AP Physics C · 电磁学

学生用书

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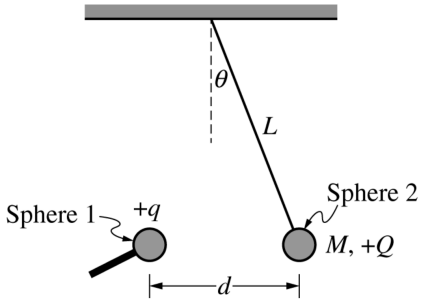
电荷与静电平衡

先认作用对象，再画真实受力

受力图的圆点代表一个物体，每个箭头代表另一个物体对它施加的力。接触产生支持力或张力，地球产生重力，另一个带电球产生电力。

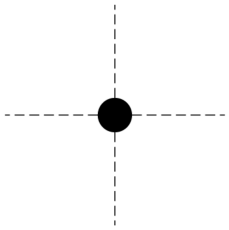
箭头从圆点出发；只画真实力，不把一个力的水平、竖直分量画成额外的力。同号相斥，所以电力沿两球球心连线远离另一球。细线的张力沿线指向悬点；平台的支持力垂直平台。

例题 2023 Set 1 Q1 (a)



1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

(a) On the following dot that represents Sphere 2 at the position shown in the previous figure, draw and label the forces (not components) that act on Sphere 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

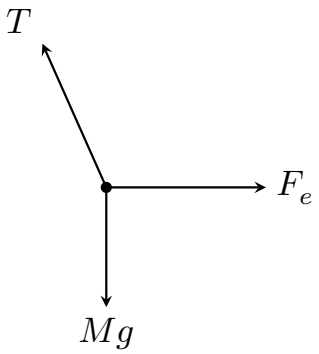


解：

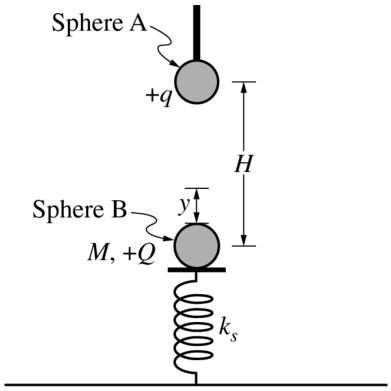
(a) 球2的受力图同号电荷排斥，所以球2受到向右的电力F_e；重力

$$Mg$$

向下；张力T沿线向左上方。图上只画这三支真实的力，不把张力分量再画成额外的力。



练习 2023 Set 2 Q1 (a)

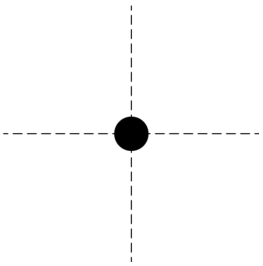


Note: Figure not drawn to scale.

1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

   Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

(a) On the following dot that represents Sphere B in the figure on the previous page, draw and label the forces (not components) that are exerted on Sphere B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



悬球偏角：用两方向平衡消去张力

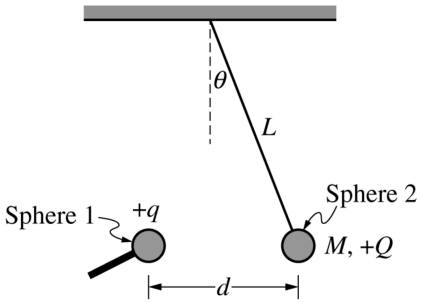
球静止，水平和竖直合力分别为零。题中两球同高，使电力完全水平；细线与竖直夹角θ，因此Tcosθ向上，Tsinθ向左。

两式相除消去同一个未知张力，再把库仑力代入。M是球质量，Q、q是两球电荷，d是球心距；ε_0是真空介电常数。

$$T\cos\theta = Mg, \quad T\sin\theta = F_e, \quad F_e = \frac{Qq}{4\pi\varepsilon_0 d^2}.$$

例题 2023 Set 1 Q1 (b)

本问承接 2023 Set 1 Q1 (a)。



1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

(b) Derive the relationship between the distance d and the angle θ to show that $d = \sqrt{\dfrac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$.

解：

(b) 推导距离与偏角的关系球静止，两个方向的合力分别为零。张力与竖直方向夹角是θ，因此竖直分量是

$$T\cos\theta,$$

水平分量大小是

$$T\sin\theta:$$

$$T\cos\theta = Mg, \qquad T\sin\theta = F_e = \frac{Qq}{4\pi\varepsilon_0 d^2}.$$

把水平式除以竖直式，消去测不到的张力；本题

$$M > 0$$

且

$$\cos\theta > 0,$$

分母非零:

$$\tan\theta = \frac{Qq}{4\pi\varepsilon_0 Mg\,d^2} \quad \Rightarrow \quad \boxed{d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}}.$$

距离取正根。若只从力式中移项而忘记最后开平方，所得量的单位会是平方米，不能作为d。

弹簧压缩量：保留重力的常数项

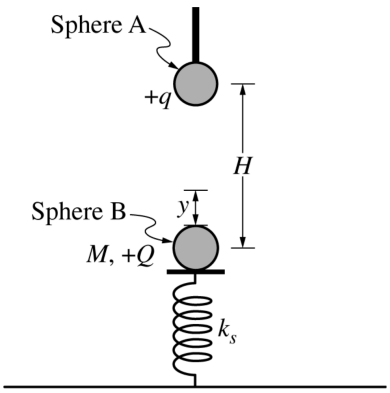
上球排斥下球，下球同时受向下的重力。平台质量可忽略时，向上支持力等于弹簧力k_sy；y从自然长度起算。

先写向上力等于全部向下力，再代入库仑力。把y写成关于1/H^2的一次关系时，重力仍存在，因此图线有非零截距。

$$k_s y = Mg + \frac{Qq}{4\pi\varepsilon_0 H^2}.$$

例题 2023 Set 2 Q1 (b)

本问承接 2023 Set 2 Q1 (a)。



Note: Figure not drawn to scale.

1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

   Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

(b) Derive the relationship between y and H to show that $y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$.

解：

(b) 压缩量与距离的关系平衡时向上支持力等于两个向下力之和：

$$N - Mg - F_e = 0, \qquad k_s y = Mg + \frac{Qq}{4\pi\varepsilon_0 H^2}.$$

除以正的劲度系数后得到题目要求的关系：

$$\boxed{y = \frac{Qq}{4\pi\varepsilon_0 k_s}\frac{1}{H^2} + \frac{Mg}{k_s}}.$$

当电力趋于零，即H很大时，压缩量仍是

$$Mg/k_s,$$

这个极限可以检查常数项有没有被误删。

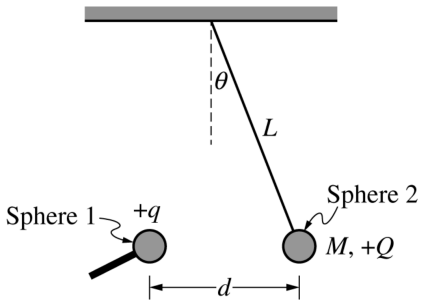
角度与距离都已知：算张力

此问没有给球质量，可先由库仑定律求水平电力，再用水平平衡得到张力。不要引入未知质量作为数值计算的前提。θ仍相对竖直，故电力对应张力的正弦分量。

$$T = \frac{F_e}{\sin\theta}, \qquad F_e = \frac{Qq}{4\pi\varepsilon_0 d^2}.$$

例题 2023 Set 1 Q1 (c)

本问承接 2023 Set 1 Q1 (b)。



1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

(c) These values are collected in one trial: $Q = q = 6.0 \times 10^{-8}$ C, $\theta = 12°$, and $d = 0.057$ m. Calculate the expected force of tension exerted on Sphere 2 by the string.

解：

(c) 张力的数值本次试验给

$$Q = q = 6.0 \times 10^{-8}\ \text{C}、$$

$$\theta = 12^\circ、$$

$$d = 0.057\ \text{m},$$

此小问没有给质量。用水平平衡式即可求张力，不必借用(d)中另一组实验条件的质量：

$$T\sin 12^\circ = \frac{k(6.0\times10^{-8})^2}{(0.057)^2},$$

$$T = \frac{(9.0\times10^{9})(6.0\times10^{-8})^2}{(0.057)^2\sin 12^\circ} \approx \boxed{0.048\ \text{N}}.$$

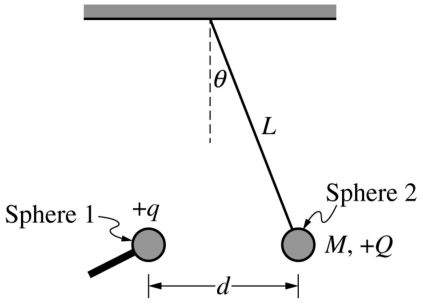
电力约为$0.00997\ \text{N}$，比张力小，与小偏角对应的较小水平分量一致。

导体中电荷重排：定位后解释

导体中的电子可以移动，外来的正球会吸引电子到近侧。近侧负电子较多，远侧电子不足，过剩正电荷因而集中在远侧。

画符号时标最高浓度的位置；解释时说清移动的是电子，孤立导体的净电荷仍保持不变。

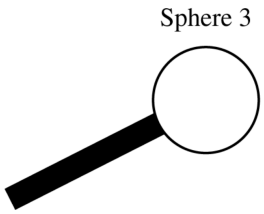
例题 2023 Set 1 Q1 (e)(i)



1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

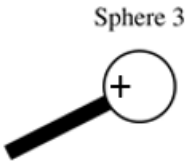
(e) The students modify the experiment by replacing Sphere 1 with a conducting Sphere 3 that has the same size and charge $+q$. The experiment is repeated.

i. The circle in the following figure represents Sphere 3 when spheres 2 and 3 are at equilibrium. On the circle, draw a single "+" sign to represent the location of highest concentration of the excess positive charges.



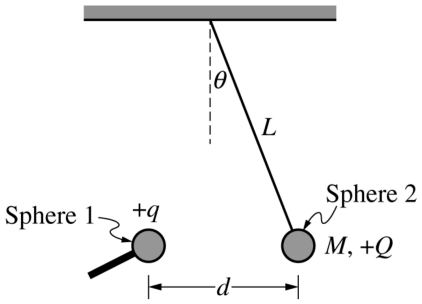
解:

球3位于球2的左侧。球3带正电且是导体，电子能够在球内移动；靠近正球2的一侧吸引更多电子，所以过剩正电荷最集中在远离球2的左侧。在代表球3的圆的最左侧标一个“+”。



练习 2023 Set 1 Q1 (e)(ii)

本问承接 2023 Set 1 Q1 (e)(i)。



1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

(e) The students modify the experiment by replacing Sphere 1 with a conducting Sphere 3 that has the same size and charge $+q$. The experiment is repeated.

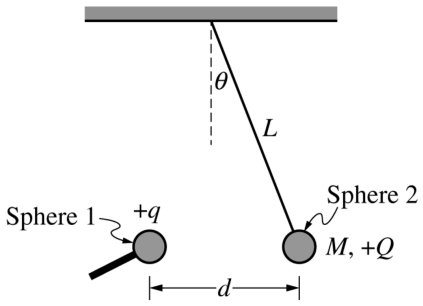
ii. Briefly explain your reasoning for the sketch drawn in part (e)(i).

净电荷相同、分布不同：比较平衡位置

把非导体换成同净电荷导体，不能只比较总电荷。外球使导体近侧电子较多，远侧正电荷较多；较近的异号作用削弱原来排斥力。

随后回到物体的平衡方程：悬球电力较小则偏角较小；弹簧托球电力较小则压缩量较小。

例题 2023 Set 1 Q1 (e)(iii)



1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

(e) The students modify the experiment by replacing Sphere 1 with a conducting Sphere 3 that has the same size and charge $+q$. The experiment is repeated.

iii. In the original experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_1 from the vertical. In the modified experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_2 from the vertical.

Is θ_2 greater than, less than, or equal to θ_1 ?

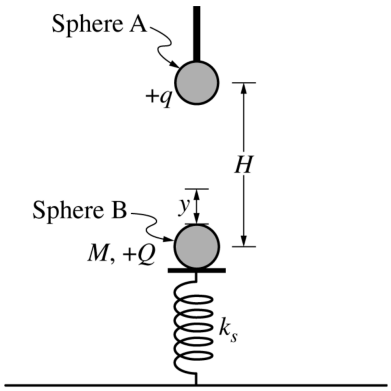
_____ $\theta_2 > \theta_1$　　_____ $\theta_2 < \theta_1$　　_____ $\theta_2 = \theta_1$

Briefly justify your answer.

解：

(e)(iii) 相同中心距下的偏角　导体极化使近侧电子的吸引与远侧剩余正电荷的排斥共同作用；与原均匀非导体球相比，净排斥力减小。球2的质量不变，且仍有 $\tan\theta = F_e/(Mg)$，所以电力减小时偏角减小，选择 $\boxed{\theta_2 < \theta_1}$。不能说“总电荷相同所以力相同”，因为导体在外场中的分布已经不再保持原来的球对称。

练习 2023 Set 2 Q1 (d)(i)



Note: Figure not drawn to scale.

1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

   Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

(d) The students modify the experiment by replacing nonconducting Sphere B with conducting Sphere C that has the same charge $+Q$ and mass M. Sphere A is brought near Sphere C without touching, compressing the spring. Sphere C comes to rest.

i. In the original experiment, when the centers of spheres A and B are a vertical distance H_1 apart, the spring is compressed a distance y_1. In the modified experiment, when the centers of spheres A and C are a vertical distance H_1 apart, the spring is compressed a distance y_2.

Is y_2 greater than, less than, or equal to y_1 ?

____ $y_2 > y_1$ ____ $y_2 < y_1$ ____ $y_2 = y_1$

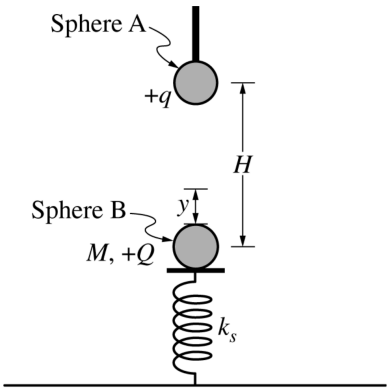
Justify your answer.

接地后最初运动：先比较刚变化的力

接地允许电子与大地交换。附近正球使导体从大地得到电子；电力因此由原来的排斥转为吸引。

刚接地的一瞬间，平台还未移动，弹簧长度及向上支持力来不及改变。先用接地前的平衡判断支持力，再比较接地后向下力是否减少。

例题 2023 Set 2 Q1 (d)(ii)



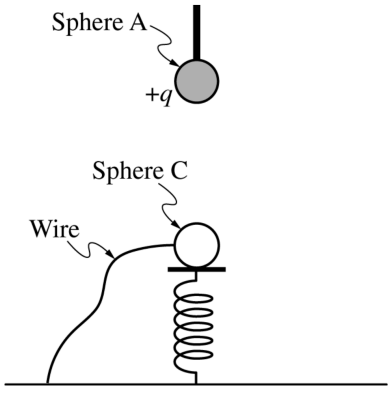
Note: Figure not drawn to scale.

1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

   Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

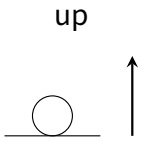
(d) The students modify the experiment by replacing nonconducting Sphere B with conducting Sphere C that has the same charge $+Q$ and mass M. Sphere A is brought near Sphere C without touching, compressing the spring. Sphere C comes to rest.

ii. Sphere C is then grounded with a wire. On the following figure, draw an arrow indicating the direction that the platform will move immediately after being grounded. If the platform remains stationary, write "does not move."



解：

(d)(ii) 接地后平台开始怎样运动？选择向上，并在平台旁画上箭头。接地使电子可由大地进入C，上方正球继续吸引这些电子；原来的向下电排斥减小，接地后的电吸引也朝上。接地瞬间压缩量还来不及改变，弹簧向上的支持力仍保持原值，所以原平衡被打破，平台向上运动。



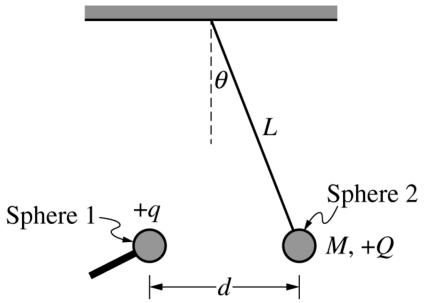
实验图线：拟合、斜率与截距

已有散点：画总体趋势线

最佳拟合线表达全部测量的总体趋势，不把点依次连成折线，也不要求穿过每个点。沿整个横轴范围看趋势，使点分布在线的两侧。

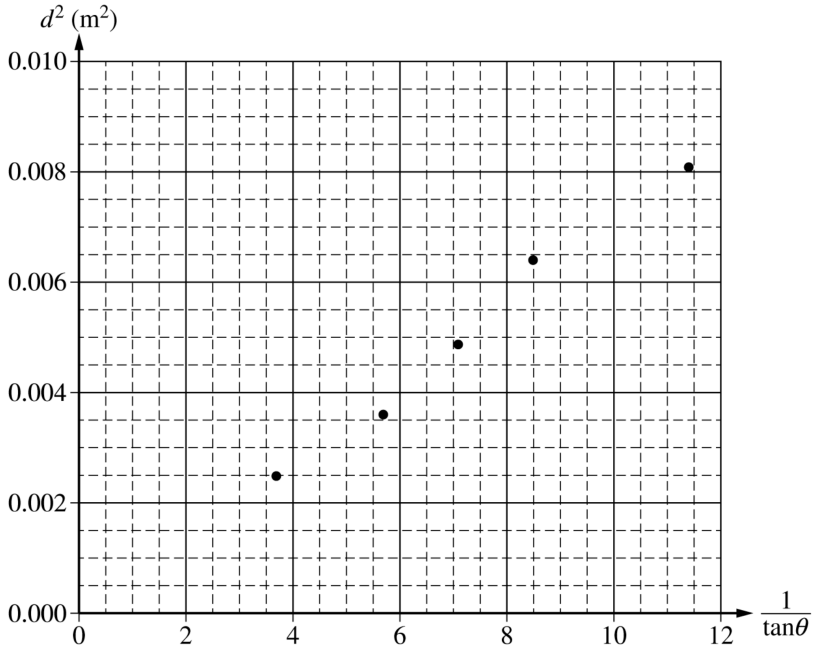
球偏角实验的坐标是d^2对1/tanθ；弹簧实验是y对1/H^2。按原图已有坐标拟合，不把图上的量颠倒。

例题 2023 Set 1 Q1 (d)(i)



1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

(d) The students vary d and measure θ after equilibrium is reached. The students use the collected data to plot the following graph of d^2 vs. $\frac{1}{\tan\theta}$.



i. Draw the best-fit line for the data.

解：

(d)(i) 拟合

$$d^2$$

对

$$1/\tan\theta$$

的直线将(b)平方后的式子与

$$y = mx + b$$

比较：纵轴

$$y = d^2,$$

横轴

$$x = 1/\tan\theta,$$

理论斜率为

$$m = Qq/(4\pi\varepsilon_0 Mg)。$$

画一条概括所有散点趋势的上升直线，不连接相邻点。下面采用同一条近似线上的两点

$$(2.0, 0.0010\ \mathrm{m}^2)$$

与

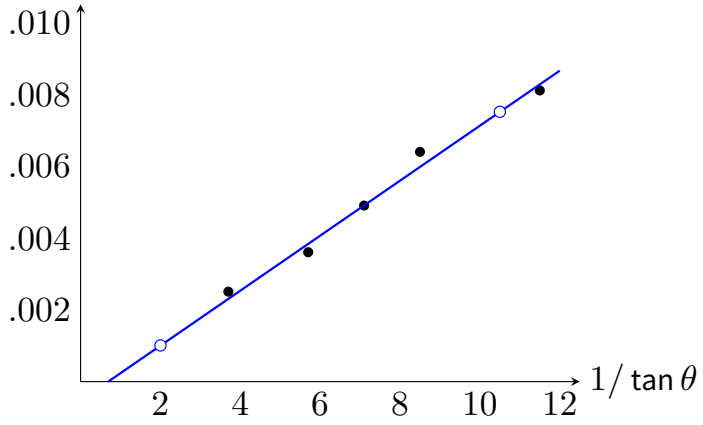
$$(10.5, 0.0075\ \mathrm{m}^2),$$

其斜率为

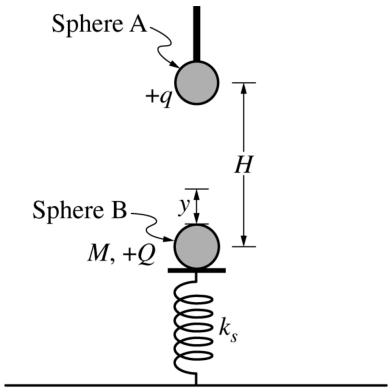
$$7.647 \times 10^{-4}\ \mathrm{m}^2。$$

图中散点位置按原图近似读出，线的微小非零截距属于实验拟合，不改变斜率的物理解释。

$$d^2\ (\mathrm{m}^2)$$



练习 2023 Set 2 Q1 (c)(i)

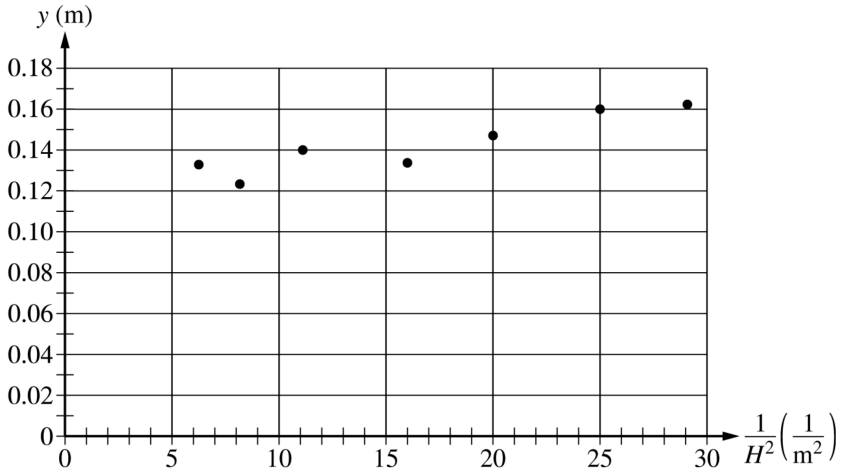


Note: Figure not drawn to scale.

1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

   Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

(c) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph.



i. Draw the best-fit line for the data.

