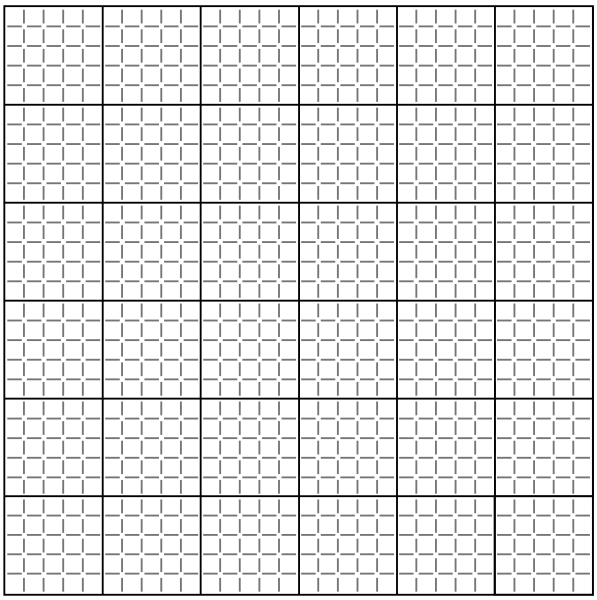
B (T)	r (m)
0.04	1.8
0.06	1.2
0.14	0.5
0.16	0.4
0.20	0.3

ii. On the grid, create a graph of the quantities indicated in part C (i).

- **Label** the vertical and horizontal axes with numerical scales.
- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work, but the table will not be scored.

Quantity (units, if appropriate)

_____ _____



Quantity (units, if appropriate)

_____ _____

描述全体数据趋势：画最佳拟合直线

实验点有误差，不应依次用折线连接，也不要求穿过每个点。一条直线应代表所有点的总体趋势，使偏离在上下两侧大致均衡；除非模型或原数据要求，不机械强迫通过原点。之后求斜率要在这条拟合线上选相距较远的两个点。

例题 2025 Q3 C(iii)

本问承接 2025 Q3 C(i)、2025 Q3 C(ii)。

C. In Experiment 2, the student is asked to determine the capacitance C of a new parallel-plate capacitor. For each trial, the absolute value $|\Delta V|$ of the potential difference across the capacitor is varied and the charge q stored on one plate of the fully charged capacitor is measured. Table 1 contains the data collected.

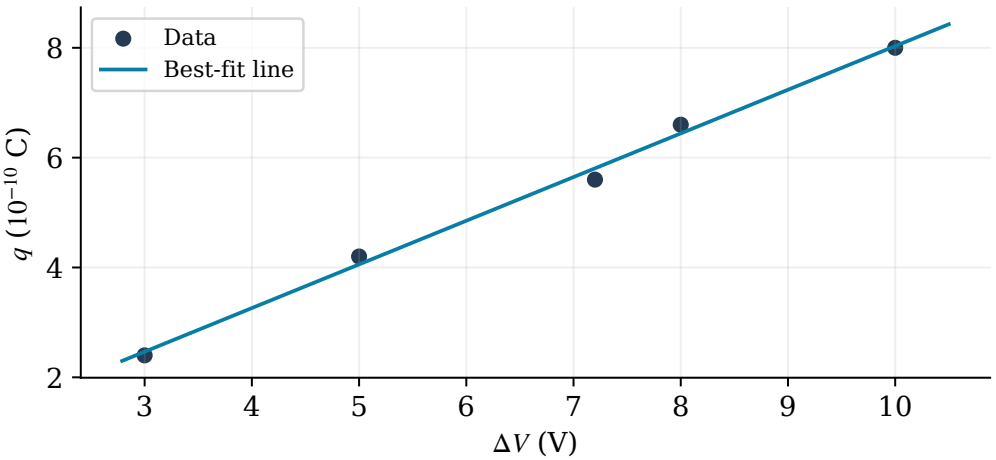
Table 1

$\lvert\Delta V\rvert$ (V)	q $(\times 10^{-10}$ C$)$
3.0	2.4
5.0	4.2
7.2	5.6
8.0	6.6
10.0	8.0

iii. **Draw** a best-fit line for the data graphed in part C (ii).