已知直线关系：从斜率反求常量

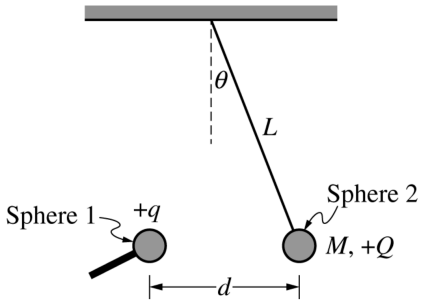
斜率是纵坐标变化除以横坐标变化。在拟合线上选相距较远的两点；不把斜率直接当成所求常量。

悬球实验由平衡式得到d^2=b(1/tanθ)，其中b=Qq/($4\pi\varepsilon_0$Mg)。弹簧实验由平衡式得到y=b(1/H^2)+Mg/k_s，其中b=Qq/($4\pi\varepsilon_0 k_s$)。先把各自的斜率式解成ε_0，再代数值。

$$b=\frac{\Delta y}{\Delta x},\qquad \varepsilon_0=\frac{Qq}{4\pi Mgb}\ \text{（悬球），}\quad \varepsilon_0=\frac{Qq}{4\pi k_s b}\ \text{（弹簧）．}$$

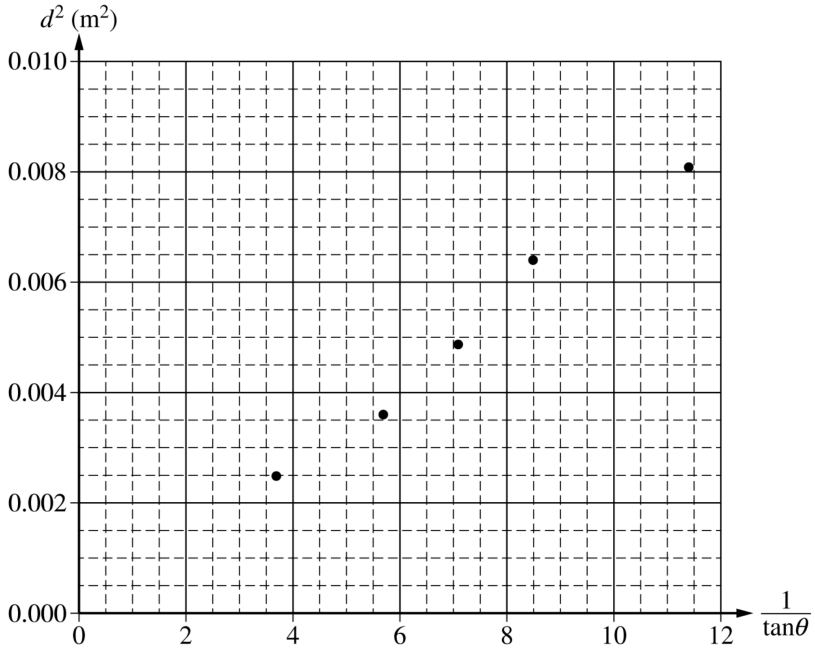
例题 2023 Set 1 Q1 (d)(ii)

本问承接 2023 Set 1 Q1 (d)(i)。



1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

(d) The students vary d and measure θ after equilibrium is reached. The students use the collected data to plot the following graph of d^2 vs. $\frac{1}{\tan\theta}$.



ii. Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $M = 0.0050\,\text{kg}$ and $Q = q = 6.0 \times 10^{-8}\,\text{C}$.

解：

(d)(ii) 由斜率反求真空介电常数从所画线取点算出m后，应先把斜率式解成所需物理量，而不是把斜率误当成ε_0：

$$m = \frac{0.0075 - 0.0010}{10.5 - 2.0} = 7.647 \times 10^{-4}\ \text{m}^2, \qquad \varepsilon_0 = \frac{Qq}{4\pi M g\, m}.$$

本小问

$$M = 0.0050\ \text{kg},$$

所以

$$\varepsilon_0 = \frac{(6.0 \times 10^{-8})^2}{4\pi(0.0050)(9.8)(7.647 \times 10^{-4})} \approx \boxed{7.6 \times 10^{-12}\ \text{C}^2/(\text{N m}^2)}.$$

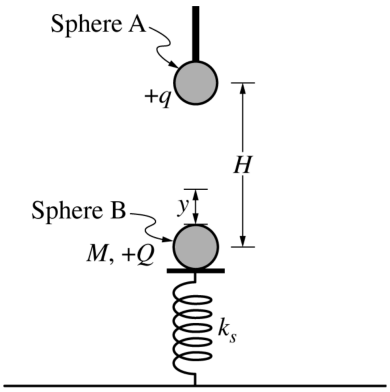
实验值不必等于标称值

$$8.85 \times 10^{-12};$$

换用另一条合理拟合线会给小量差异，但斜率、取点和公式必须来自同一幅图。

练习 2023 Set 2 Q1 (c)(ii)

本问承接 2023 Set 2 Q1 (c)(i)。

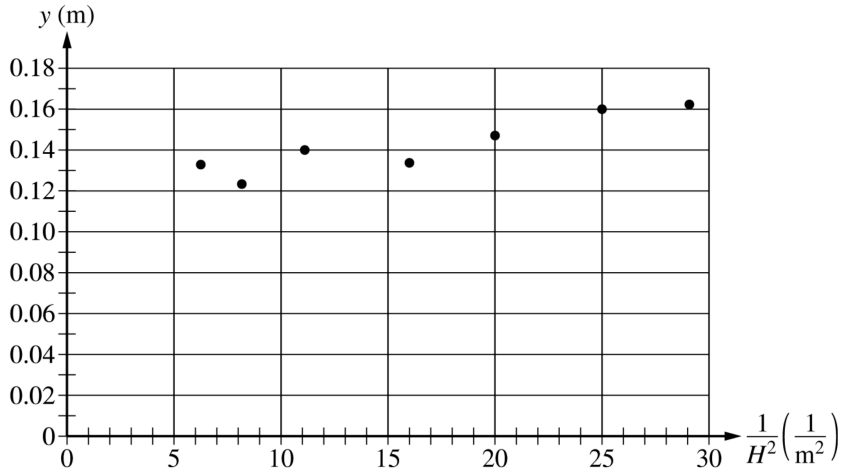


Note: Figure not drawn to scale.

1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

   Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

(c) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph.



ii. Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $Q = q = 2.00 \times 10^{-6}\,\text{C}$ and $k_s = 25\,\text{N/m}$.

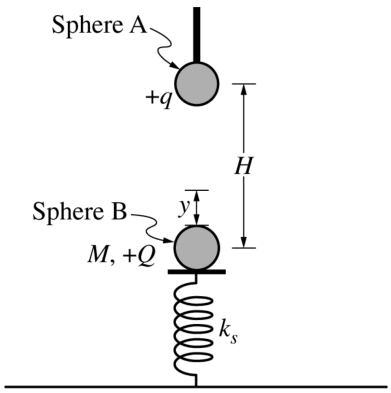
截距保留原有支撑：反求质量

当$1/H^2$趋于零时，上球很远，库仑力趋于零，但弹簧仍支撑重量。图的纵截距y_0因此满足$k_s y_0 = Mg$。读的是拟合线与纵轴交点，不能取任一个数据点的高度。

$$M = \frac{k_s y_0}{g}.$$

例题 2023 Set 2 Q1 (c)(iii)

本问承接 2023 Set 2 Q1 (c)(i)。

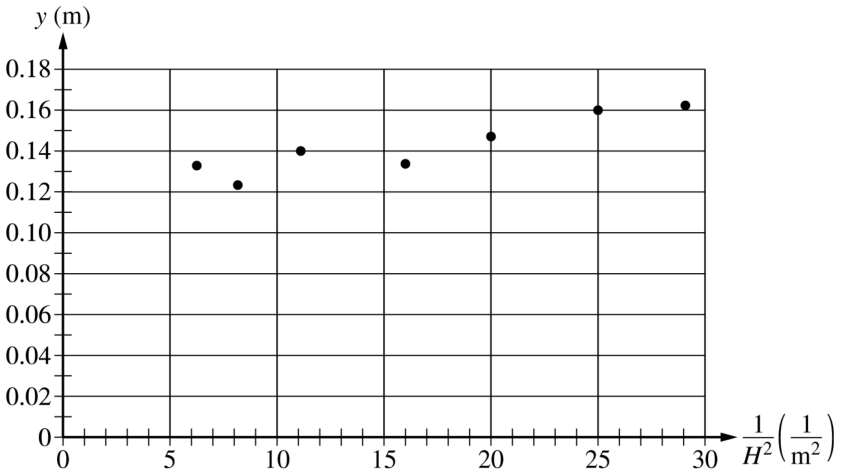


Note: Figure not drawn to scale.

1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

   Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

(c) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph.



iii. Using the best-fit line, calculate an experimental value for the mass of Sphere B.

解：

(c)(iii) 球的质量质量在截距中，而不是在斜率中。用

$$b = Mg/k_s,$$

$$M = \frac{bk_s}{g} = \frac{(0.12)(25)}{9.8} \approx \boxed{0.31\ \text{kg}}.$$

合理读图通常给约$0.30\ \text{kg}$。这是由拟合外推到无电力时的压缩量所求的实验质量。

高斯定律与球、柱的电场

闭合面总通量：只数包围电荷

想象把闭合面分成许多小面积。dA表示一小片面积，向量dA指向该片外法线；E · dA表示电场沿外法线的分量乘面积。闭合积分 $\oint$ 把整个封闭面所有小片贡献相加。

高斯定律说总和只由面内净电荷决定。面外电荷会改变局部电场，但对闭合面的净通量贡献为零。等势值帮助认出题中的面，不能把伏特数直接当成电通量。

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}.$$

例题 2024 Set 1 Q1 (a)

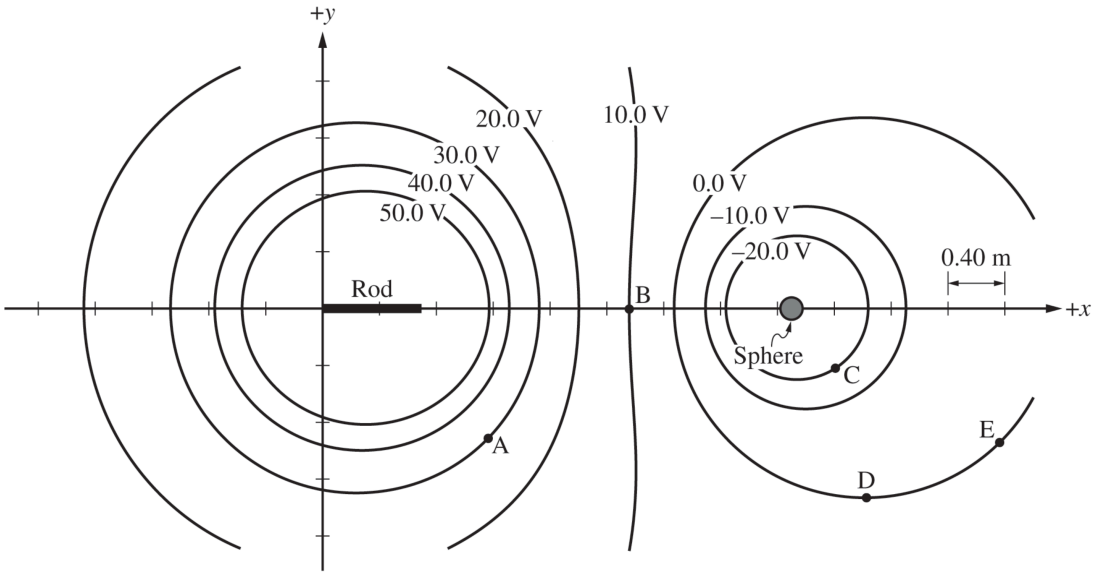


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the -20.0 V equipotential line.

解：

(a)　闭合面的电通量绝对值先看指定的$-20.0\ \mathrm{V}$等势线围住什么：它围住球，没有围住正电杆。高斯定律计算的是穿过闭合面的总电通量，由面内净电荷决定；面外杆可以改变某处的电场，却不改变这一个面的总通量。

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0} = \frac{-2.0\times10^{-9}}{8.85\times10^{-12}}\ \mathrm{N\,m^2/C}.$$

负号表示净通量向内。题目要绝对值，因此

$$\boxed{|\Phi_E| \approx 226\ \mathrm{N\,m^2/C}}.$$

等势值$-20.0\ \mathrm{V}$用来识别面，不能直接代替电荷量或电通量。

练习 2024 Set 2 Q1 (a)

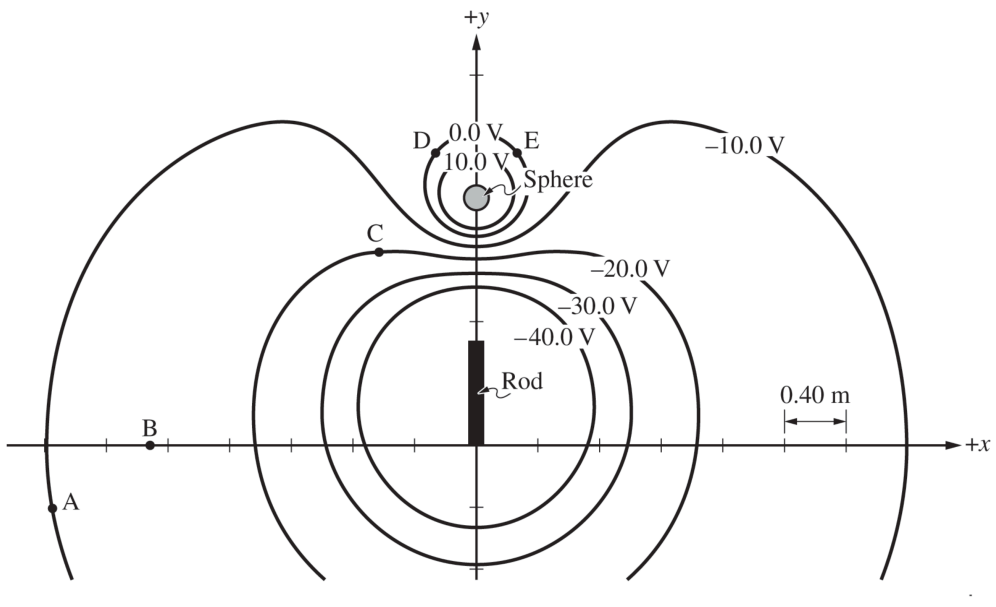


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge +1.0 nC. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are 0.40 m apart.

(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

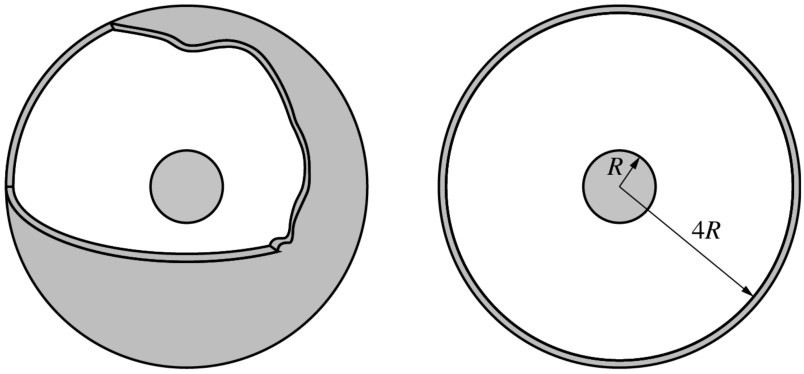
导体壳内场为零：求外表面电荷

静电平衡时金属材料内部场为零。在金属层里选一个包围内带电体的闭合面，高斯定律要求面内净电荷为零；因此内壁电荷必须抵消内带电体电荷。

壳的净电荷等于内壁与外壁电荷之和。知道净电荷后，再减去已求出的内壁电荷，得到外壁电荷。

$$Q_{\text{inner surface}} = -Q_{\text{inside}}, \qquad Q_{\text{outer surface}} = Q_{\text{shell}} - Q_{\text{inner surface}}.$$

例题 2022 Set 1 Q1 (a)



Cutout View　　Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.

解：

(a) 导体壳的外表面电荷静电平衡时导体材料内部电场为零。在壳的材料中选一个包住内球的球形高斯面，通量为零，因此包围的总电荷也为零。内球电荷为

$$-Q,$$

所以壳内表面必须带

$$+Q。$$

壳的净电荷是其两表面电荷之和，故

$$q_{\text{outer}} = 3Q - (+Q) = \boxed{+2Q}.$$

这里不能把

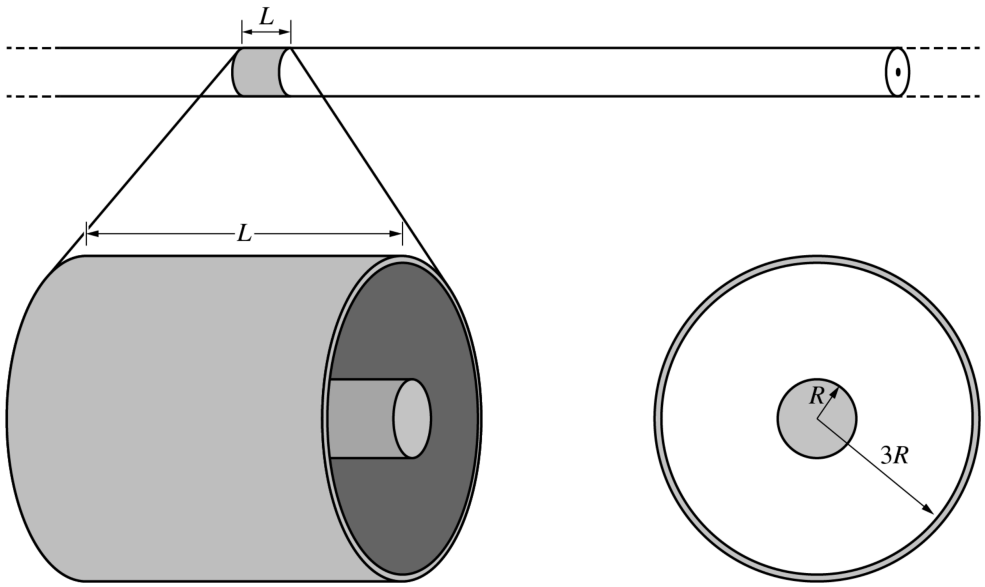
$$+3Q$$

全放在外表面；内表面的感应电荷先由导体内部

$$E = 0$$

确定。

练习 2022 Set 2 Q1 (a)



Cutout View　　Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the cylindrical shell within length L.

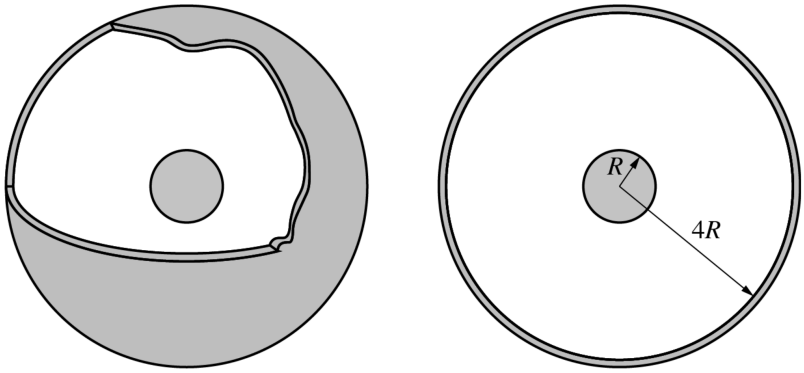
球内均匀体电荷：用体积找包围量

下题球带−Q，其中Q>0；取径向向外为正，k=1/($4\pi\varepsilon_0$)是库仑常数。球对称使半径r的球形高斯面上场强处处相同、沿径向；每一小片法线也沿径向，所以通量之和是E乘球面积$4\pi r^2$。

r小于带电球半径R时，只包围内半径r的体电荷。均匀分布意味着电荷比例等于体积比例，不能把整个球的电荷都装入此高斯面。

$$E_r(4\pi r^2)=\frac{Q_{\text{enc}}}{\varepsilon_0},\qquad Q_{\text{enc}}=-Q\frac{r^3}{R^3}.$$

例题 2022 Set 1 Q1 (b)



Cutout View Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

解:

(b) 球内部的电场球对称使电场在半径为r的球面上大小相同且沿径向，因而高斯通量可写为

$$E_r 4\pi r^2。$$

均匀分布意味着包围电荷按体积比例计算:

$$\rho = \frac{-Q}{\frac{4}{3}\pi R^3}, \qquad q_{\text{enc}} = \rho\frac{4}{3}\pi r^3 = -Q\frac{r^3}{R^3}.$$

代入高斯定律：

$$E_r 4\pi r^2 = \frac{-Qr^3/R^3}{\varepsilon_0} \quad \Rightarrow \quad \boxed{E_r = -\frac{Qr}{4\pi\varepsilon_0 R^3}} \quad (0 < r < R).$$

负号表示向球心；电场大小是

$$kQr/R^3。$$

同心球壳在其内部的电场为零，故没有额外的壳场项。中心处由对称性得

$$E = 0,$$

也与上述公式的极限一致。

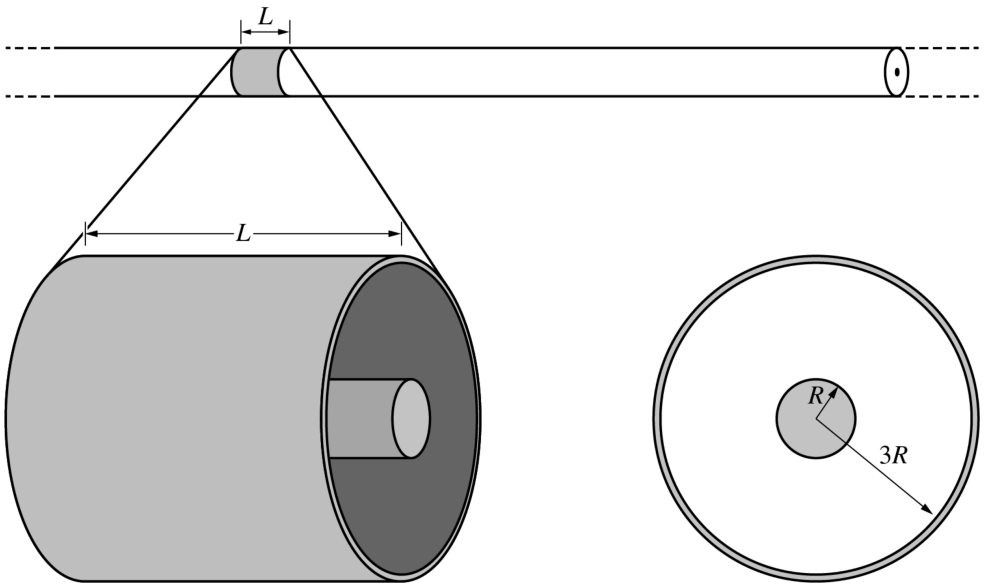
柱对称电场：先找带电体与高斯柱面积

选同轴半径r、长度ℓ的柱面。侧面法线与径向电场同向，端面法线沿轴而电场沿径向，端面通量为零；侧面通量为E(2πrℓ)。

均匀带电实心柱内部的包围电荷按截面积πr^2取比例。空心电容的两壳之间，高斯柱则包围内壳的全部电荷；内壳电荷=面电荷密度σ_1乘内壳面积$2\pi R_1\ell$。两种电荷表达式不同，面积化简动作相同。

$$E(2\pi r\ell) = \frac{Q_{\text{enc}}}{\varepsilon_0}.$$

例题 2022 Set 2 Q1 (b)



Cutout View　　Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

解：

$$\text{(b) } r < R$$

的电场柱对称使电场只沿径向；在柱形高斯面的侧面大小恒定，在两个端面与面法向垂直，所以端面通量为零。侧面积为

$$2\pi r L。$$

均匀体电荷给出

$$\rho = \frac{Q}{\pi R^2 L}, \qquad q_{\text{enc}} = \rho \pi r^2 L = Q\frac{r^2}{R^2}.$$

高斯定律于是成为

$$E_r(2\pi r L) = \frac{Q r^2}{\varepsilon_0 R^2} \quad \Rightarrow \quad \boxed{E_r = \frac{Q r}{2\pi \varepsilon_0 L R^2}}.$$

电场向外，大小随r线性增加。注意这里侧面积只随r增长；若误用球面面积

$$4\pi r^2,$$

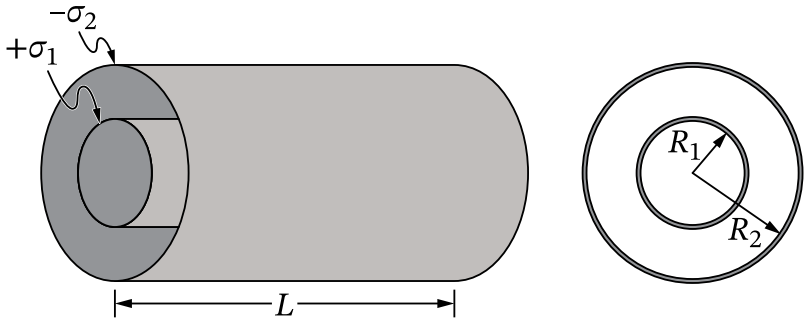
会把柱对称问题变成另一个问题。

例题 2025 Q1 (a)(i)

Question 1: Version J

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1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Side View Cross-Sectional View

Note: Figures not drawn to scale.