解：

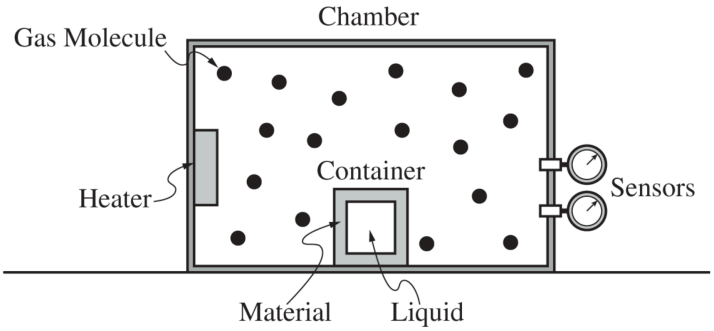
C(iii) 用一条直线表示全部点的趋势，不逐点连折线。



五个原始数据点及最佳拟合直线；纵轴刻度每单位代表 10^{-10} C。

练习 2024 Q2 c(iii)

本问承接 2024 Q2 c(i)、2024 Q2 c(ii)。



Note: Figure not drawn to scale.

Figure 2

In Experiment 2, shown in Figure 2, a liquid-filled container that is completely wrapped with a material of uniform thickness 0.01 m is inside the sealed chamber that is filled with an ideal gas. The material has a total area of 0.06 m^2 in contact with the gas. The heater is turned on. As the temperature T_G of the gas increases, the following data for the temperature T_L of the liquid and the rate $\frac{Q}{\Delta t}$ of energy transfer are collected.

	T_G (K)	T_L (K)	$\frac{Q}{\Delta t}\left(\frac{J}{s}\right)$	
	295	295	0.0	
	371	303	26.3	
	425	308	43.1	
	475	313	60.0	
	528	323	75.0	

iii. **Draw** the best-fit line for the data graphed in part (c)(ii).

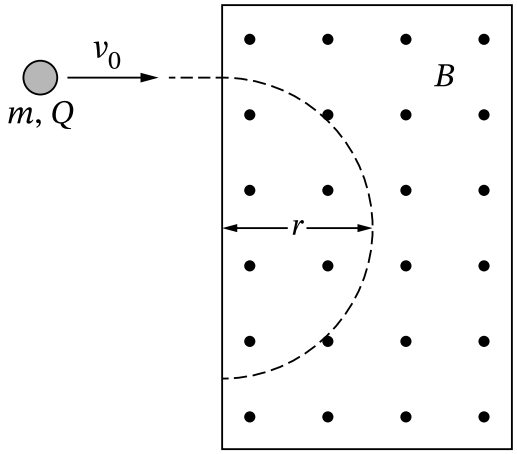
练习 2026 Q3 C(iii)

本问承接 2026 Q3 C(i)、2026 Q3 C(ii)。

In Experiment 2, scientists want to use a graph to determine the mass m of identical, charged particles that are emitted from a device. Each particle has charge $Q = +6.4 \times 10^{-19}$ C.

In each trial, a particle is emitted with speed $v_0 = 3.0 \times 10^{6}$ m/s toward a region of external, uniform magnetic field of variable magnitude B. The radius r of the path along which the particle moves while in the field, shown in Figure 2, is recorded.

Figure 2



In subsequent trials, B is increased and the resulting r is recorded. Gravitational effects are negligible. Collected data are provided in Table 1.

Table 1

B (T)	r (m)
0.04	1.8
0.06	1.2
0.14	0.5
0.16	0.4
0.20	0.3

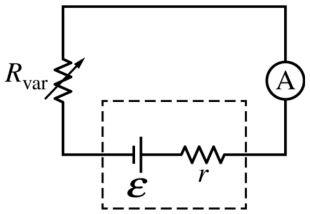
iii. **Draw** a best-fit line for the data graphed in part C (ii).

自行完成变换、描点与拟合

前面已经分别完成坐标变换、描点和拟合。本组原小问把这些动作同时要求：沿用自己前一问的坐标选择，先列表算所有坐标，再统一标尺、描点，最后画整体趋势线。内阻实验要先加内阻再取倒数；串联电容先由已知电容求共同电荷。

例题 2023 Q2 b(iii)

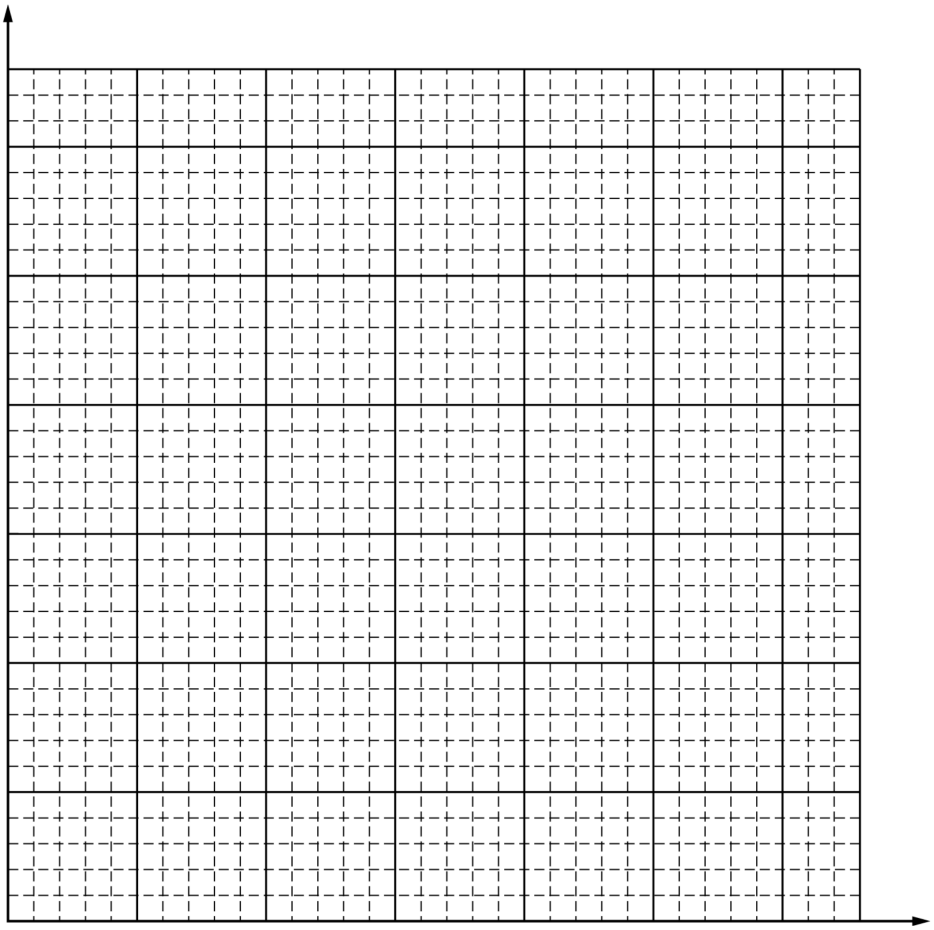
本问承接 2023 Q2 b(ii)。



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\,\Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

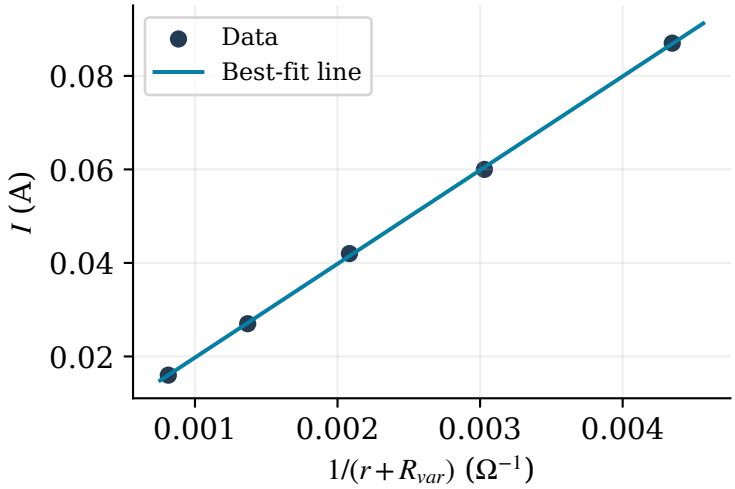
I (A)	R_{var} (Ω)		
0.087	200		
0.060	300		
0.042	450		
0.027	700		
0.016	1200		

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



解：

五个横轴坐标约为 0.004348、0.003030、0.002083、0.001370、0.0008130 Ω^{-1}；分别与原表五个电流相配。标刻度、量和单位，画全部点及最佳拟合直线。