Figure 1

A.

i. Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of R_1, σ_1, r, and physical constants, as appropriate.

解：

A(i) 两壳之间的电场高斯定律把穿出闭合面的电通量与面内电荷联系起来：

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}.$$

选半径r、长度ℓ的同轴圆柱高斯面，其中

$$R_1 < r < R_2。$$

圆柱对称性使侧面上电场大小相同且沿侧面法线；在两端盖上，电场平行于端盖，所以端盖通量为零。穿出面积是

$$2\pi r\ell,$$

包围的是内壳同样长度上的电荷

$$\sigma_1(2\pi R_1\ell)。$$

于是

$$E(2\pi r\ell) = \frac{\sigma_1(2\pi R_1\ell)}{\varepsilon_0}, \qquad \boxed{E(r) = \frac{\sigma_1 R_1}{\varepsilon_0 r}}.$$

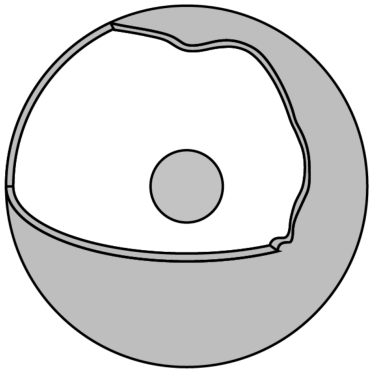
ℓ是为选取高斯面引入的辅助长度，会消去；不能把整个壳的L与高斯面的ℓ混用。

已知一个半径的场：比较相同区间的比例

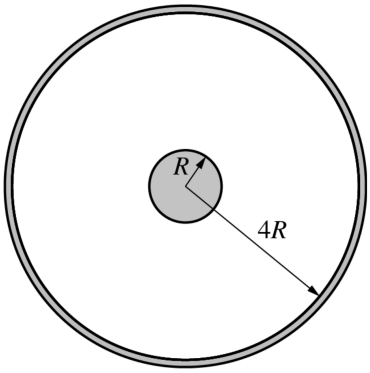
先确认两个半径处属于哪个区域。在带电球或带电柱之外，高斯面已包围相同电荷，场随高斯面侧面积改变：球外按1/r^2，长柱外按1/r。

如果另一个半径落在金属中，电场为零；如果落在仍有新电荷被包围的区域，不能照用外部比例。

例题 2022 Set 1 Q1 (c)



Cutout View



Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(c) The magnitude of the electric field at $r = R$ is $8\,\mathrm{N/C}$. Calculate the value of the electric field at $r = 2R$.

解：

$$\text{(c) } r = 2R$$

的电场

$$R < r < 4R$$

的高斯面包围全部

$$-Q,$$

尚未包围导体壳电荷。因此

$$|E| = kQ/r^2。$$

从R到

$$2R,$$

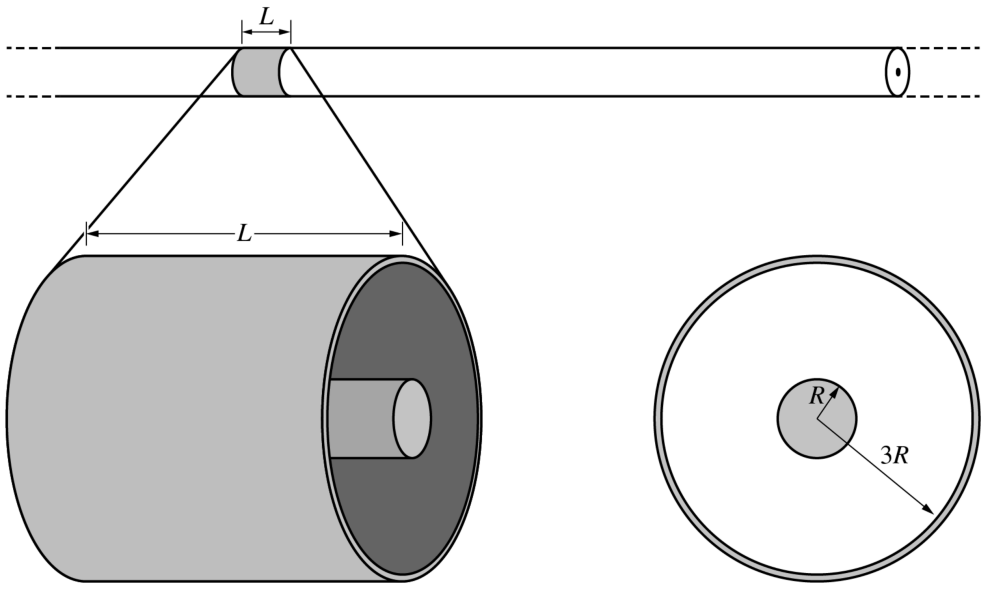
电场大小变为四分之一：

$$|E(2R)| = 8\left(\frac{R}{2R}\right)^2 = \boxed{2\ \mathrm{N/C}}.$$

方向仍然指向球心；若使用径向有符号分量，则

$$E_r(2R) = -2\ \mathrm{N/C}。$$

练习 2022 Set 2 Q1 (c)



Cutout View　　Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

(c) The magnitude of the electric field at $r = R$ is $12\ \mathrm{N/C}$. Calculate the value of the electric field at $r = 2R$.

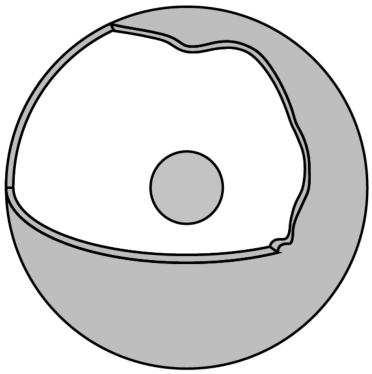
电场分段图：每个区域分别用包围电荷

先标实际材料边界。均匀带电球或柱内部，包围电荷随半径增加，场的大小从零线性增加；有符号分量按电荷正负判定，本题负球内的径向分量画在横轴下方。金属厚壳里场为零；真空外部由全部包围电荷决定。

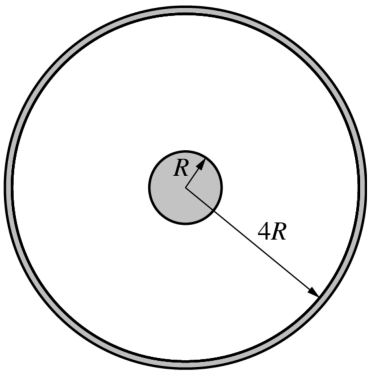
空心同轴电容内空腔和两壳外部的净包围电荷都是零；两壳之间场按1/r下降。表面电荷可使电场在边界跳变，不要强行把不同段画连续。

例题 2022 Set 1 Q1 (e)(i)

本问承接 2022 Set 1 Q1 (a)、2022 Set 1 Q1 (b)。



Cutout View



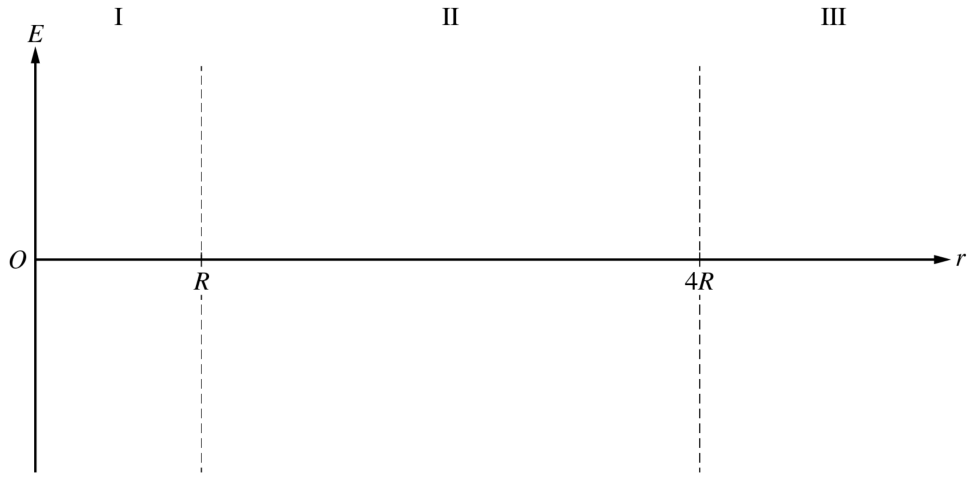
Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



解：

(e)(i) 三个区域的电场图区域III 的高斯面包围整个系统，其净电荷为

$$-Q + 3Q = 2Q。$$

完整有符号电场为

$$E_r(r) = \begin{cases} -kQr/R^3, & 0 \le r < R, \\ -kQ/r^2, & R < r < 4R, \\ +2kQ/r^2, & r > 4R. \end{cases}$$

从中心到R是过原点的负斜率直线；在R连续。区域II 仍为负，但向零靠近。在薄壳

$$4R$$

处由

$$-kQ/(16R^2)$$

跳到

$$+kQ/(8R^2),$$

区域III 是正的递减曲线。壳处的跳变源于表面电荷，不应把两侧画成连续曲线。下面横轴用

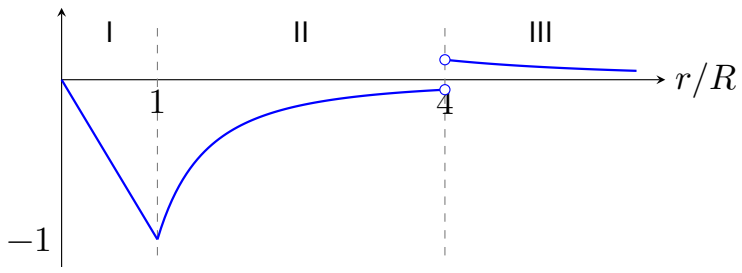
$$r/R,$$

纵轴用

$$E_r/(kQ/R^2);$$

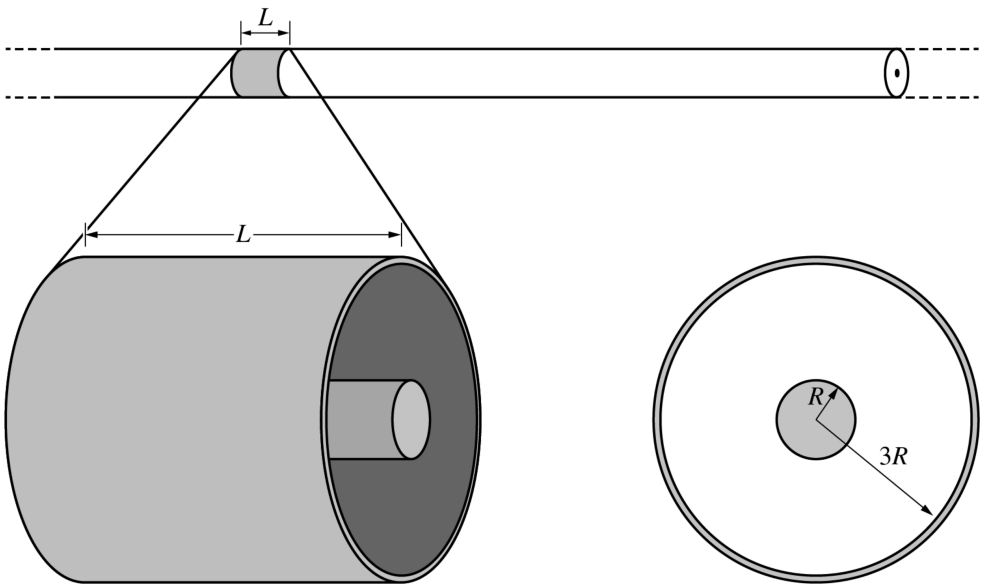
空心点标记薄壳两侧的极限。

$$E_r/(kQ/R^2)$$



练习 2022 Set 2 Q1 (e)(i)

本问承接 2022 Set 2 Q1 (a)、2022 Set 2 Q1 (b)。



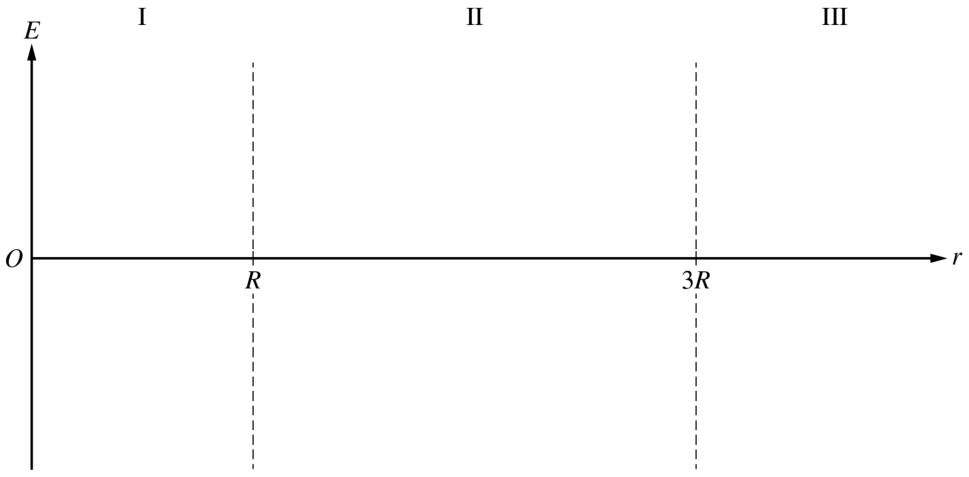
Cutout View　　Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

(e)

i. On the following axes that include regions I, II, and III, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.



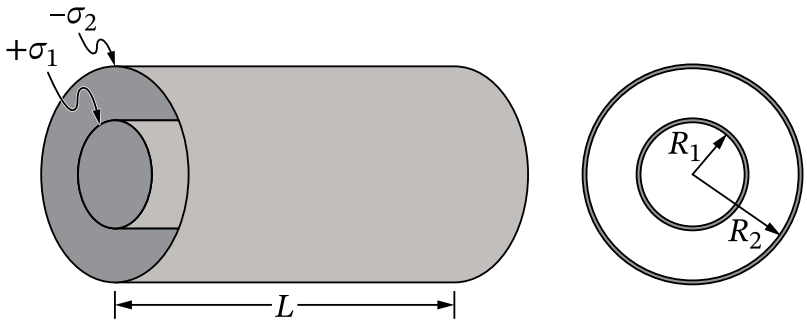
练习 2025 Q1 (a)(iii)

本问承接 2025 Q1 (a)(i)。

Question 1: Version J

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1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Side View Cross-Sectional View

Note: Figures not drawn to scale.

Figure 1

A.

iii. On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

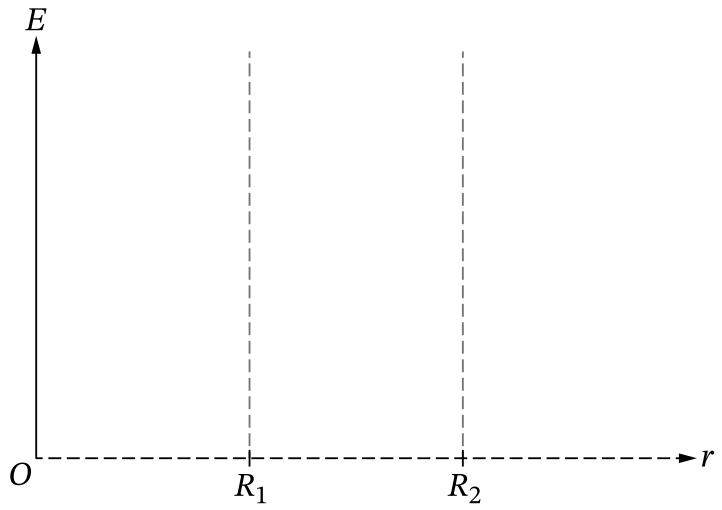


Figure 2

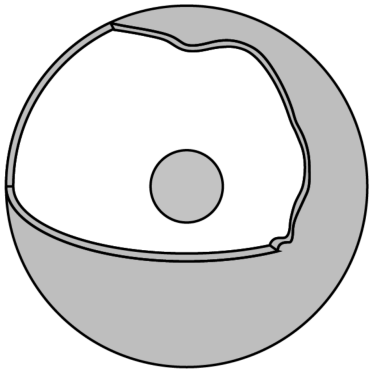
电势差：沿指定区间积分电场

电势差不是场强乘任意长距离。沿r方向移动一小段dr，电势改变dV=−$E_r$dr；总变化把起点至终点各小段相加。

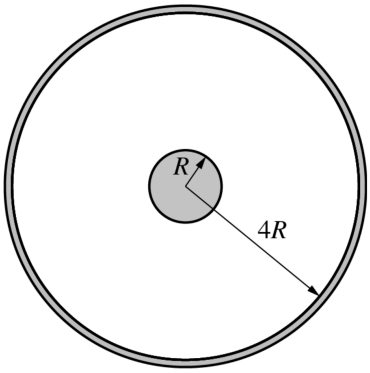
球外的1/r^2积分给1/r差；柱外的1/r积分给对数。题目求绝对值时，先确认方向和上下限，再取大小。

$$V(b)-V(a)=-\int_a^b E_r\,dr,\quad \int r^{-2}dr=-r^{-1},\quad \int \frac{dr}{r}=\ln r.$$

例题 2022 Set 1 Q1 (d)



Cutout View



Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

解：

(d) 球表面与壳内表面间电势差的绝对值沿径向由R移到

$$4R,$$

这一段使用的是区域II 的电场，而不是球内部的线性电场。电势差来自沿路电场积分：

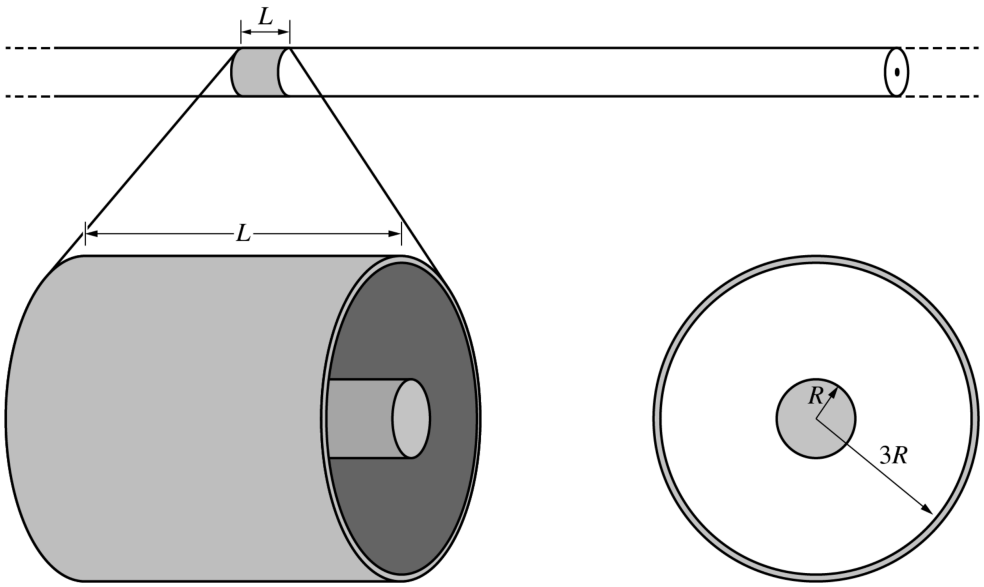
$$V(4R) - V(R) = -\int_R^{4R} E_r\, dr = kQ \int_R^{4R} \frac{dr}{r^2} = kQ \left[-\frac{1}{r}\right]_R^{4R} = \frac{3kQ}{4R}.$$

这个差为正，取绝对值不改变结果：

$$\boxed{|\Delta V| = \frac{3Q}{16\pi\varepsilon_0 R}}.$$

负电荷的电场向内，向外走时电势上升，正号因此与物理方向一致。

例题 2022 Set 2 Q1 (d)



Cutout View　　Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

解:

为把长柱场的系数写短，令α=Q/($2\pi\varepsilon_0$L)；Q是原题长度L内的内柱正电荷。α单位为伏特，径向向外取正。

(d) 两表面间电势差的绝对值由内柱表面R到壳内表面

$$3R,$$

电势变化为

$$V(3R) - V(R) = -\int_R^{3R} \frac{\alpha}{r}\, dr = -\alpha \left[\ln r\right]_R^{3R} = -\alpha \ln 3.$$

向外走顺着电场方向，电势降低，因而上述有符号差为负。题目要求绝对值:

$$\boxed{|\Delta V| = \frac{Q}{2\pi\varepsilon_0 L} \ln 3}.$$

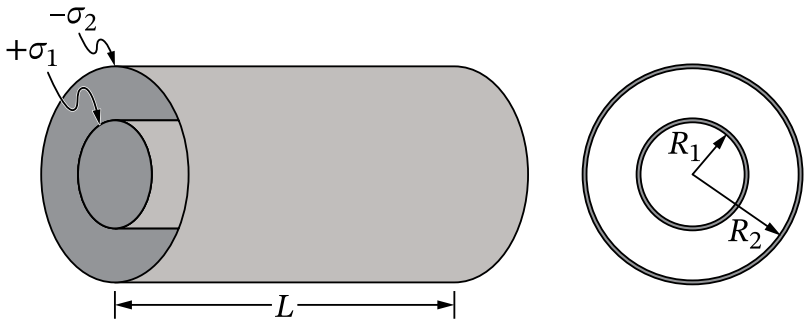
练习 2025 Q1 (a)(ii)

本问承接 2025 Q1 (a)(i)。

Question 1: Version J

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1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Side View Cross-Sectional View

Note: Figures not drawn to scale.