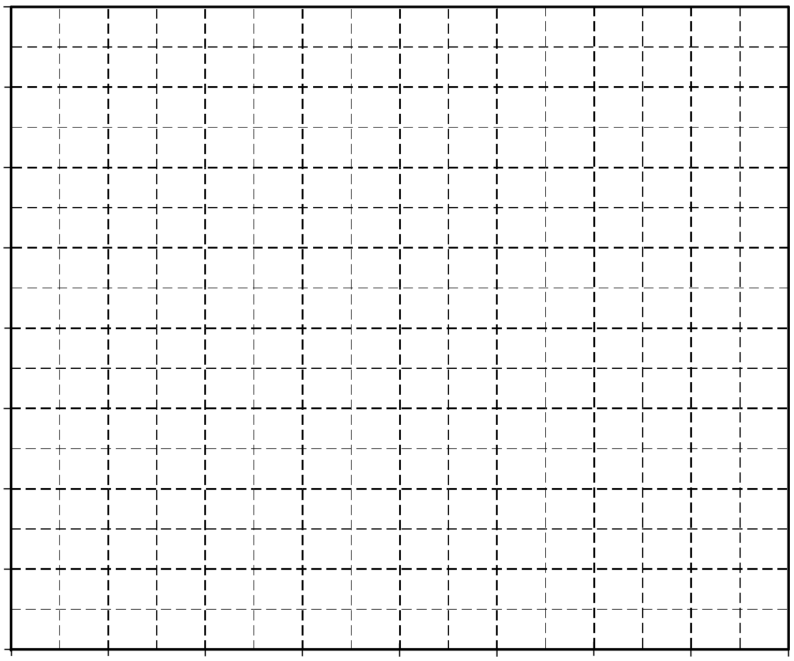
练习 2022 Q2 c(ii)

本问承接 2022 Q2 c(i)。

In another experiment, the students have a capacitor with unknown capacitance C_U. They want to determine C_U by using a battery of potential difference 4.5 V and several other capacitors of known capacitance. They create circuits with the battery, the unknown capacitor, and one of the capacitors of known capacitance. The students wait until the capacitors are fully charged and then record the potential difference ΔV_{known} across the known capacitor and the potential difference ΔV_U across the unknown capacitor. Their data are shown in the table on the following page.

Known Capacitance of Capacitors (μF)	ΔV_{known} (V)	ΔV_U (V)		
200	0.91	3.53		
300	0.65	3.74		
400	0.51	3.95		
500	0.42	4.06		
600	0.36	4.17		

ii. Label the axes below and provide an appropriate scale with units. Plot the data points for the quantities indicated in part (c)(i) on the axes and draw a best-fit line.



拟合线斜率：先辨物理系数，再求参数

斜率s是纵坐标改变量除横坐标改变量，取自拟合线上两个相距较远的点。它代表什么，须回到各自的线性物理式；q对ΔV图的斜率直接是电容，传热率对温差图的斜率却是kA/L，r对1/B图的斜率是mv0/Q。先写对应关系并解目标参数，再代数值和单位，不能看到直线就把斜率当答案。

$$s=\frac{y_2-y_1}{x_2-x_1}.$$

例题 2025 Q3 D

本问承接 2025 Q3 C(i)、2025 Q3 C(ii)、2025 Q3 C(iii)。

C. In Experiment 2, the student is asked to determine the capacitance C of a new parallel-plate capacitor. For each trial, the absolute value $|\Delta V|$ of the potential difference across the capacitor is varied and the charge q stored on one plate of the fully charged capacitor is measured. Table 1 contains the data collected.

Table 1

$\lvert\Delta V\rvert$ (V)	$q\ (\times 10^{-10}\ \mathrm{C})$
3.0	2.4
5.0	4.2
7.2	5.6
8.0	6.6
10.0	8.0

D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for capacitance C.

解：

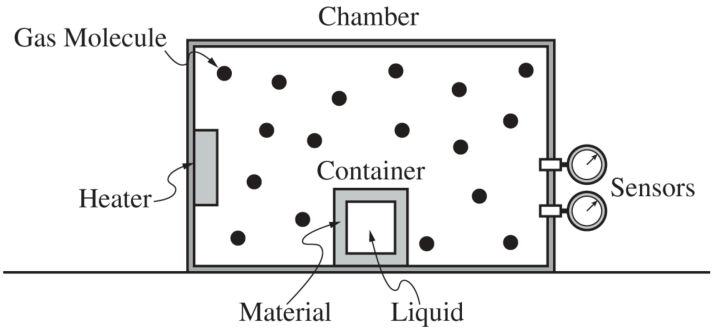
图中全点拟合斜率约为 0.7950 个纵轴单位每伏特。例如在这条拟合线上取电势差 3.0、9.0 V，电荷约为 2.466、7.236×10^{-10} C：

$$C=\frac{\Delta q}{\Delta(\Delta V)}\approx\frac{(7.236-2.466)\times10^{-10}\ \mathrm{C}}{(9.0-3.0)\ \mathrm{V}}\approx 7.95\times10^{-11}\ \mathrm{F}.$$