Figure 1

A.

ii. **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1, R_2, σ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

从电场到电势图：用斜率与连续性

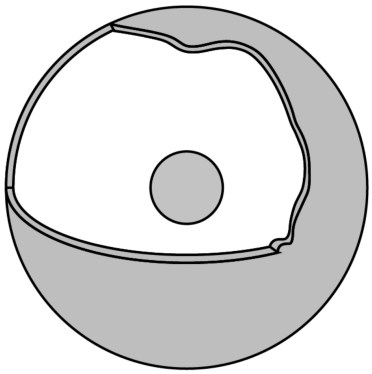
电势曲线的斜率是电场径向分量的负值。正的向外电场使V随r下降；金属内E=0所以V水平；E线性增大对应二次下降的电势。

电势在正常带电表面连续；可以先任选一个参考高度，再按各段斜率接起来。无限长带电柱的电势不能把无穷远取为零，题中允许的参考点才是高度基准。

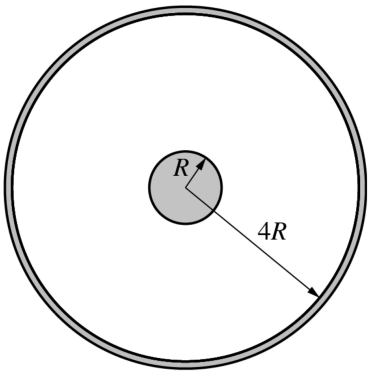
$$\frac{dV}{dr} = -E_r.$$

例题 2022 Set 1 Q1 (e)(ii)

本问承接 2022 Set 1 Q1 (e)(i)。



Cutout View



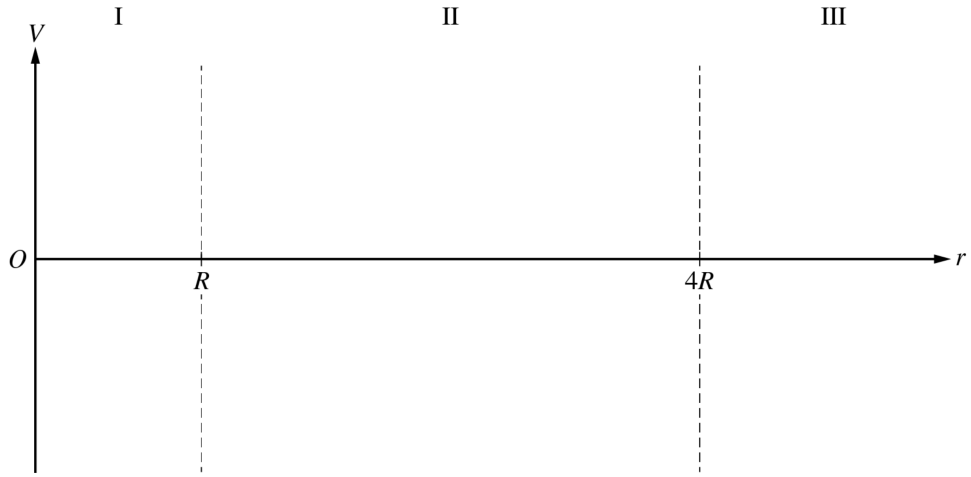
Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(e)

ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



解：

(e)(ii) 电势图电势斜率满足

$$dV/dr \;=\; -E_r,$$

所以

$$r \;<\; 4R$$

时电势上升，

$$r \;>\; 4R$$

时下降，且电势跨越薄壳连续。为固定图的高度，选择

$$V(\infty) \;=\; 0;$$

本题为有限总电荷系统，这一选择可行。区域III 得

$$V = 2kQ/r,$$

因此

$$V(4R) = kQ/(2R)。$$

往内积分并要求边界连续，得到

$$V(r) = \begin{cases} \dfrac{kQ}{R}\left(-\dfrac{3}{4} + \dfrac{r^2}{2R^2}\right), & r \le R, \\ \dfrac{3kQ}{4R} - \dfrac{kQ}{r}, & R \le r \le 4R, \\ \dfrac{2kQ}{r}, & r \ge 4R. \end{cases}$$

可核对

$$V(0) = -3kQ/(4R)、$$

$$V(R) = -kQ/(4R)、$$

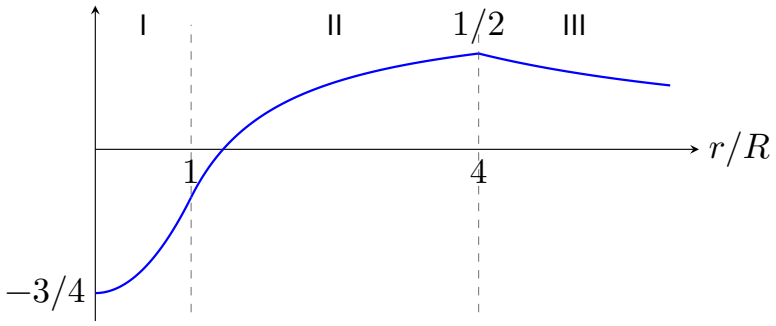
$$V(4R) = kQ/(2R)。$$

最大值在

$$4R;$$

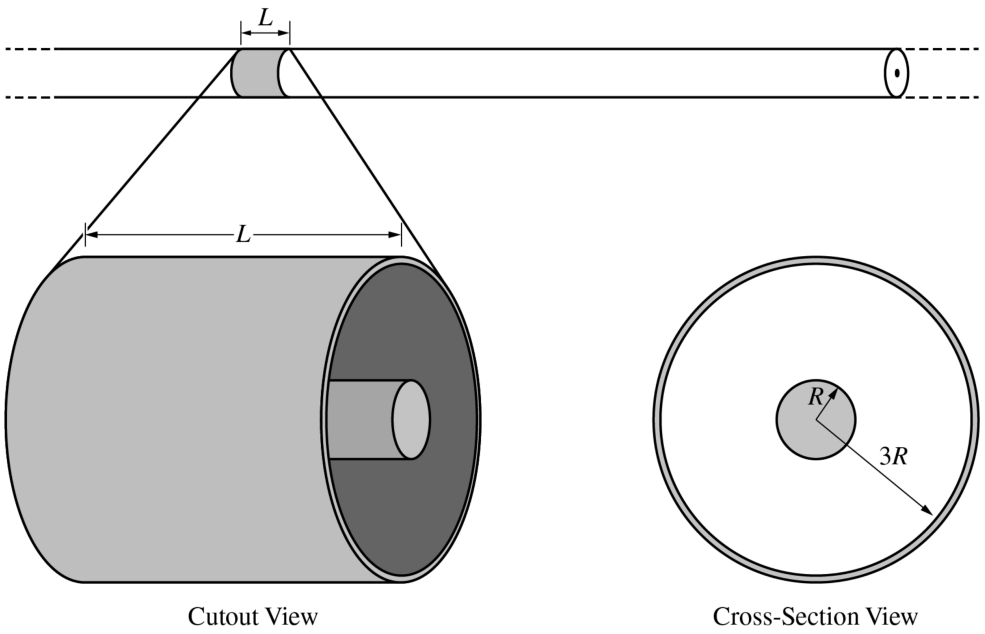
区域I 从水平切线开始向上弯曲，区域II 上升但斜率越来越小，区域III 下降而斜率绝对值越来越小。更换电势零点只会把整条图竖直平移。

$$V/(kQ/R)$$



练习 2022 Set 2 Q1 (e)(ii)

本问承接 2022 Set 2 Q1 (e)(i)。



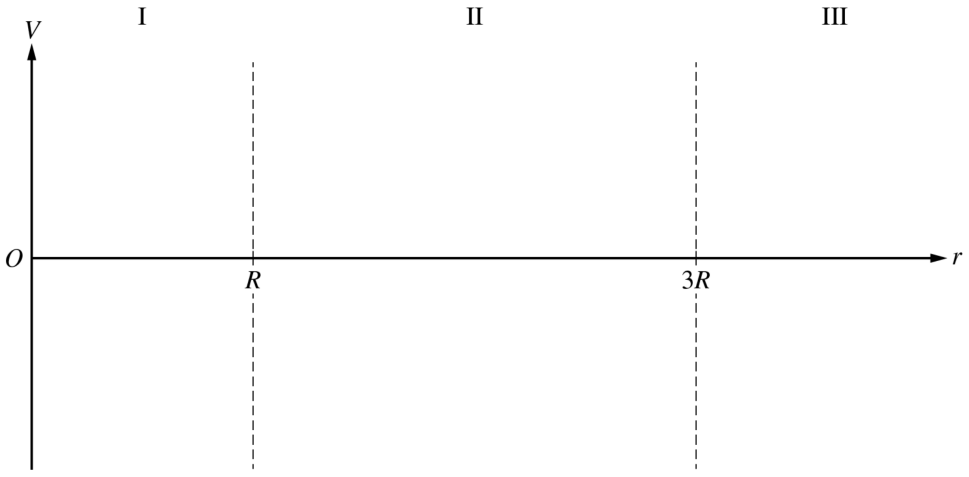
Cutout View　　Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

(e)

ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.



电介质电容：电荷与电势差之比

电容描述给定电荷产生多少电势差，定义C=Q/|ΔV|。完全填入介电常数κ的介质后，理想模型中的场和电势差都缩小为原来的1/κ。

在孤立电容中总电荷保持；用内壳Q=$\sigma_1(2\pi R_1 L)$除以刚才求出的电势差，再乘κ。不要把κ当成增加的自由电荷量。

$$C = \frac{Q}{|\Delta V|}.$$

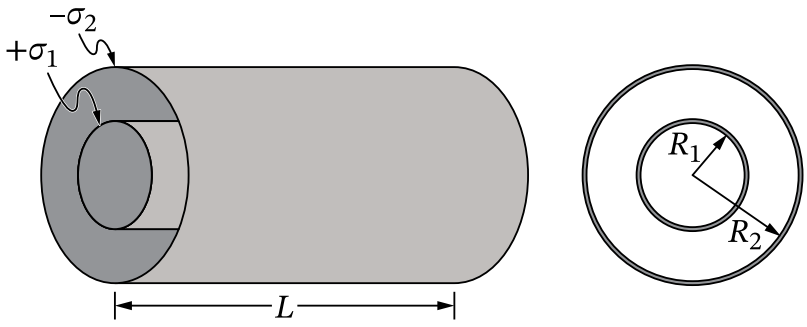
例题 2025 Q1 (b)

本问承接 2025 Q1 (a)(ii)。

Question 1: Version J

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1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.

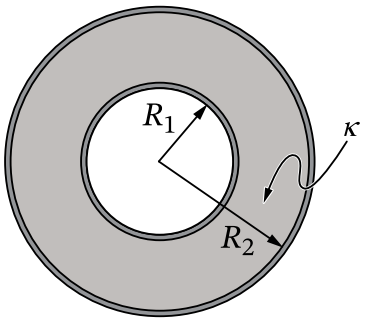


Side View　　Cross-Sectional View

Note: Figures not drawn to scale.

Figure 1

B. A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.



Cross-Sectional View

Figure 3

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L, R_1, R_2, κ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

B　填满电介质后的电容电容定义为一极板上的电荷量与两极板电势差之比：

$$C = Q/\Delta V。$$

本题圆柱壳的有效侧面积是

$$2\pi R_1 L,$$

所以

$$Q = \sigma_1(2\pi R_1 L)。$$

先求空气电容：

$$C_0 = \frac{2\pi\sigma_1 R_1 L}{(\sigma_1 R_1/\varepsilon_0)\ln(R_2/R_1)} = \frac{2\pi\varepsilon_0 L}{\ln(R_2/R_1)}.$$

介质完全填满，介电常数为κ，电容乘以κ，故

$$\boxed{C = \frac{2\pi\kappa\varepsilon_0 L}{\ln(R_2/R_1)}}.$$

电容器孤立，插入介质时自由电荷Q不变，电势差减为原来的

$$1/\kappa,$$

这也直接验证电容增为

$$\kappa C_{0^\circ}$$

等势图、带电杆与电势叠加

等势线给起终值：画外力功柱状图

试探电荷每段起止都静止，动能改变为零，所以外力功等于电势能改变q(Vf−Vi)。路线的长短不决定此功。

题目画功的绝对值，把各段|ΔV|比较；同一等势值之间功为零，电势下降的有符号功虽为负，其绝对值柱仍向上。

$$W_{\text{ext}} = \Delta U = q(V_f - V_i), \qquad |W_{\text{ext}}| = |q|\,|V_f - V_i|.$$

例题 2024 Set 1 Q1 (b)(i)

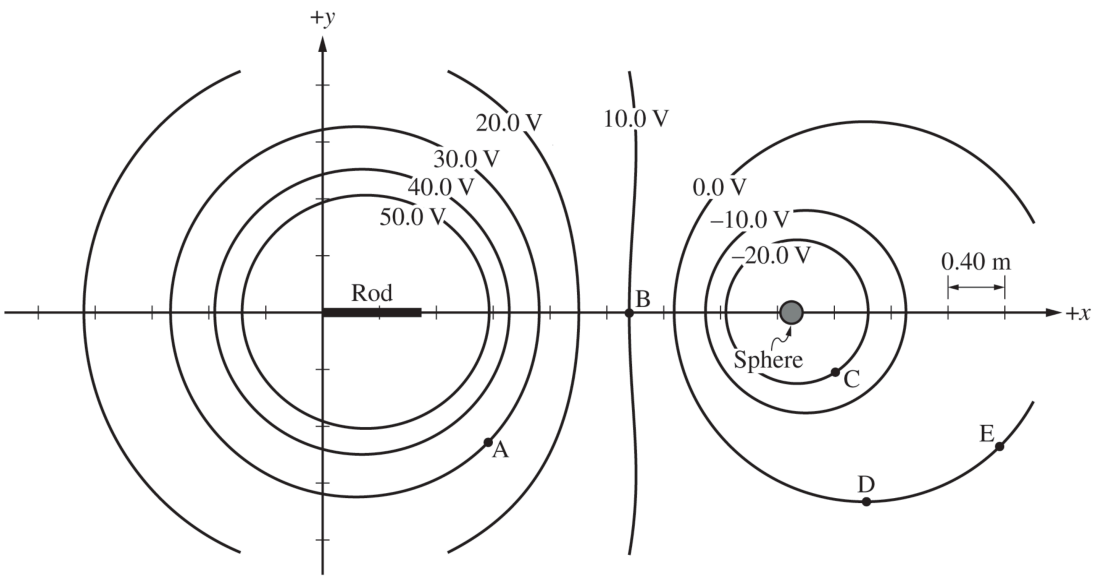


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

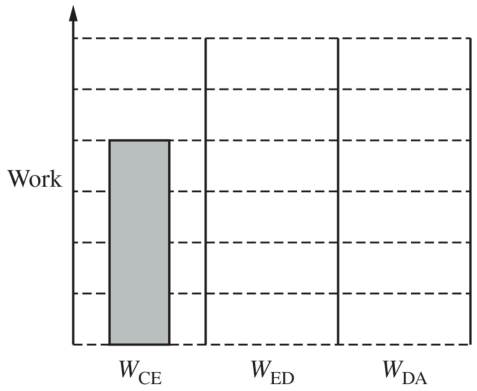


Figure 2

解：

(b)(i) 补全外力功的柱状图试探电荷

$$q > 0$$

在每个位置都瞬间静止，所以每段起止动能相同。外力功与电场功相抵，而电场功等于势能的减少量，故

$$W_{\text{ext}} = \Delta U = q(V_f - V_i).$$

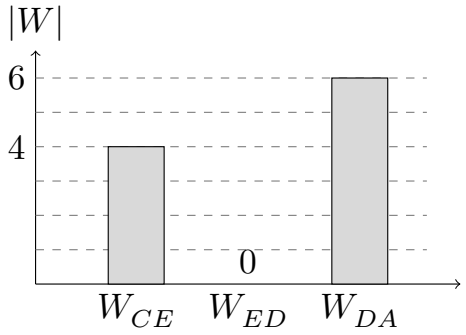
这只用起止电势，不用路线长短。由原图读出

$$V_C = -20.0\,\text{V}, \qquad V_E = V_D = 0.0\,\text{V}, \qquad V_A = 30.0\,\text{V}.$$

于是三个绝对值的比例是

$$|W_{CE}| : |W_{ED}| : |W_{DA}| = 20 : 0 : 30 = 2 : 0 : 3.$$

原图给出的W_{CE}柱高为4 个格，因此W_{ED}写“0”，W_{DA}画到6 个格，即给定柱高的1.5 倍。



练习 2024 Set 2 Q1 (b)(i)

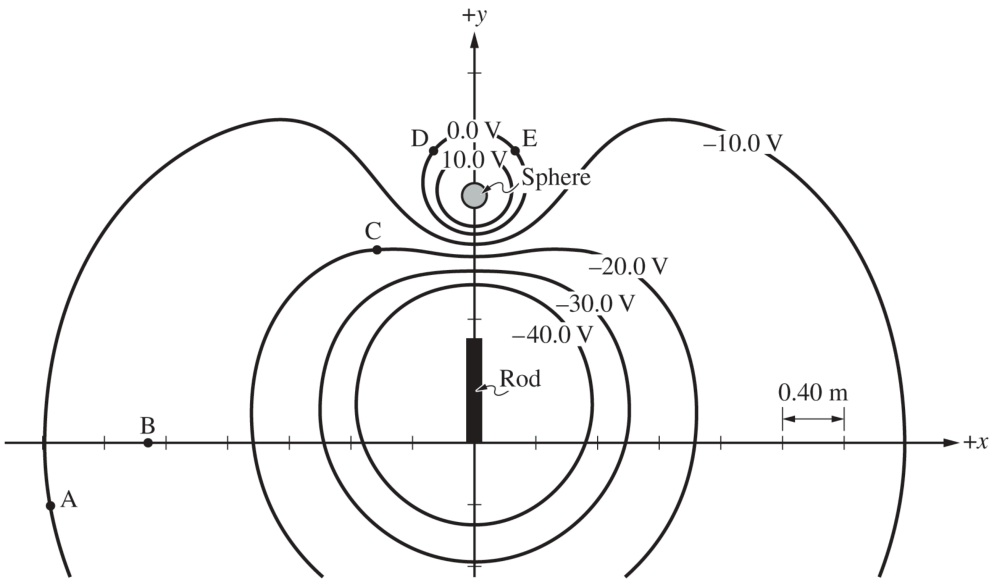


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge +1.0 nC. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are 0.40 m apart.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

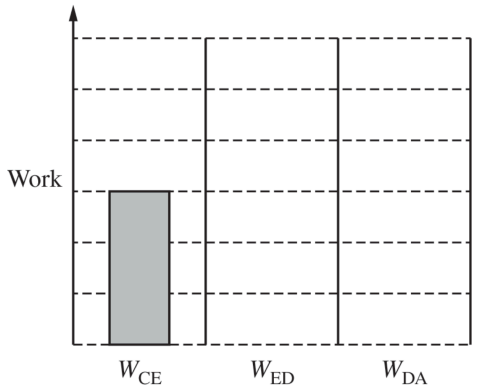


Figure 2

局部等势间距：估计电场分量

电场指向电势下降，x分量对应沿水平方向的变化率。沿穿过指定点的水平线读相邻等势线的电势差和水平距离，用−ΔV/Δx估算Ex。

曲线弯曲时这是局部近似。不能沿竖直量距离后把结果称x分量；题求大小时最后才取绝对值。

$$E_x \approx -\frac{\Delta V}{\Delta x}.$$

例题 2024 Set 1 Q1 (b)(ii)

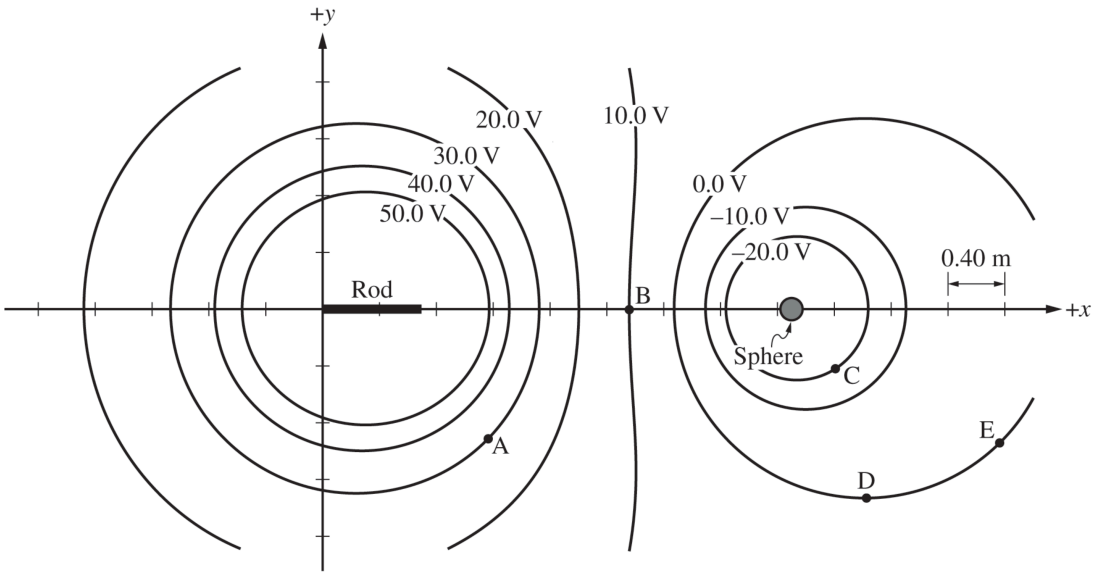


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

解：

(b)(ii)　估算B 点电场的水平分量电场指向电势降低的方向。沿x方向移动一小段，电势变化与电场的关系为

$$E_x = -dV/dx;$$

图上只提供有限间距，便用相邻等势线之间的平均变化近似B 点附近的变化率。

B 附近的20.0 V 与0.0 V 等势线在水平线上相距约0.65 m。向右电势下降，因此

$$E_x \approx -\frac{0.0 - 20.0}{0.65} \approx +31\,\text{V/m}.$$

题求大小，答

$$\boxed{|E_x| \approx 31\,\text{V/m}}.$$

这里的距离应沿所求分量的x方向读，不是沿弯曲等势线量弧长。图像估读允许合理的小差异。

练习 2024 Set 2 Q1 (b)(ii)

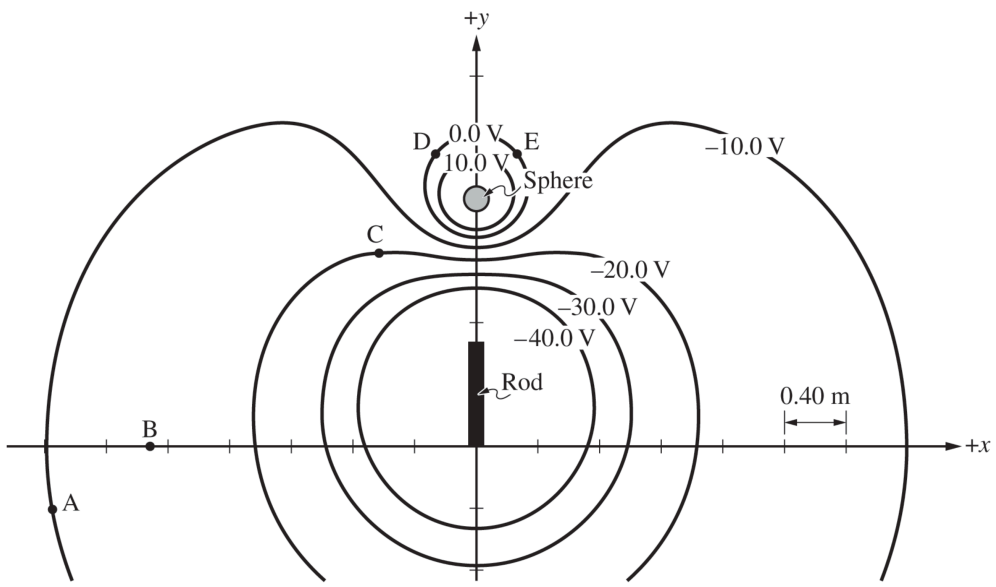


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge +1.0 nC. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are 0.40 m apart.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

等势线法向：判断正电荷初始受力

电场与等势线的局部切线垂直，指向电势更低的一侧。先在指定点找法线，再比较法线两侧的电势。

正试探电荷受力与电场同向；负电荷相反。只说被某个负电荷吸引，不能替代杆和球合场的实际方向。

例题 2024 Set 1 Q1 (c)

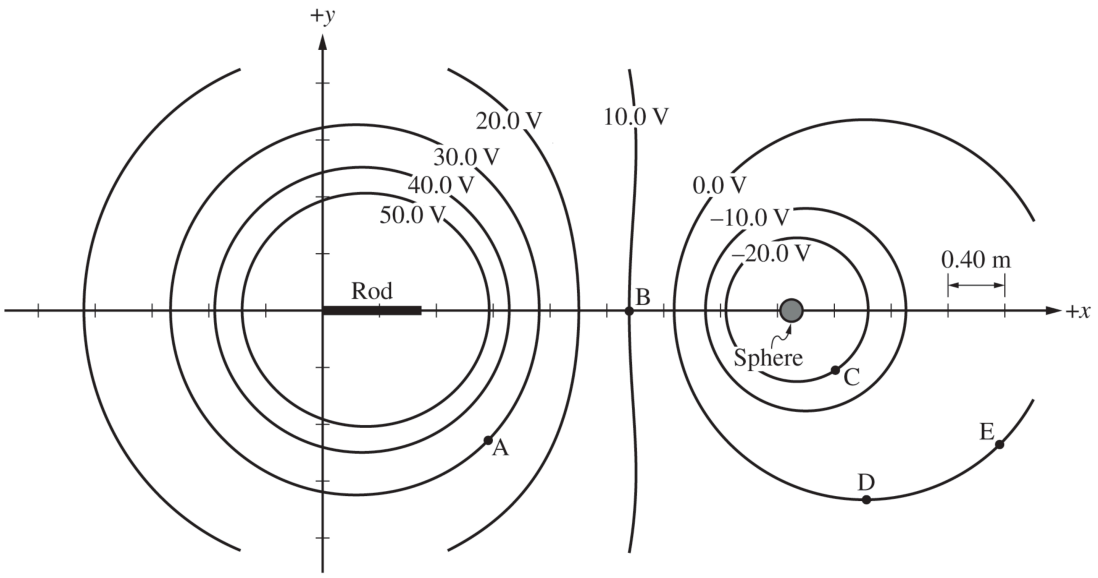


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

The positive test charge is placed at Position D. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

解：

(c)　D 点刚释放时的电力方向　选择 $\boxed{+y}$。

D 处的等势线切线近似水平，电场与等势线的切线垂直，所以方向在竖直线上。D 上方、朝球的一侧电势更低，正试探电荷受到的电力指向电势降低的一侧，即向上。这一判断同时考虑了杆与球的合场；只说“正电荷受负球吸引”不足以确定合力的精确方向。

练习 2024 Set 2 Q1 (c)

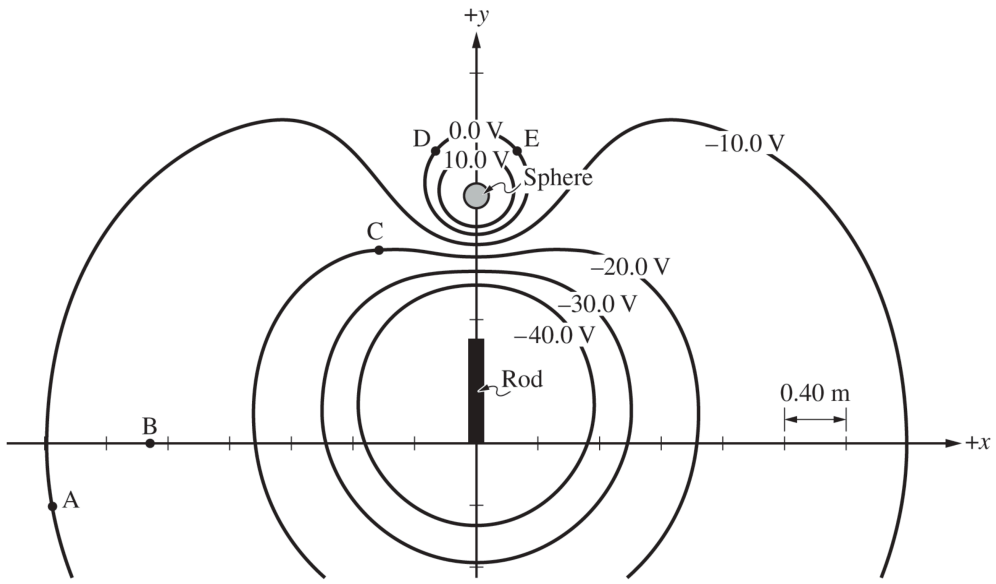


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge +1.0 nC. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are 0.40 m apart.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$　　_____ $+y$　　_____ Directly away from the sphere

_____ $-x$　　_____ $-y$　　_____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

对称杆的矢量场：先抵消，再合成

在一根均匀杆的垂直平分线上，关于中点对称的小段横向场分量互相抵消，剩余分量沿垂直平分线。位于杆延长线上则各小段场同向。

两杆贡献先分别找方向再相加。正杆的场远离杆；小负球的受力与合电场相反。

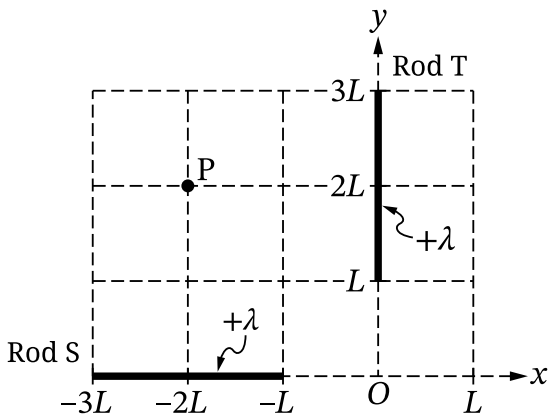
例题 2026 Q2 (a)

Question 2: Version J

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2. Thin, nonconducting, charged Rods S and T are held fixed in the *xy*-plane, as shown in Figure 1. Rod S is oriented along the *x*-axis in the region $-3L \le x \le -L$. Rod T is oriented along the *y*-axis in the region $L \le y \le 3L$. Each rod has uniform, positive linear charge density $+\lambda$. Point P is located at $(-2L,\ 2L)$.

Figure 1

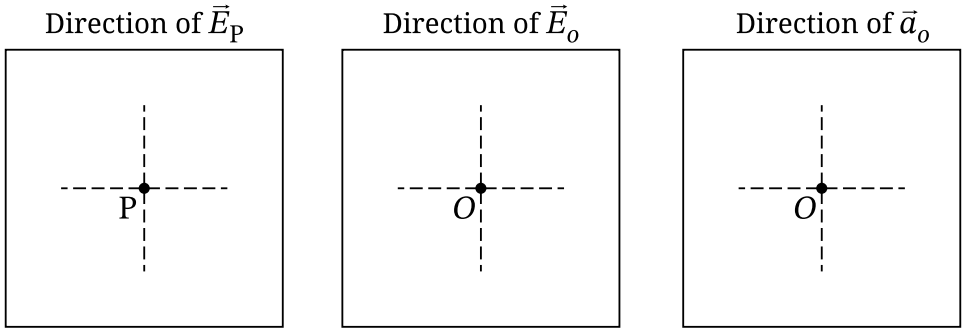


Part A

Complete the following tasks in Figure 2.

- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_{\mathrm{P}}$ at Point P due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_O$ at the origin *O* due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net acceleration $\vec{a}_O$ of a small, negatively charged sphere (not shown) immediately after the sphere is released from rest at the origin *O*.

Figure 2



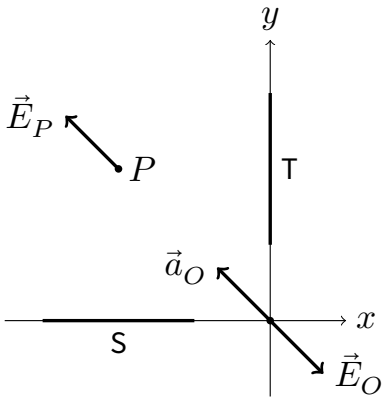
解：

A 三个方向箭头P 在S 的垂直平分线上；S 上关于中点对称的电荷所产生的水平分量抵消，剩余向上。P 也在T 的垂直平分线上，T 的上下分量抵消，剩余向左。由于两杆等长、等密度且距离相同，两个剩余分量等大，净电场向左上45°。

O 位于S 的右方，S 的场向右；O 位于T 的下方，T 的场向下。两个分量仍等大，所以净电场向右下45°。负电荷的力

$$\vec{F} = q\vec{E}$$

与电场相反，因此在O 释放的负球加速度向左上。



杆产生的场：用微元积分再作矢量合成

λ是每单位长度的电荷。杆上一小段dx带电dq=λdx；它离观察点距离r，产生场大小k dq/r^2，k=1/$(4\pi\varepsilon_0)$。

固定观察点，把距离写成杆上积分坐标的函数，然后沿杆的两个端点积分。只有方向相同才直接把大小相加；垂直的两杆场须用分量合成。

$$dE = \frac{k\,dq}{r^2}, \qquad dq = \lambda\,dx.$$

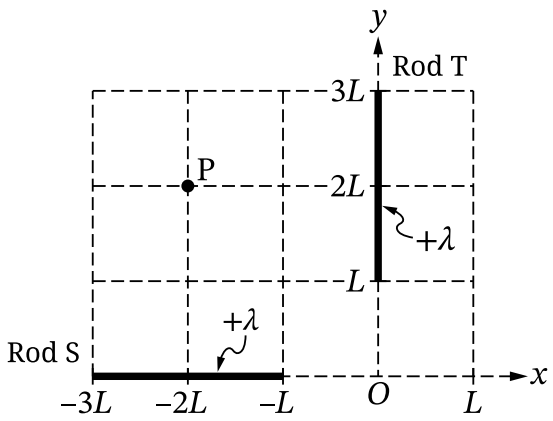
例题 2026 Q2 (b)

Question 2: Version J

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2. Thin, nonconducting, charged Rods S and T are held fixed in the *xy*-plane, as shown in Figure 1. Rod S is oriented along the *x*-axis in the region $-3L \le x \le -L$. Rod T is oriented along the *y*-axis in the region $L \le y \le 3L$. Each rod has uniform, positive linear charge density $+\lambda$. Point P is located at $(-2L,\ 2L)$.

Figure 1



Part B

Derive an expression for the magnitude E_O of the net electric field at the origin *O* due to Rods S and T in terms of L, λ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

B　O 处净场的大小连续电荷的场要把各小段贡献相加：

$$\vec{E} = k \int \frac{dq}{r^2} \hat{r}.$$

以T 为例，距离O 为y的小段带电

$$dq = \lambda \, dy,$$

场沿

$$-y_{\circ}$$

其场大小为

$$E_T = k\lambda \int_L^{3L} \frac{dy}{y^2} = k\lambda \left[-\frac{1}{y} \right]_L^{3L} = \frac{2k\lambda}{3L}.$$

S 因同样的距离分布给出大小相同的

$$+x$$

分量。两个场互相垂直，合成大小用勾股，而不是直接加：

$$\boxed{E_O = \sqrt{E_S^2 + E_T^2} = \frac{2\sqrt{2}k\lambda}{3L}}.$$

杆的电势：距离作分母、上下限取杆端

电势是标量，每一小段的贡献直接相加，不先分解方向。正杆dq=λdx；负杆dq=−λdy，电荷符号必须保留。

观察点固定，积分变量在杆上跑，因此先表达从小段到观察点的真实正距离，再给杆端坐标作为上下限。∫ 表示所有小段贡献的极限总和。

$$dV = \frac{k \, dq}{r}, \qquad V = \int_{\text{rod}} \frac{k \, dq}{r}.$$

例题 2024 Set 1 Q1 (d)(i)

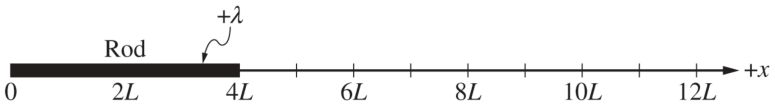


Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x-axis in the orientation shown in Figure 3. Position P (not shown) is located on the x-axis a distance x_P from the origin, where $x_P > 4L$.

(d) The electric potential V_P at x_P is $V_P = k\lambda \ln\left(\dfrac{x_P}{x_P - 4L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

解：

(d)(i) 用积分推导杆的电势现在只有正电杆，杆占据

$$0 \le x \le 4L;$$

观察点固定在

$$x_P > 4L。$$

取杆上坐标为x、长度为

$$dx$$

的一小段。它带电量

$$dq = \lambda\, dx,$$

到P 的距离为

$$x_P - x > 0,$$

因此它贡献的电势为

$$dV_P = k\frac{dq}{x_P - x} = k\lambda\frac{dx}{x_P - x}.$$

电势是标量，所有小段的贡献直接相加。观察点x_P在积分过程中保持不变，变化的是电荷小段的坐标x：

$$V_P = k\lambda \int_0^{4L} \frac{dx}{x_P - x}.$$

求原函数时要处理负号：对

$$\ln(x_P - x)$$

求导，得到

$$-1/(x_P - x),$$

所以所需原函数为

$$-\ln(x_P - x)。$$

上下限分别代入再相减：

$$\begin{aligned} V_P &= k\lambda[-\ln(x_P - x)]_0^{4L} \\ &= k\lambda[-\ln(x_P - 4L) + \ln x_P] \\ &= \boxed{k\lambda \ln\left(\frac{x_P}{x_P - 4L}\right)}. \end{aligned}$$

比值大于1，因此电势为正，与正电杆一致；对数中的比值没有单位。远离杆时，这个比值趋近1，电势趋于零，与无穷远为零的约定一致。

练习 2024 Set 2 Q1 (d)(i)

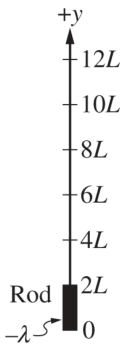


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\frac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

杆外电场图：从正负与远近判断

正杆右侧的场向+x，负杆上方的场向−y。距带电体很远时场趋于零；靠近理想细杆端点时场大小急剧增大。

图上要求带符号分量时，正值画在轴上方、负值画在下方。不能只画场强大小后忽略方向。

$$E_x = -\frac{dV}{dx}, \qquad E_y = -\frac{dV}{dy}.$$

例题 2024 Set 1 Q1 (d)(ii)

本问承接 2024 Set 1 Q1 (d)(i)。

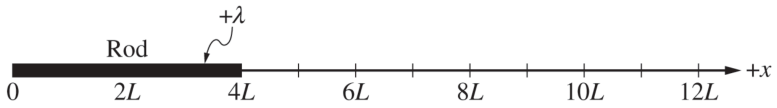


Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x-axis in the orientation shown in Figure 3. Position P (not shown) is located on the x-axis a distance x_P from the origin, where $x_P > 4L$.

(d) The electric potential V_P at x_P is $V_P = k\lambda \ln\left(\dfrac{x_P}{x_P - 4L}\right)$.

ii. On Figure 4, **sketch** a graph of the x-component E_x of the electric field from the rod as a function of x in the region $4L < x < 12L$.

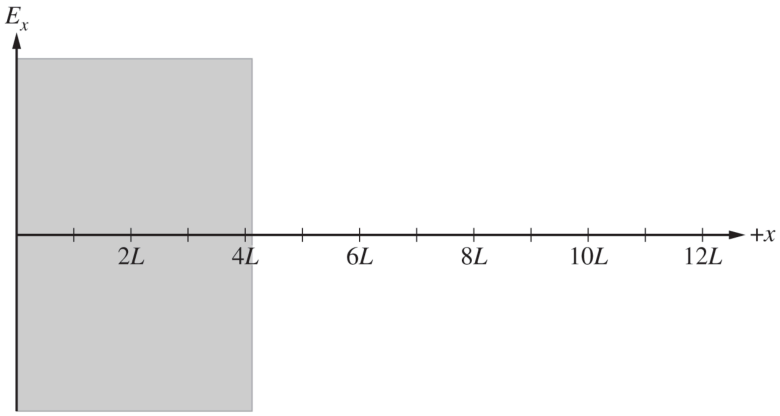


Figure 4

解：

(d)(ii) 杆右侧电场的图电场向右，

$$E_x > 0。$$

把上一式的观察坐标改写为

$$x > 4L,$$

再用

$$E_x = -dV/dx:$$

$$E_x(x) = -k\lambda\left(\frac{1}{x} - \frac{1}{x-4L}\right)$$

$$= \boxed{k\lambda\left(\frac{1}{x-4L} - \frac{1}{x}\right) = \frac{4k\lambda L}{x(x-4L)}}.$$

刚靠近杆的右端，分母很小，场很大；离杆越远，场越小，但仍为正。曲线向上凹并逐渐接近横轴。

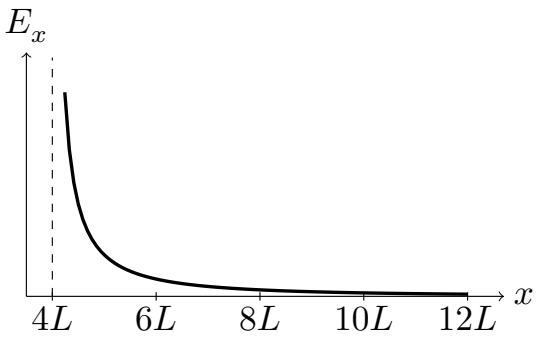
$$4L < x < 12L$$

内没有零点，也不能画成穿过横轴的直线。

下图横轴按L缩放，纵轴用任意正比例表示场强；

$$x = 4L$$

是杆端，不是有限场值的点。



练习 2024 Set 2 Q1 (d)(ii)

本问承接 2024 Set 2 Q1 (d)(i)。

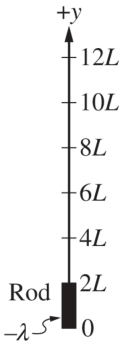


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

ii. On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

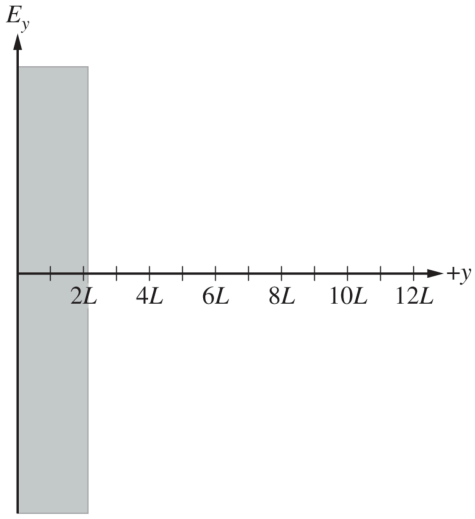


Figure 4

带电杆移远：电势随距离的曲线

固定观察点，杆离开时每一小段到观察点的距离都增大，正电势因此下降。长度2L的正杆近端距离为r时，沿杆的距离从r到r+2L。

用杆电势积分看极限：r趋于零时理想杆端点给发散的正电势；r很大时V趋于零，曲线下降且逐渐变平。

$$V = k\lambda \int_{r}^{r+2L} \frac{du}{u} = k\lambda \ln \frac{r+2L}{r}.$$

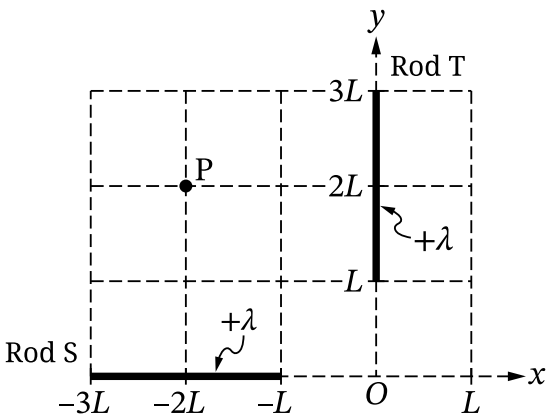
例题 2026 Q2 (c)

Question 2: Version J

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2. Thin, nonconducting, charged Rods S and T are held fixed in the *xy*-plane, as shown in Figure 1. Rod S is oriented along the *x*-axis in the region $-3L \le x \le -L$. Rod T is oriented along the *y*-axis in the region $L \le y \le 3L$. Each rod has uniform, positive linear charge density $+\lambda$. Point P is located at $(-2L,\ 2L)$.

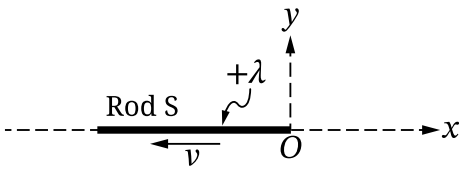
Figure 1



Part C

Rod T is removed. Rod S is then moved such that the right end of Rod S is at the origin *O*. Rod S is then moved with speed *v* in the –*x*-direction, as shown in Figure 3.

Figure 3



On the axes shown in Figure 4, **sketch** a graph of the electric potential V_O at the origin *O* due to Rod S as a function of the distance *r* between the origin and the right end of Rod S.

Figure 4



解：

C 杆离开原点时的电势图S 长度

$$2L,$$

近端到O 距离为

$$r > 0;$$

其电荷到O 的距离从r至

$$r + 2L。$$

电势是标量，采用

$$dV = k\,dq/u,$$

其中u是积分所用的距离变量：

$$V_O(r) = k\lambda \int_r^{r+2L} \frac{du}{u} = \boxed{k\lambda \ln \frac{r+2L}{r}}.$$

对

$$r > 0,$$

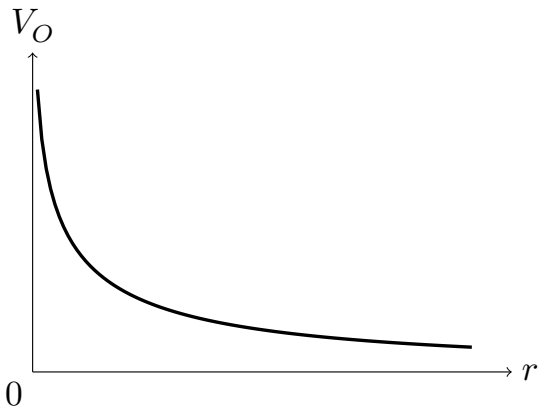
它为正且递减；越往右越平，最终趋近零。理想零粗细杆近端趋近原点时，

$$V_O \to +\infty;$$

因此图上

$$r = 0$$

为竖直渐近线。移动速度v在画V对距离图时不出现。



等距等量异号杆：电势逐项抵消

电势可以带正负号直接相加。两根等长杆的线密度绝对值相同、符号相反，且在每个时刻对应小段到原点的距离相同。

把两杆对应的小段配对，每对贡献大小相等、符号相反，因此总电势在全部r处都为零。此判断只涉及标量电势，不能由此断言矢量电场也为零。

$$V_{\text{total}} = V_S + V_W.$$

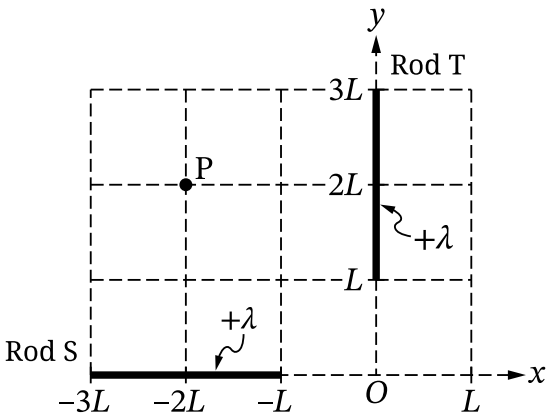
例题 2026 Q2 (d)

Question 2: Version J

---

2. Thin, nonconducting, charged Rods S and T are held fixed in the *xy*-plane, as shown in Figure 1. Rod S is oriented along the *x*-axis in the region $-3L \le x \le -L$. Rod T is oriented along the *y*-axis in the region $L \le y \le 3L$. Each rod has uniform, positive linear charge density $+\lambda$. Point P is located at $(-2L, 2L)$.

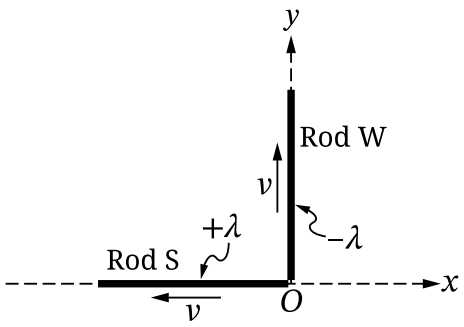
Figure 1



Part D

A new thin, nonconducting, charged Rod W has the same length as Rod S. Rod W has uniform, negative linear charge density $-\lambda$ and is oriented along the *y*-axis. Rods S and W are initially held fixed with one end at the origin. Starting at the same time, Rod S is moved with speed *v* in the –*x*-direction and Rod W is moved with speed *v* in the +*y*-direction, as shown in Figure 5.

Figure 5



Indicate how a sketch of V_O at the origin *O* due to both Rods S and W as a function of *r* would be different from the sketch made in part C. Briefly **justify** your answer.