手绘最佳拟合线给出约 8.0×10^{-11} F 同样恰当。斜率取自拟合线上两个较远点；不能漏掉纵轴的 10^{-10}，也不能只取一个原始点的 q/ΔV。

练习 2024 Q2 d

本问承接 2024 Q2 c(i)、2024 Q2 c(ii)、2024 Q2 c(iii)。



Note: Figure not drawn to scale.

Figure 2

In Experiment 2, shown in Figure 2, a liquid-filled container that is completely wrapped with a material of uniform thickness 0.01 m is inside the sealed chamber that is filled with an ideal gas. The material has a total area of 0.06 m^2 in contact with the gas. The heater is turned on. As the temperature T_G of the gas increases, the following data for the temperature T_L of the liquid and the rate $\frac{Q}{\Delta t}$ of energy transfer are collected.

	T_G (K)	T_L (K)	$\frac{Q}{\Delta t}\left(\frac{J}{s}\right)$	
	295	295	0.0	
	371	303	26.3	
	425	308	43.1	
	475	313	60.0	
	528	323	75.0	

(d) Using the best-fit line, **calculate** an experimental value for k.

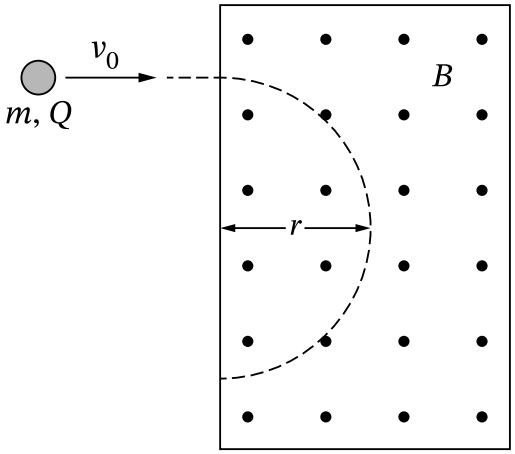
练习 2026 Q3 D

本问承接 2026 Q3 C(i)、2026 Q3 C(ii)、2026 Q3 C(iii)。

In Experiment 2, scientists want to use a graph to determine the mass m of identical, charged particles that are emitted from a device. Each particle has charge $Q = +6.4 \times 10^{-19}$ C.

In each trial, a particle is emitted with speed $v_0 = 3.0 \times 10^{6}$ m/s toward a region of external, uniform magnetic field of variable magnitude B. The radius r of the path along which the particle moves while in the field, shown in Figure 2, is recorded.

Figure 2



In subsequent trials, B is increased and the resulting r is recorded. Gravitational effects are negligible. Collected data are provided in Table 1.

Table 1

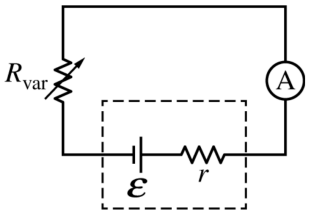
B (T)	r (m)
0.04	1.8
0.06	1.2
0.14	0.5
0.16	0.4
0.20	0.3

Part D

Calculate m from the best-fit line drawn on the grid.

练习 2023 Q2 b(iv)

本问承接 2023 Q2 b(ii)、2023 Q2 b(iii)。



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\,\Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)		
0.087	200		
0.060	300		
0.042	450		
0.027	700		
0.016	1200		

iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

练习 2022 Q2 c(iii)

本问承接 2022 Q2 c(i)、2022 Q2 c(ii)。

In another experiment, the students have a capacitor with unknown capacitance C_U. They want to determine C_U by using a battery of potential difference 4.5 V and several other capacitors of known capacitance. They create circuits with the battery, the unknown capacitor, and one of the capacitors of known capacitance. The students wait until the capacitors are fully charged and then record the potential difference ΔV_{known} across the known capacitor and the potential difference ΔV_U across the unknown capacitor. Their data are shown in the table on the following page.

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400	0.51	3.95		
500	0.42	4.06		
600	0.36	4.17		

iii. Using your best-fit line, determine the capacitance of capacitor C_U.