解：

D 再加入等长负杆W两杆始终以相同速度从O 移走；对于每个

$$r > 0,$$

各自的距离分布都是

$$[r, r + 2L]。$$

W 的密度为

$$-\lambda,$$

所以其电势恰为S 电势的负值：

$$V_W(r) = -k\lambda \ln \frac{r + 2L}{r}, \qquad \boxed{V_S(r) + V_W(r) = 0}.$$

图改为整个定义域上的水平零线。这里之所以相消，是电势为标量且等距离电荷逐段等量异号；不能据此说电场也相消，因为两杆电场方向不同。

电阻率实验

测电压、电流和几何尺寸：设计收集程序

圆柱元件的电阻同时受材料和几何影响。电阻率ρ通过R=ρℓ/A定义；ℓ为长度，A为截面积，ρ单位Ω·m。圆截面需测直径d，再算A=$\pi d^2/4$。

用电流表串联测流过元件的电流、电压表并联测其两端电压。改变电源获得多组(V,I)，重复读数并控制温度；发热会改变材料电阻率，不能把升温造成的变化当作几何关系。

$$R = \rho\frac{\ell}{A}, \qquad A = \frac{\pi d^2}{4}.$$

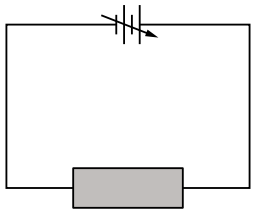
例题 2025 Q3 (a)

Question 3

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3. In Experiment 1, students are asked to use a graph to determine the resistivity ρ_1 of a circuit element that is connected to a variable power supply, as shown in Figure 1. The circuit element is cylindrical and has uniform resistivity. The students have access to a voltmeter, an ammeter, and a ruler.

Figure 1



A. Describe a procedure for collecting data that would allow the students to use a graph to determine ρ_1, including any steps necessary to reduce experimental uncertainty.

解：

A 第一组实验的收集程序用尺测圆柱长度ℓ与直径d，算出

$$A = \pi d^2/4。$$

电流表串联在元件所在支路，电压表并联在元件两端；每次改变电源设置后，记录一对

$$(I, V),$$

至少取若干不同电压，不只测一个工作点。每个设置重复测量并取平均，直径在几个位置测量；避免长时间大电流加热。

这些测量分别对应待求量的两部分：

$$(I, V)$$

用来求R,

$$(\ell, A)$$

用来把R转成ρ_1。只测长度和电流，缺少电压或几何截面积，不能确定电阻率。

把材料关系写成纵轴等于斜率乘横轴

想从图求未知量，先将物理关系写成y=bx+a。对一个元件改变电压，V=RI可用V对I的斜率求R，然后乘A/ℓ求ρ。

对同材料、同截面积而不同长度的元件，R=(ρ/A)L。此时选R对L，斜率是ρ/A，既不是R也不是ρ本身。

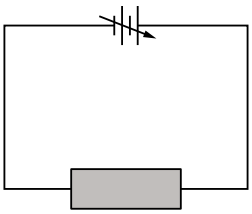
$$V = RI, \quad \rho = \frac{RA}{\ell}; \qquad R = \frac{\rho}{A}L.$$

例题 2025 Q3 (b)

Question 3

3. In Experiment 1, students are asked to use a graph to determine the resistivity ρ_1 of a circuit element that is connected to a variable power supply, as shown in Figure 1. The circuit element is cylindrical and has uniform resistivity. The students have access to a voltmeter, an ammeter, and a ruler.

Figure 1



Circuit Element

B. Describe how the collected data could be graphed and how that graph would be analyzed to determine ρ_1.

解：

B　第一组实验的图与分析横轴画I（A），纵轴画V（V）。欧姆定律为

$$V = IR = \left(\frac{\rho_1 \ell}{A}\right) I.$$

因此最佳拟合直线斜率

$$m_1 = \Delta V / \Delta I$$

等于R，于是

$$\boxed{\rho_1 = \frac{m_1 A}{\ell} = \frac{m_1 \pi d^2}{4\ell}}.$$

取拟合线上相距较远的两点求斜率可减小读数误差。若反过来画I对V，斜率为

$$1/R,$$

必须相应取倒数。

练习 2025 Q3 (c)(i)

In Experiment 2, the students are asked to use a graph to determine the resistivity ρ_2 of solid, cylindrical resistors made of the same material but of different lengths L. The cross-sectional area of each resistor is $5.0\times10^{-6}\ \text{m}^2$. The students directly measure the resistance R between the ends of each resistor. Table 1 provides L and R for each resistor.

Table 1

L (m)	$R\ (\Omega)$
0.010	0.90
0.020	1.6
0.030	2.5
0.040	3.2
0.050	4.0

C.

i. **Indicate** two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine ρ_2.

Vertical axis: _____ Horizontal axis: _____

选定坐标：先标单位与均匀刻度再描点

描点前确定横纵轴量与单位。横轴取L（m）、纵轴取R（Ω）时，将表内每一组(L,R)当作一个点。

刻度必须等间隔；使数据覆盖图的主要区域，同时留出读数空间。这里只描真实数据点，拟合直线留给下一问。

例题 2025 Q3 (c)(ii)

本问承接 2025 Q3 (c)(i)。

In Experiment 2, the students are asked to use a graph to determine the resistivity ρ_2 of solid, cylindrical resistors made of the same material but of different lengths L. The cross-sectional area of each resistor is $5.0\times10^{-6}\ \text{m}^2$. The students directly measure the resistance R between the ends of each resistor. Table 1 provides L and R for each resistor.

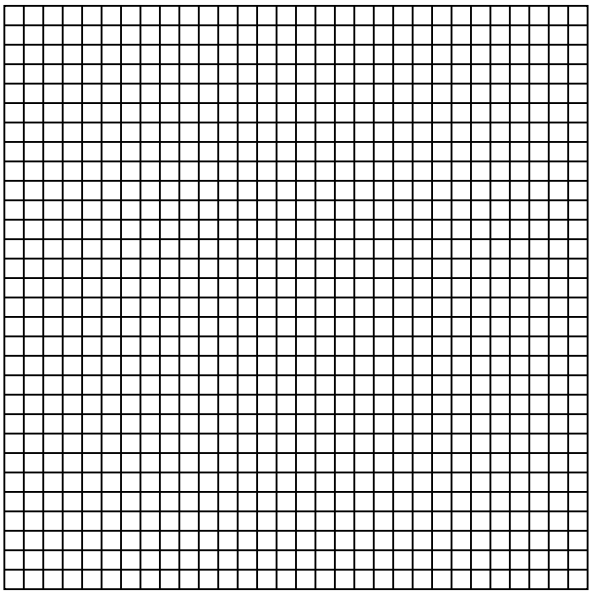
Table 1

L (m)	$R\ (\Omega)$
0.010	0.90
0.020	1.6
0.030	2.5
0.040	3.2
0.050	4.0

C.

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.
- **Plot** the points you recorded in Table 2.



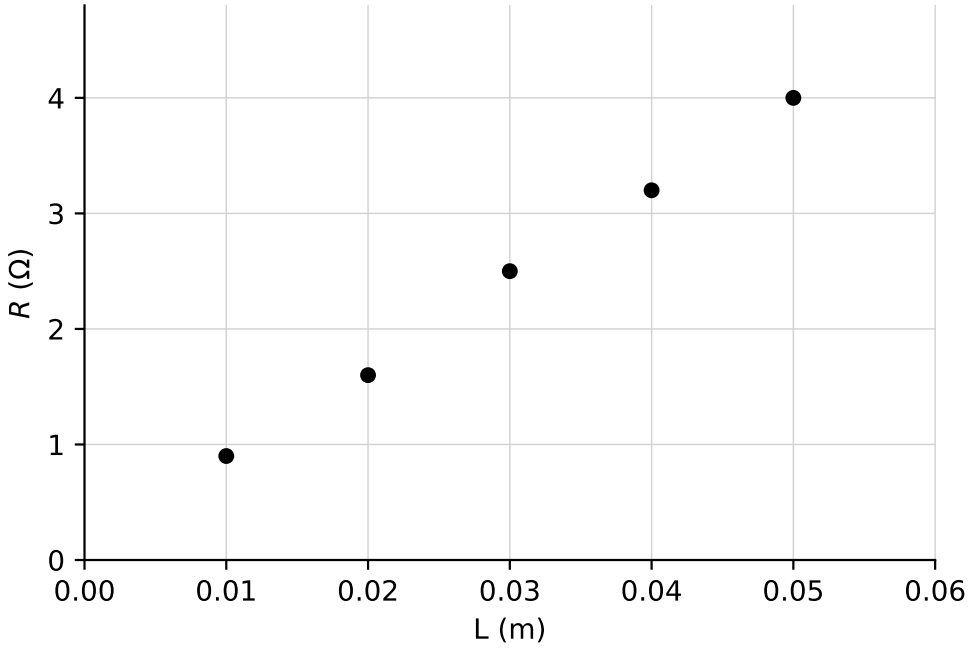
解：

C(ii) 数据点题给共同截面积为

$$5.0 \times 10^{-6}\ \mathrm{m}^2,$$

数据如下，不必作非线性变换。

L（m）	R（Ω）
0.010	0.90
0.020	1.6
0.030	2.5
0.040	3.2
0.050	4.0



各点按原测量表逐行对应；此图只描点。

已描出的散点：只画一条最佳拟合线

沿点群的整体趋势画直线，使各点在两侧较均衡。不要通过逐点连线把测量误差当成规律；也不要因为物理模型截距为零就忽视明显的系统偏差。

例题 2025 Q3 (c)(iii)

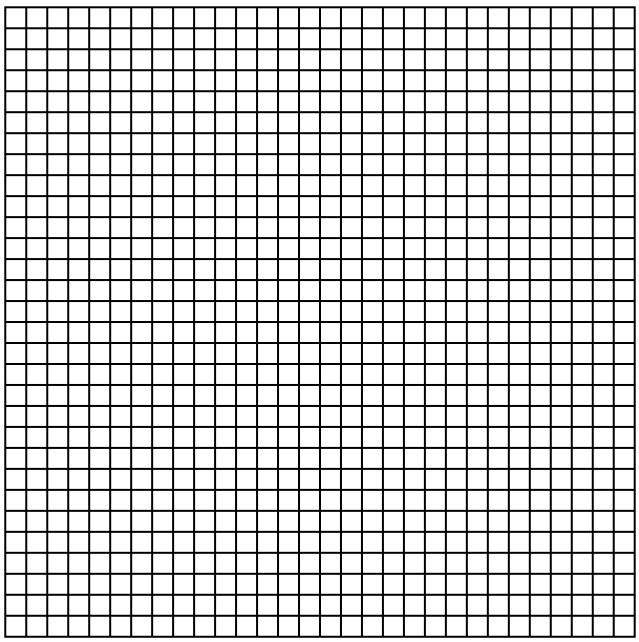
本问承接 2025 Q3 (c)(ii)。

In Experiment 2, the students are asked to use a graph to determine the resistivity ρ_2 of solid, cylindrical resistors made of the same material but of different lengths L. The cross-sectional area of each resistor is 5.0×10^{-6} m^2. The students directly measure the resistance R between the ends of each resistor. Table 1 provides L and R for each resistor.

Table 1

L (m)	$R\,(\Omega)$
0.010	0.90
0.020	1.6
0.030	2.5
0.040	3.2
0.050	4.0

C.



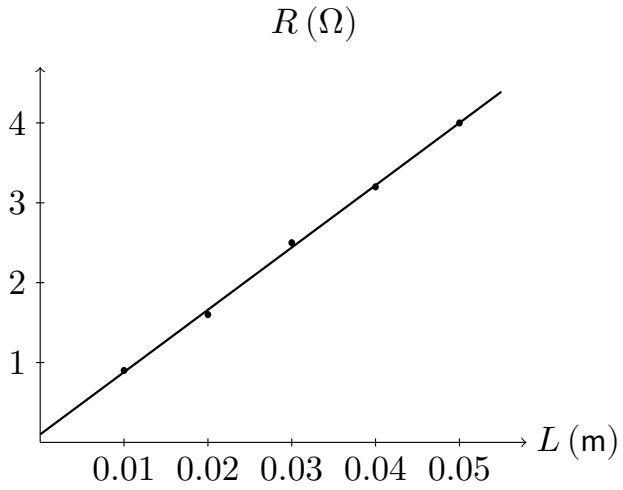
iii. **Draw** a best-fit line for the data graphed in part C (ii).

解：

C(iii) 最佳拟合线用线性刻度在图上标出全部五点，画一条经过点群中间的直线。不要把五点折线连接。下图的最小二乘直线为

$$R \approx 78L + 0.10;$$

小截距体现有限实验误差，不需要强迫直线经过原点。



由R对L斜率：算电阻率

R对L图的斜率b=ρ/A。先从拟合线上挑两个相距较远的点计算b，再乘题给共同截面积A。两点来自拟合线，不必是原表中的两个测量点。

$$b = \frac{\Delta R}{\Delta L}, \qquad \rho = bA.$$

例题 2025 Q3 (d)

本问承接 2025 Q3 (c)(iii)。

In Experiment 2, the students are asked to use a graph to determine the resistivity ρ_2 of solid, cylindrical resistors made of the same material but of different lengths L. The cross-sectional area of each resistor is 5.0×10^{-6} m^2. The students directly measure the resistance R between the ends of each resistor. Table 1 provides L and R for each resistor.

Table 1

L (m)	$R\,(\Omega)$
0.010	0.90
0.020	1.6
0.030	2.5
0.040	3.2
0.050	4.0

D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for ρ_2.

解：

D 第二组实验的电阻率斜率

$$m_2 = \rho_2/A,$$

所以

$$\rho_2 = Am_2。$$

在上述拟合线上，选

$$(0.010, 0.88)$$

与

$$(0.050, 4.00),$$

有

$$m_2 = \frac{4.00-0.88}{0.050-0.010} = 78\,\Omega/\text{m},$$

$$\boxed{\rho_2 = (5.0\times10^{-6})(78) = 3.9\times10^{-4}\,\Omega\cdot\text{m}}.$$

注意

$$R/L$$

不是一般拟合线的斜率；有非零截距时应取

$$\Delta R/\Delta L$$。

电容充放电与电荷共享

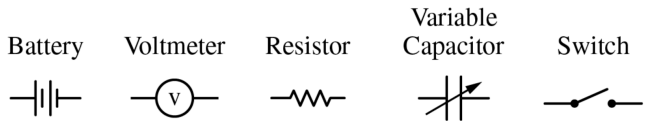
充、放两条路径：把测量仪接到正确位置

电容充分充电时应与电池构成路径；随后放电要让电池退出放电回路，否则它会继续维持电势差。可用两个开关选择两条路径。

测电容电压用并联电压表；测放电电流用串联电流表。原图允许的导线、开关数量和器件都要按题意保留。

例题 2022 Set 1 Q2 (a)

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



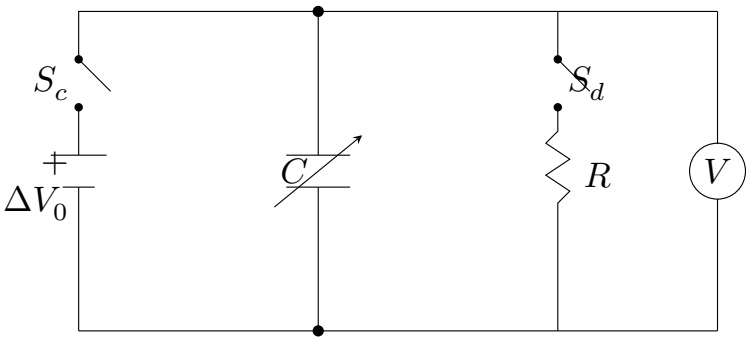
(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

解：

(a) 可充电也可放电的电路电压表要测电容两端，所以与电容并联。放电时必须让电池退出回路，否则电池会继续维持电压。下面用两个开关：充电时闭合S_c、断开S_d，使电池给电容充电；充满后断开S_c，在

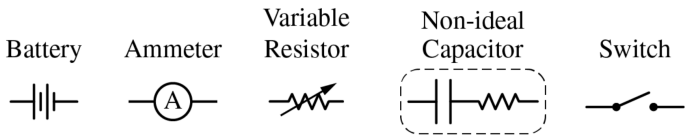
$$t = 0$$

闭合S_d，使电容和电阻构成放电回路。任何时刻都不要把两个开关同时闭合来代替这一操作。



练习 2022 Set 2 Q2 (a)

2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the current through the capacitor as it discharges through the resistor.

放电指数式：回路关系加电荷变化率

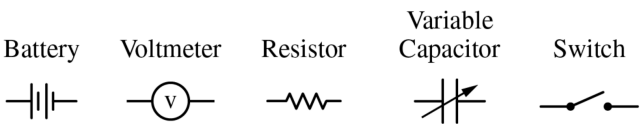
令Q是正极板剩余电荷，放电电流I取沿放电方向的正值，故I=−dQ/dt。电容电压Q/C等于放电路径电阻的电压IR。

总电阻包括路径中的全部内阻。得到dQ/Q=−dt/(R总C)，从初始Q_0积分到Q、从0积分到t，然后取指数。电压或电流再由Q推得。

$$\frac{dQ}{dt} = -\frac{Q}{R_{\text{total}}C}, \qquad Q = Q_0 e^{-t/(R_{\text{total}}C)}.$$

例题 2022 Set 1 Q2 (b)

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

解：

(b) 从回路关系证明指数衰减令

$$q(t) > 0$$

为开始时正极板上剩余的电荷，并把由该极板流向电阻的放电电流定义为

$$I > 0。$$

于是

$$I = -dq/dt,$$

负号表示极板电荷在减少。放电回路内，电容的电压等于电阻压降：

$$\frac{q}{C} = IR = -R\frac{dq}{dt}.$$

分离变量时，对有限放电时间

$$q > 0,$$

可以除以q。把初始电荷记为

$$q_0 = C\Delta V_0:$$

$$\int_{q_0}^{q(t)} \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt \quad \Rightarrow \quad \ln\frac{q(t)}{q_0} = -\frac{t}{RC}.$$

指数化，再除以不随时间改变的C：

$$q(t) = q_0 e^{-t/(RC)}, \qquad \boxed{\Delta V_C(t) = \Delta V_0 e^{-t/(RC)}}.$$

初始值是

$$\Delta V_0,$$

长期极限是零；指数中

$$t/(RC)$$

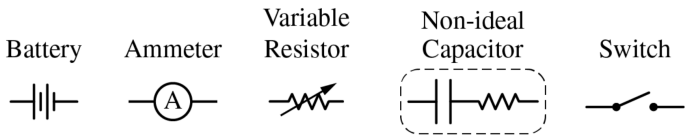
无量纲，因为

$$RC$$

的单位是秒。

练习 2022 Set 2 Q2 (b)

2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the current I through the capacitor as a function of time t is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

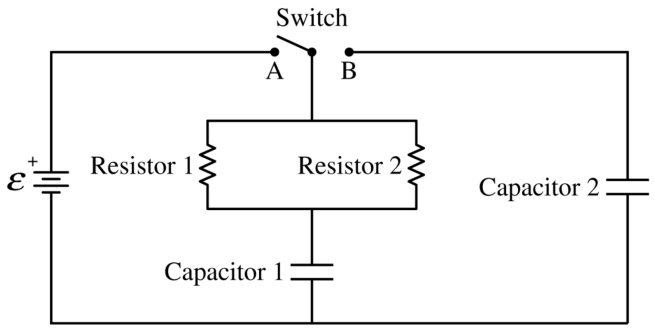
只建立充电方程：沿回路相加电势变化

充电电流使正极板电荷增加，因此I=dQ/dt。沿电流方向绕回路，电池升压$\mathcal{E}$、电阻降压IR总、电容降压Q/C之和为零。

先辨电阻组合：同一对节点是并联，不同器件位于唯一不分支路径才串联。题目要求不求解时，正确微分方程已完成目标。

$$\mathcal{E} - R_{\text{total}}\frac{dQ}{dt} - \frac{Q}{C} = 0.$$

例题 2023 Set 1 Q3 (a)



3. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   (a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

解：

(a) 充电微分方程，不求解两只R并联的等效电阻为

$$R/2;$$

流入电容的总电流

$$I = dQ/dt,$$

并非每只并联电阻的支路电流。沿充电回路列电压升降:

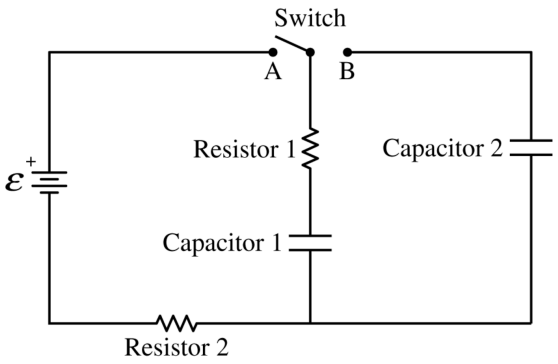
$$\mathcal{E} - I\frac{R}{2} - \frac{Q}{C} = 0.$$

所以所需方程为

$$\boxed{\mathcal{E} - \frac{R}{2}\frac{dQ}{dt} - \frac{Q}{C} = 0}, \qquad Q(0) = 0.$$

题目明确不要求求解，写清这个回路关系与电流定义即可完成(a)。

练习 2023 Set 2 Q3 (a)



3. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   (a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

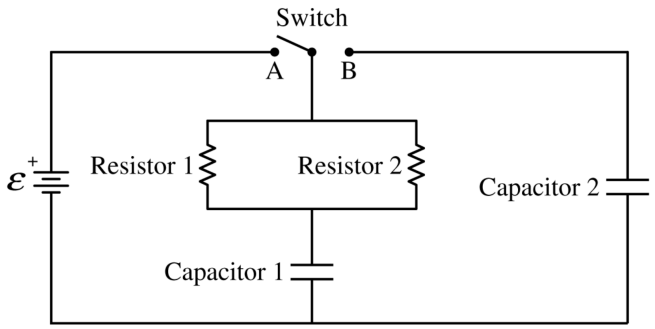
充电曲线：先看初值、终值与变化速度

电容初始未带电，充电电压由零逐渐趋于电池电压；电流从$\mathcal{E}$/R总逐渐趋于零。时间常数τ=R总C，是指数变化的时间尺度。

面电荷密度σ=Q/A，图形与Q相同。电阻功率P=I^2R总随指数更快下降；电容能量U=Q^2/(2C)初始斜率为零，之后先变陡再变平，不能把U画成全程向下凹。

$$Q = C\mathcal{E}(1 - e^{-t/\tau}), \quad I = \frac{\mathcal{E}}{R_{\text{total}}} e^{-t/\tau}, \quad U = \frac{Q^2}{2C}.$$

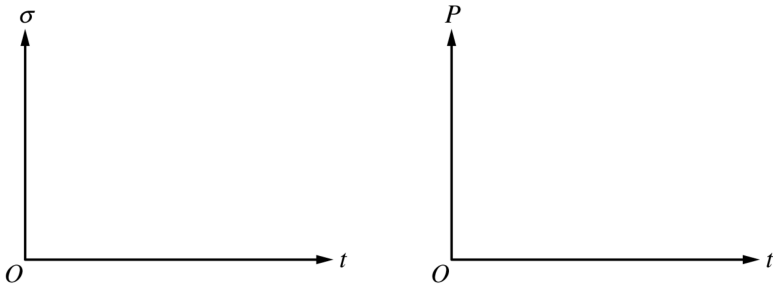
例题 2023 Set 1 Q3 (b)



3. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

(b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



解：

(b) 正极板面电荷密度与电阻总功率为了确定曲线形状，可用串联等效电阻的充电规律，时间常数

$$\tau = RC/2。$$

记电容1极板面积为A_p，不要与开关位置A 混淆：

$$Q = C\mathcal{E}(1 - e^{-t/\tau}), \qquad \sigma = \frac{Q}{A_p} = \frac{C\mathcal{E}}{A_p}(1 - e^{-t/\tau}).$$

因此σ从零开始，上升但正斜率不断减小，长期趋向

$$C\mathcal{E}/A_p。$$

总电流为

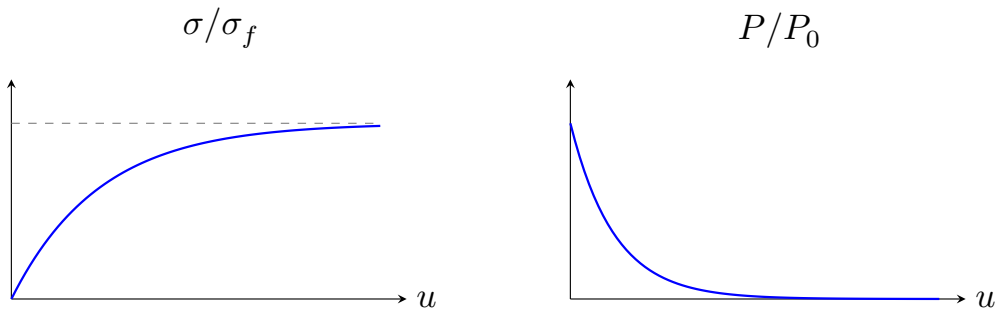
$$I = (2\mathcal{E}/R)e^{-t/\tau},$$

电阻总功率必须使用等效电阻配总电流：

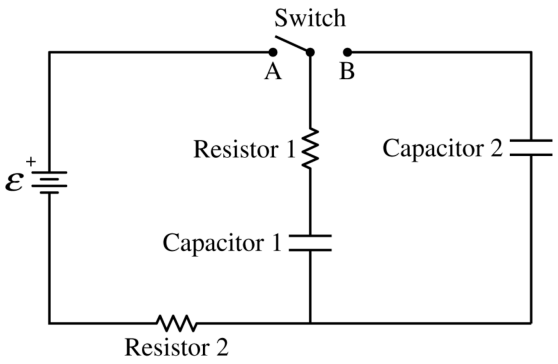
$$P = I^2\frac{R}{2} = \frac{2\mathcal{E}^2}{R}e^{-2t/\tau}.$$

它从非零初值下降到零，曲线向上弯曲。下面分别以最终面密度与初始功率归一化，横轴

$$u = t/\tau。$$



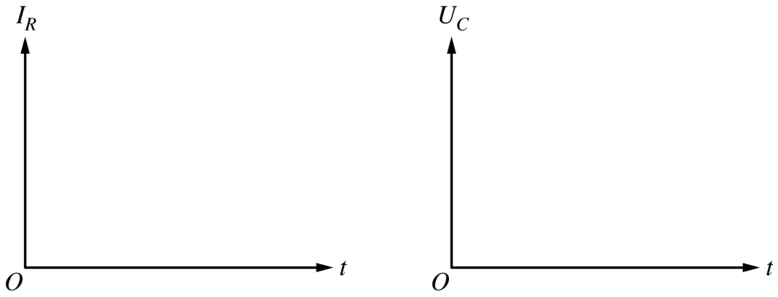
例题 2023 Set 2 Q3 (b)



3. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



解:

(b) 电阻1电流与电容1储能时间常数是

$$\tau = 2RC,$$

充电规律给

$$I_R = \frac{\mathcal{E}}{2R}e^{-t/\tau}, \qquad Q = C\mathcal{E}(1-e^{-t/\tau}),$$

$$U_C = \frac{Q^2}{2C} = \frac{C\mathcal{E}^2}{2}(1-e^{-t/\tau})^2.$$

电流从非零初值下降，趋零且负斜率逐渐减小。能量从零上升并趋于

$$C\mathcal{E}^2/2,$$

其初始斜率为零：初始电荷

$$Q = 0,$$

而能量与

$$Q^2$$

有关。令

$$u = t/\tau,$$

能量的归一化曲线为

$$(1-e^{-u})^2;$$

二阶导数为

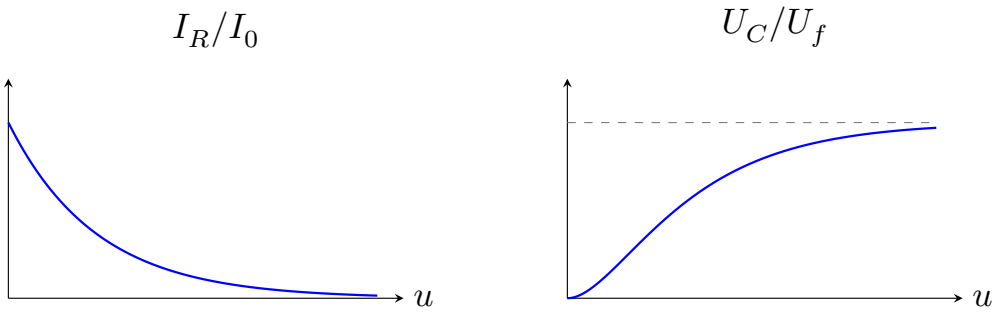
$$2e^{-u}(2e^{-u}-1),$$

所以初期向上弯曲，

$$u = \ln 2$$

后才向下弯曲并趋于水平。精确曲线不能把初期平切线省成一条从起点就始终向下弯的指数曲线。

官方评分示例使用了全程向下弯曲的简化示意；它与这里从能量式求得的初期曲率有差别。本图保留精确的初期形状，评分图的近似不能作为忽略平方因子的依据。



对数或时间常数图：拟合衰减规律

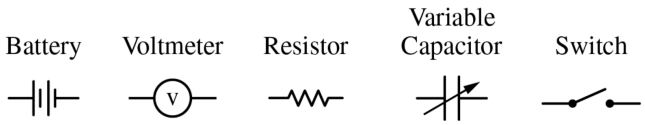
电压指数衰减时，先除以初始电压V_0再取自然对数，得到ln(VC/V_0)=−t/(RC)，比值无量纲，图的斜率为−1/(RC)、理论截距为零。

含电容内阻rC时，时间常数τ=(R+rC)C=CR+CrC，所以τ对外电阻R是直线。画已给散点的整体趋势，不把时间常数误当作电流。

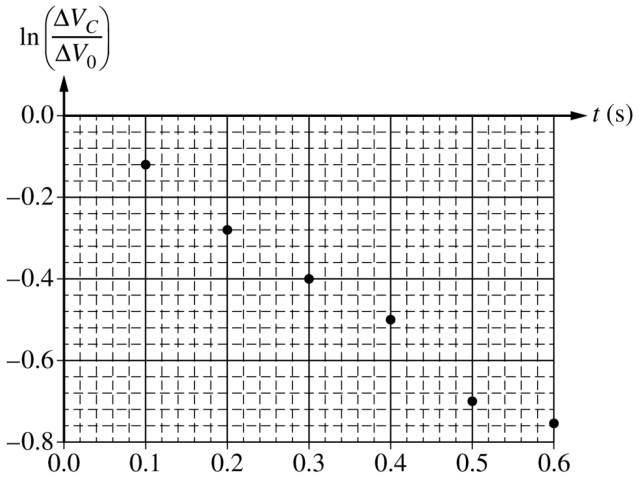
$$\ln\frac{V_C}{V_0}=-\frac{t}{RC},\qquad \tau=CR+Cr_C.$$

例题 2022 Set 1 Q2 (c)(i)

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



(c) The experiment is performed using a resistor of $R = 150\ \text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

t

i. Draw the best-fit line for the data.

解：

(c)(i) 最佳拟合直线将结果取对数：

$$y = \ln \frac{\Delta V_C}{\Delta V_0} = -\frac{t}{RC}.$$

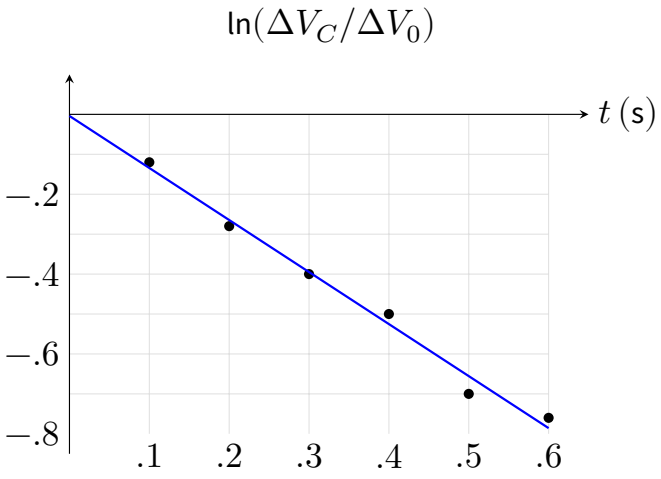
因此要画一条代表所有点总体趋势的下降直线，而不是依次连接散点。原图近似读数如下；图读数有小量误差。

t (s)	$\ln(\Delta V_C/\Delta V_0)$
0.1	-0.12
0.2	-0.28
0.3	-0.40
0.4	-0.50
0.5	-0.70
0.6	-0.76

对这些点的拟合为

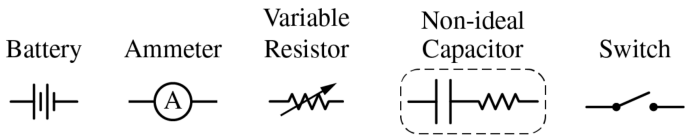
$$y \approx -1.303t - 0.004,$$

截距接近理论值零。下面给出这条与计算一致的拟合线。

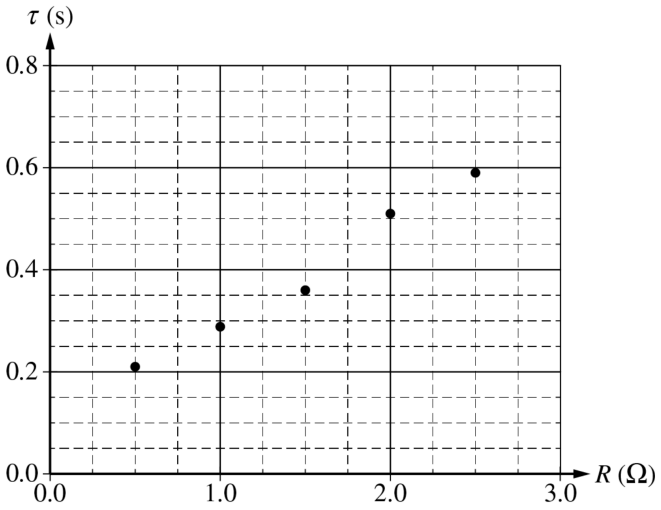


练习 2022 Set 2 Q2 (c)(i)

2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.



(c) The students determine the time constant τ for the circuit as a function of the resistance R. The students' data are shown in the following graph.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

$-t$

i. Draw the best-fit line for the data.

衰减图反求C或内阻：分开斜率和截距

ln(VC/V_0)对t图的负斜率b给C=−1/(Rb)。τ对R图的斜率b给C=b，而纵截距a给rC=a/b。

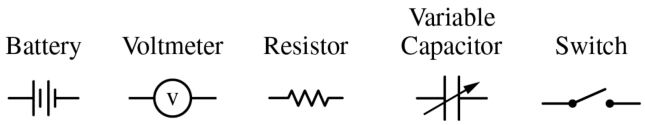
先写图上哪一个系数对应哪个量，再取拟合线上两点。只有时间常数实验同时有斜率和截距，不能拿单个测量点的τ/R当作内阻。

$$C = -\frac{1}{Rb}\ \text{（对数图）；}\qquad C = b,\quad r_C = \frac{a}{b}\ \text{（时间常数图）．}$$

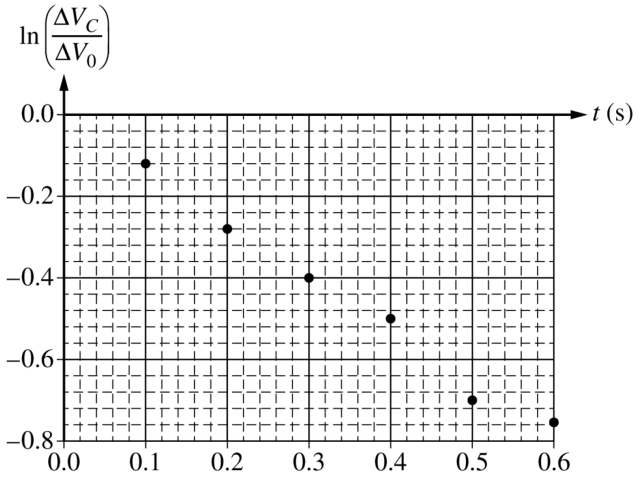
例题 2022 Set 1 Q2 (c)(ii)

本问承接 2022 Set 1 Q2 (c)(i)。

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



(c) The experiment is performed using a resistor of $R = 150\ \text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

t

ii. Using the best-fit line, calculate a value for the unknown capacitance C.

解：

(c)(ii) 由斜率求电容斜率包含的是

$$RC,$$

而不是电池电压；取对数时初始电压已被比值消去。选拟合线上的两个相距较远的点，比如

$$(0, -0.004)$$

与

$$(0.6, -0.7858):$$

$$m = \frac{-0.7858 - (-0.004)}{0.6 - 0} \approx -1.303\,\mathrm{s}^{-1}. \qquad m = -\frac{1}{RC}.$$

$$C = -\frac{1}{Rm} = -\frac{1}{(150 \times 10^3)(-1.303)} \approx \boxed{5.1 \times 10^{-6}\,\mathrm{F} = 5.1\,\mu F}.$$

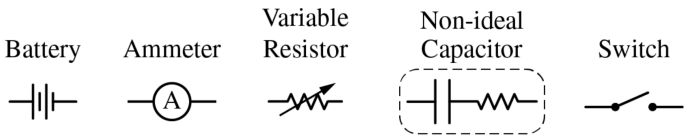
合理的手绘拟合线会给略有差异的结果；官方示例采用另一条近似拟合线，给出约

$$5\,\mu F。$$

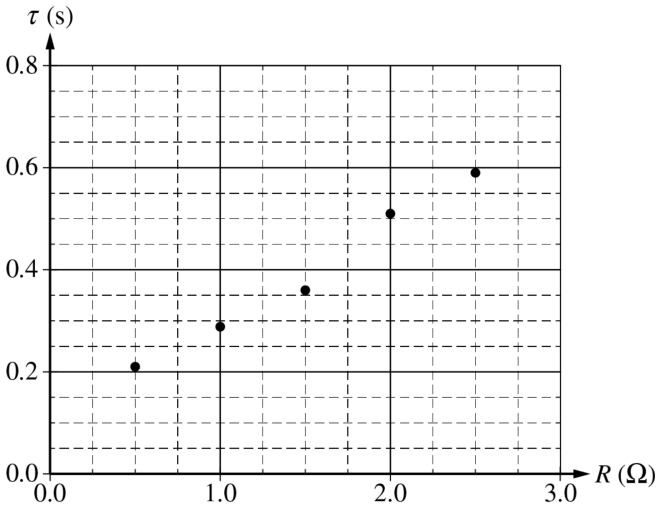
取斜率时用拟合线上的点，不必强求两个点都恰好是实验散点。

例题 2022 Set 2 Q2 (c)(ii)

本问承接 2022 Set 2 Q2 (c)(i)。

> 2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.
>
> 
>

(c) The students determine the time constant τ for the circuit as a function of the resistance R. The students' data are shown in the following graph.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

$-t$

ii. Using the best-fit line, calculate a value for the internal resistance r_C of the capacitor.

解：

(c)(ii) 求内部电阻应从同一条拟合线取斜率与截距。上述线在

$$R = 0$$

与

$$R = 3.0\,\Omega$$

的拟合值分别为0.098、0.686 s，故

$$C = m = \frac{0.686 - 0.098}{3.0 - 0} = 0.196\,\text{s}/\Omega = 0.196\,\text{F}.$$

纵截距

$$b = Cr_C,$$

于是

$$\boxed{r_C = \frac{b}{m} = \frac{0.098}{0.196} = 0.50\,\Omega}.$$

这一数值属于本页所画拟合线，读图和手绘会产生小量差异。官方示例给约$0.4\,\Omega$；例如另一条合理近似线

$$\tau = 0.20R + 0.08$$

会给

$$0.08/0.20 = 0.4\,\Omega。$$

核验时始终检查斜率、截距和图是同一条线，不能混用不同直线上的读数。两种近似都抓住了

$$r_C = b/m$$

这一实验关系。

改变器件后看图：只追踪闭合路径的参数

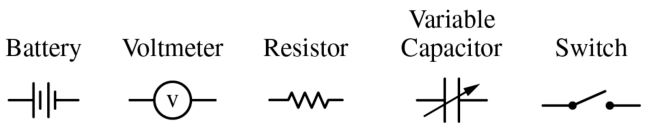
先确认正在充电还是放电，以及电池、仪表是否在那条闭合路径上。增大平行板面积使C增大；非理想电流表的串联内阻会被误计入电容内阻。

充分充电后电流趋于零，电池内阻无稳态压降；脱离电池的放电路径也不含电池内阻。图的斜率或截距改变与否，须分别用对应表达式判断，不能说“非理想”就全部改变。

$$C = \varepsilon\frac{A}{d}, \qquad \tau = (R + r_C + r_A)C.$$

例题 2022 Set 1 Q2 (d)

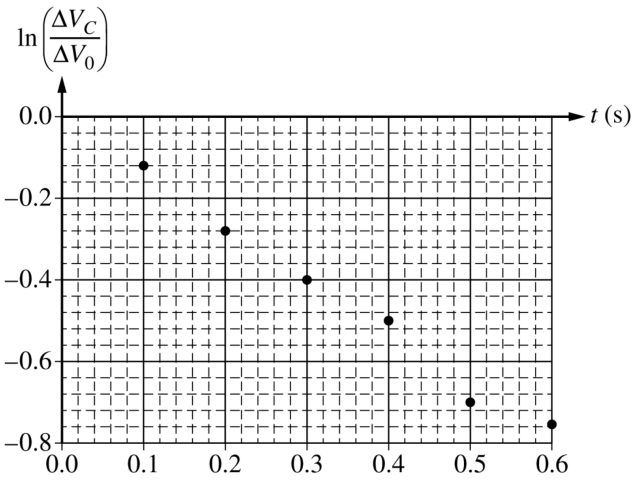
2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

t

(c) The experiment is performed using a resistor of $R = 150\,\text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

___ More steep ___ Less steep ___ Unchanged

Briefly justify your answer.

解：

(d) 极板面积增加平行板电容在板距和介质不变时满足

$$C = \varepsilon A/d;$$

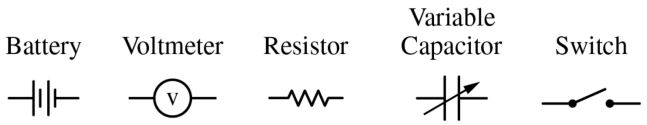
本题只调整极板面积，故C增大。由于

$$|m| = 1/(RC),$$

其绝对值减小，负斜率更接近零。因此选择Less steep（较平缓）。从放电过程看，更大的电容在相同电压下存储更多电荷，经同一电阻放电需要更久。

练习 2022 Set 1 Q2 (e)(i)

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

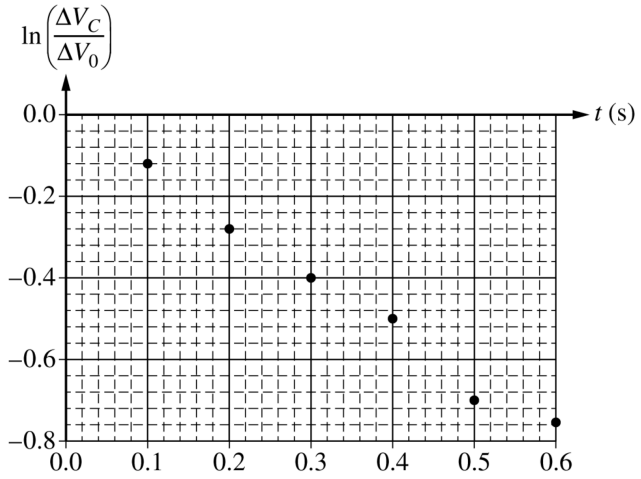


(e) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

t

(c) The experiment is performed using a resistor of $R = 150\ \text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



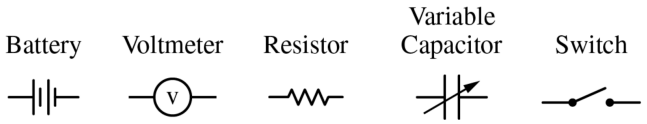
i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

练习 2022 Set 1 Q2 (e)(ii)

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

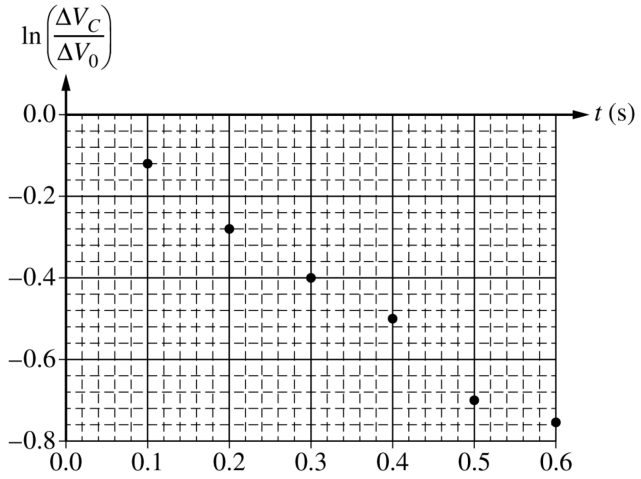


(e) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

t

(c) The experiment is performed using a resistor of $R = 150\ \text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



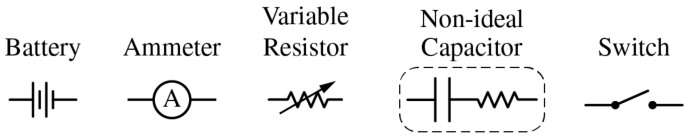
ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

例题 2022 Set 2 Q2 (d)

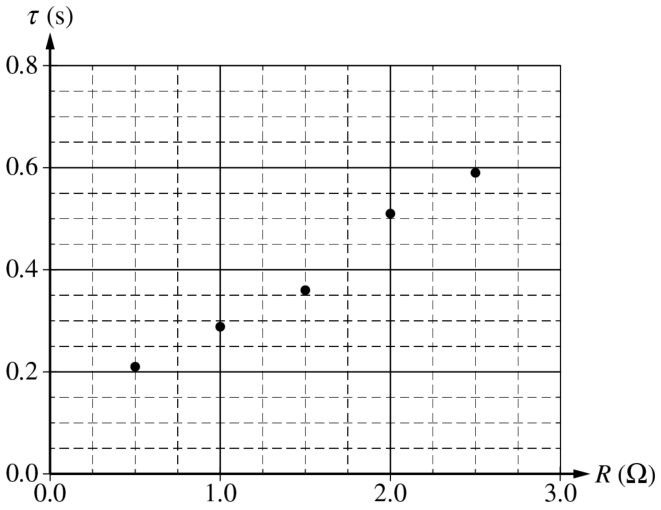
2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

$-t$

(c) The students determine the time constant τ for the circuit as a function of the resistance R. The students' data are shown in the following graph.



(d) The ammeter is found to be nonideal. Is the actual value for the internal resistance r_C for the capacitor greater than, less than, or equal to the experimental internal resistance of the capacitor calculated in part (c)?

___ Greater than ___ Less than ___ Equal to

Briefly justify your answer using features of the graph in part (c).

解：

(d) 电流表非理想时，真实r_C与实验值的关系设电流表内阻为

$$r_A > 0。$$

串联总电阻实际是

$$R + r_C + r_A,$$

所以

$$\tau = CR + C(r_C + r_A).$$

斜率仍为C，但纵截距多了

$$Cr_A;$$

忽略电流表内阻时，把

$$b/C$$

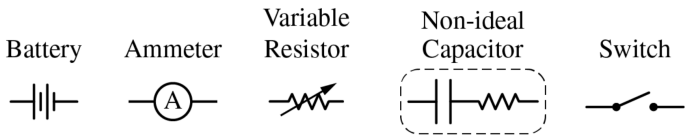
全归给电容，会得到实验值

$$r_{C,\text{exp}} = r_C + r_A。$$

因此真实电容内阻Less than（小于）实验所得值。这个结论来自图的截距增加，而不是来自电容值改变。

练习 2022 Set 2 Q2 (e)

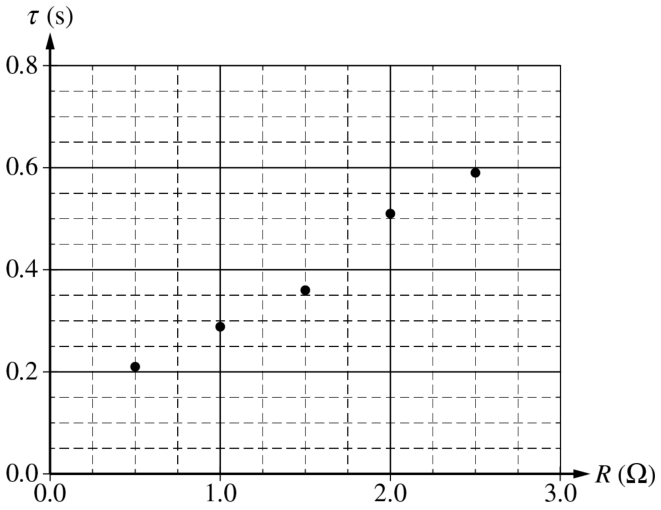
2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

$$-t$$

(c) The students determine the time constant τ for the circuit as a function of the resistance R. The students' data are shown in the following graph.



(e) The values of the variable resistor in the original experiment ranged from $0.5\,\Omega$ to $2.5\,\Omega$. The experiment is repeated with values ranging from $3.0\,\Omega$ to $6.0\,\Omega$. Would the slope of the best-fit line be more steep, be less steep, or remain unchanged compared to the graph in part (c)?

___ More steep ___ Less steep ___ Remain unchanged

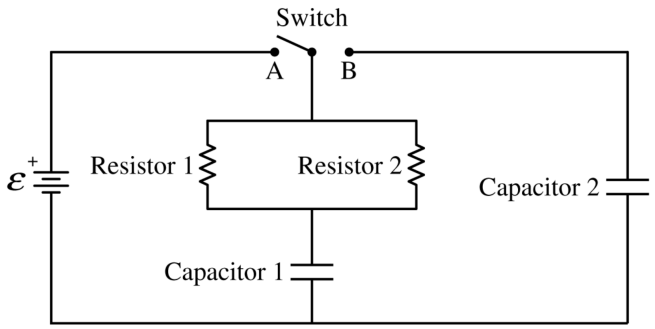
Briefly justify your answer.

接到未带电电容：看电势高低判瞬时流向

正极板储有电荷意味着它电势较高；未带电电容初始电势差为零。接通共享路径后，正电荷沿路径从高电势板向低电势板流动。

沿原图实际导线追踪经过开关或指定电阻的方向。共享路径已脱离电池时，不把电池方向当作瞬时电流方向。

例题 2023 Set 1 Q3 (c)(i)



3. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

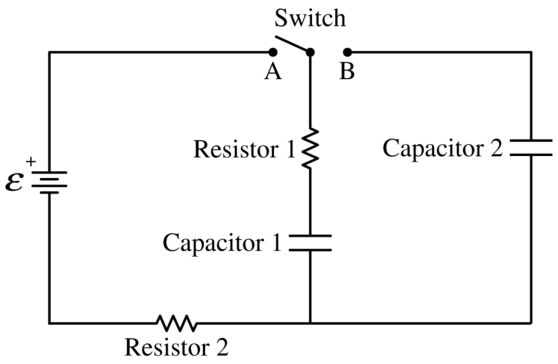
   (c) At time t_1, the switch is closed to Position B.

   i. Immediately after time t_1, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

解：

(c)(i) 接B 后开关中的电流方向充分充电后电容1上板为正、下板为负，带电Q_0；电容2尚未带电。接B 将其上端与电容1的高电势端连通。常规电流从电容1上板出发，向上经电阻后向右通过开关，给电容2上板充电，所以选择向右。

练习 2023 Set 2 Q3 (c)(i)



3. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

   (c) At time t_1, the switch is closed to Position B.

   i. Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

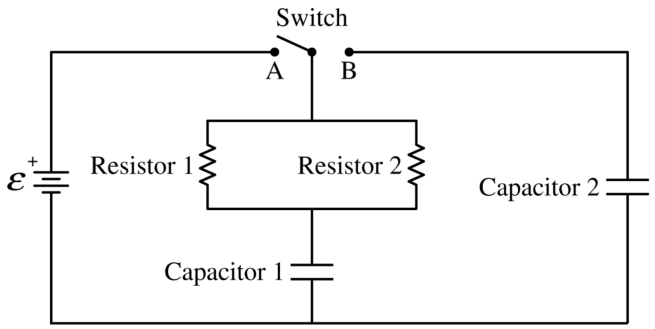
孤立电容相连：电荷守恒与最终同压

连接后两电容并联，最终两端电势差相同；电池退出时，共享两上板的总电荷守恒。设共同电压V，分别用Q_1=C_1V、Q_2=C_2V。

先由(C_1+C_2)V=Q_0求V，再求指定电容电荷。电荷按电容比共享，不是无条件各分一半。

$$Q_1 + Q_2 = Q_0, \qquad \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = V.$$

例题 2023 Set 1 Q3 (c)(ii)



3. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

   (c) At time t_1, the switch is closed to Position B.

   ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

解：

(c)(ii) 共享完成后的电容2电荷电池退出，两个上板的总电荷守恒，

$$Q_1 + Q_2 = Q_0。$$

新稳定状态中电阻电流为零，电阻压降为零，两电容电压相等：

$$Q_1/C = Q_2/(2C),$$

即

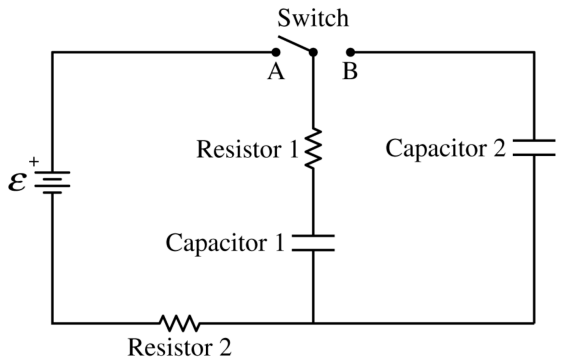
$$Q_2 = 2Q_1。$$

联立得

$$\boxed{Q_2 = \frac{2Q_0}{3}}, \qquad Q_1 = \frac{Q_0}{3}.$$

电荷按电容比例分配，不能因为有两只电容就平分。

练习 2023 Set 2 Q3 (c)(ii)



3. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

   (c) At time t_1, the switch is closed to Position B.

   ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

共享耗热：初末电容能量相减

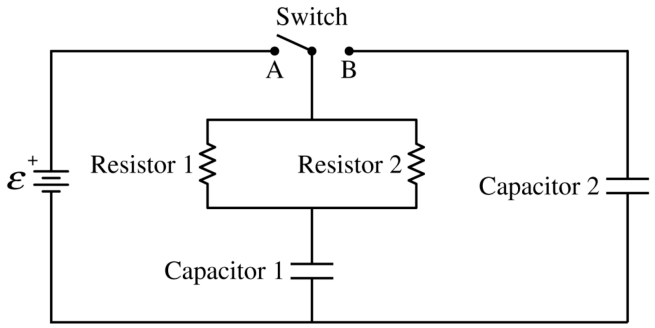
共享阶段没有电池提供能量，储能减少转为电阻焦耳热。初态只有一只电容带电；末态两只电容都带电。

先写每只电容能量$Q^2/(2C)$，把两只末态能量相加，再用初态减末态。这里不要求先算电流随时间再积分。

$$E_R = U_i - U_f, \qquad U = \frac{Q^2}{2C}.$$

例题 2023 Set 1 Q3 (c)(iii)

本问承接 2023 Set 1 Q3 (c)(ii)。



3. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

   (c) At time t_1, the switch is closed to Position B.

   iii. Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

解：

(c)(iii) 两电阻耗散的总能量这一阶段没有电池输入能量，电容储能的减少成为电阻热量。初态仅电容1有能量，末态两只都有：

$$U_i = \frac{Q_0^2}{2C}, \qquad U_f = \frac{(Q_0/3)^2}{2C} + \frac{(2Q_0/3)^2}{2(2C)} = \frac{Q_0^2}{6C}.$$

因此正的耗散能量为

$$\boxed{E_R = U_i - U_f = \frac{Q_0^2}{3C}}.$$

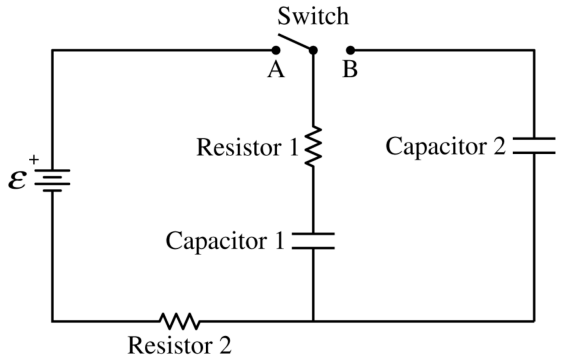
若用“电容储能变化”作为量，则

$$\Delta U = U_f - U_i = -Q_0^2/(3C);$$

它的负号表示储能减少，不能把这个负值直接称为电阻得到的热量。

练习 2023 Set 2 Q3 (c)(iii)

本问承接 2023 Set 2 Q3 (c)(ii)。



3. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

   (c) At time t_1, the switch is closed to Position B.

   iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

共享后再改板距或介质：重新分配电荷

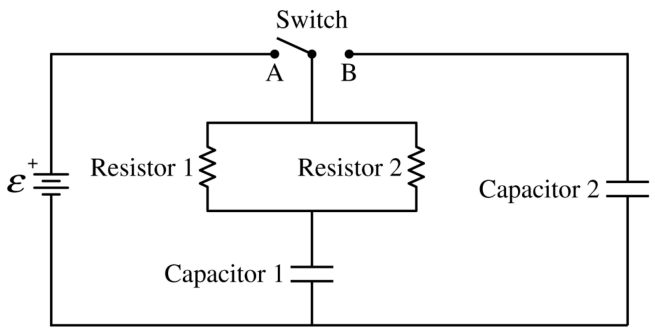
并联且孤立时，改变某一电容后总电荷仍守恒，最终电压仍共同；改变的电容值必须重新代入共享关系。

板距加倍使C减半；填入介电常数κ使C乘κ。求能量比或新电荷前，先把改变后的两电容值和共同电压写清，不能直接保持每只原电荷不变。

$$C = \varepsilon \frac{A}{d}, \qquad V = \frac{Q_0}{C_1 + C_2}.$$

例题 2023 Set 1 Q3 (d)

本问承接 2023 Set 1 Q3 (c)(ii)。



3. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

   A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

   (c) At time t_1, the switch is closed to Position B.

   (d) Determine the ratio $\dfrac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

解：

(d) 电容2板距加倍后的储能比开关仍在B，板距加倍使电容2由

$$2C$$

变为C，因为

$$C \propto 1/d。$$

两电容继续相连，因此不是分别保持原电荷；总电荷仍为Q_0，新的共同电压为

$$Q_0/(2C),$$

每只最终带

$$Q_0/2。$$

两只电容现在电容相同且电压相同，利用

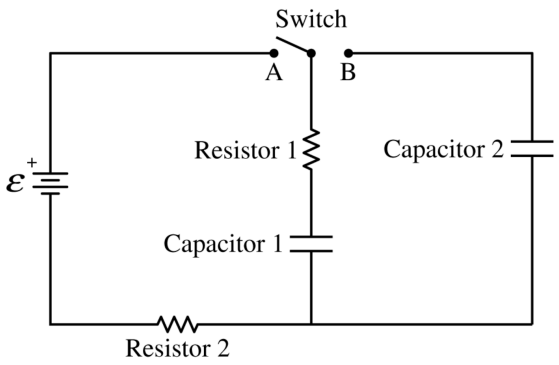
$$U = C(\Delta V)^2/2$$

得

$$\boxed{\frac{U_2}{U_1} = 1}.$$

例题 2023 Set 2 Q3 (d)

本问承接 2023 Set 2 Q3 (c)(ii)。



3. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

解：

(d) 在电容2插入介质开关仍在B，介电常数

$$\kappa = 2$$

使电容2由C变为

$$2C。$$

两电容相连，会再分配电荷，守恒的是二者总正极板电荷Q_0。最终同电压条件为

$$Q_1/C = Q_2/(2C),$$

故

$$Q_2 = 2Q_1, \qquad Q_1 + Q_2 = Q_0 \quad \Rightarrow \quad \boxed{Q_2 = \frac{2Q_0}{3}}.$$

最终共同电压为

$$Q_0/(3C)。$$

孤立单个电容插介质时电荷不变的结论，不能应用到仍与另一电容连着的电容2上。

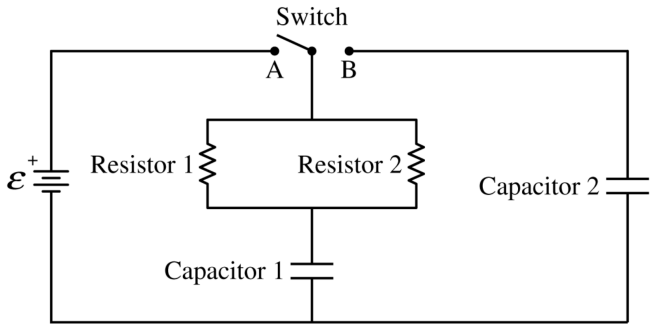
开关刚改变：电容电压连续求初始电流

电容电荷和电压不能在零时间内有限跳变。先取换接前已求的电容电压，放到换接后的真实回路；再用电池与电容的电压差除以对应电阻。

如果新增导线把节点短接，先重画等电势节点。此时路径可能由电容而非电池供电，电流方向要依电容极性判断。

例题 2023 Set 1 Q3 (e)(i)

本问承接 2023 Set 1 Q3 (d)。



3. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   With the capacitors still charged as in part (d), the switch is now closed to Position A.

   (e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

   A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

   (c) At time t_1, the switch is closed to Position B.

   With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

   i. Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.

解：

(e)(i) 再接A 瞬间的电池电流开关转回A 后电容2上端断开，电池重新给电容1供电。最初充分充电时

$$Q_0 = C\mathcal{E},$$

因此

$$\mathcal{E} = Q_0/C。$$

电容1在转接瞬间仍保持(d)的电荷

$$Q_0/2,$$

电压为

$$Q_0/(2C);$$

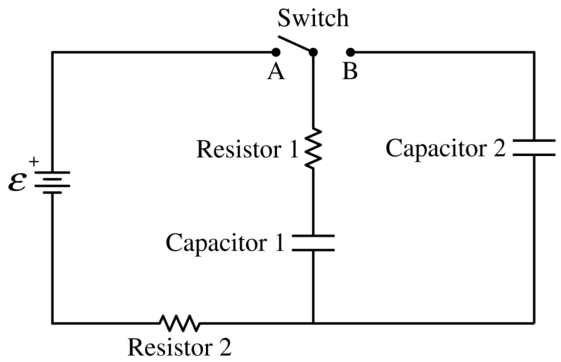
有限电阻不能让电荷瞬间跳变。回路式为

$$\frac{Q_0}{C} - I_0\frac{R}{2} - \frac{Q_0}{2C} = 0, \qquad \boxed{I_0 = \frac{Q_0}{RC}}.$$

这是总电池电流，两并联电阻各承担一半。

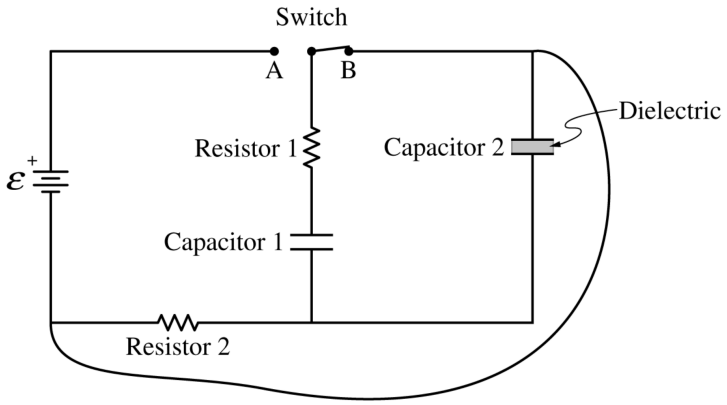
例题 2023 Set 2 Q3 (e)(i)

本问承接 2023 Set 2 Q3 (d)。



3. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

解：

(e)(i) 接入图示导线后电阻2的瞬时电流原图新增零阻导线把电容2的上端节点接到电阻2的左端，使电阻2的两端与电容2的两端成为同一对节点，形成放电路径。电容2电荷不能通过有限电阻瞬间改变，其初始电压仍为

$$\Delta V_{C_2}(0^+) = \frac{2Q_0/3}{2C} = \frac{Q_0}{3C}.$$

电阻2因此立即承受相同电压，欧姆定律给

$$\boxed{I_{R_2}(0^+) = \frac{Q_0}{3RC}}.$$

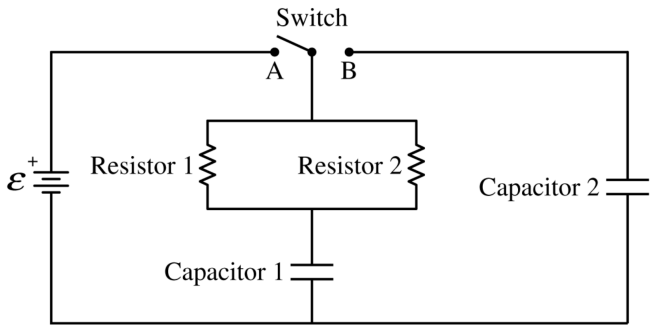
这里不需先求整个复杂网络的瞬时等效电阻；由新增导线形成的相同节点，已经能直接确定电阻2两端电压。

很久以后：电容支路不再有直流电流

最终电荷不再变化，所以电容电流dQ/dt为零。若没有剩余的电池与电阻闭合路径，整个所问支路电流为零。

判断长期电流时看最终拓扑，不把瞬时充放电电流沿用到很久以后。

例题 2023 Set 1 Q3 (e)(ii)



3. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

   At time $t = 0$, the switch is closed to Position A.

   With the capacitors still charged as in part (d), the switch is now closed to Position A.

   (e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

   A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

   (c) At time t_1, the switch is closed to Position B.

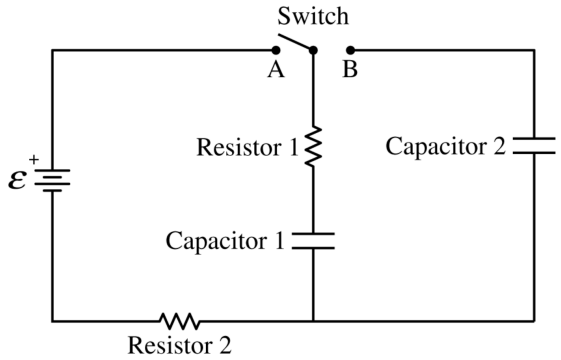
With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

ii. Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

解：

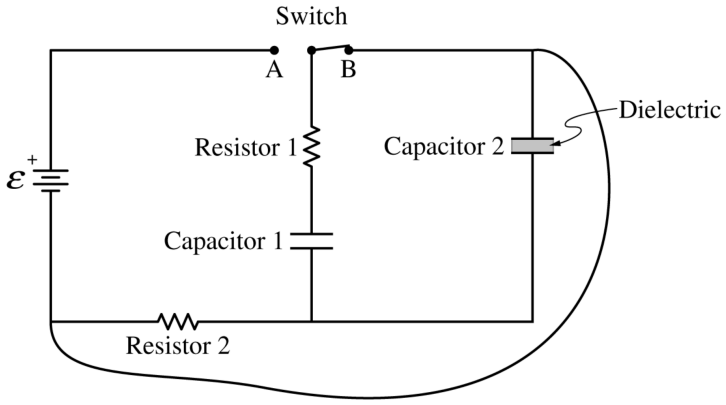
(e)(ii) 长期电池电流　电容1再次达到电池电压后，不再继续积累电荷，$dQ/dt = 0$，所以$\boxed{I_\infty = 0}$。电容2上端仍断开，不构成绕过电容1的稳定导电支路。

练习 2023 Set 2 Q3 (e)(ii)



3. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

电流的磁场与磁力

非均匀电流密度：先积分包围电流

J是每单位截面积通过的电流。以导线轴为中心，半径r、厚dr的小圆环面积是2πrdr，故该环电流dI=J(r)2πrdr。

安培定律把磁场沿闭合环路的总和与包围电流联系。柱对称时B大小处处相同、沿环路切线，因此左边化为B(2πr)。导线内部不能直接使用全部电流I_0。

$$I_{\text{enc}}(r) = \int_0^r J(u)\,2\pi u\,du, \qquad B(2\pi r) = \mu_0 I_{\text{enc}}.$$

例题 2026 Q1 (a)(i)

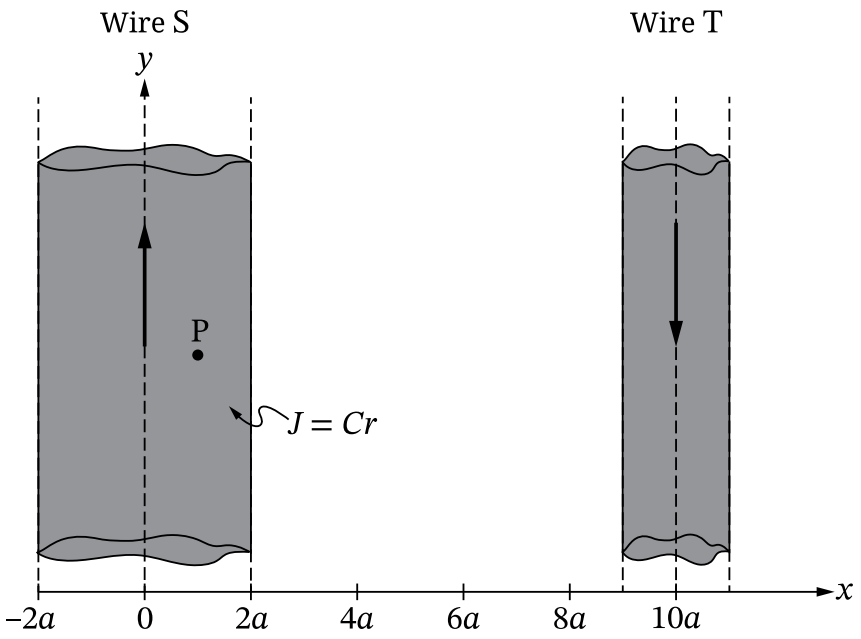
Question 1: Version J

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1. Two very long, cylindrical, current-carrying Wires S and T are oriented parallel to the *y*-axis, as shown in Figure 1. The centers of the wires are in the *xy*-plane. The wires are described as follows.

- Wire S has radius $2a$ and is centered at $x = 0$. Wire S has nonuniform current density and carries total current I_0 in the +*y*-direction. The magnitude of the current density in Wire S as a function of the radial distance *r* from the center of the wire is described by $J = Cr$, where *C* is a positive constant.
- Wire T has radius *a* and is centered at $x = 10a$. Wire T has uniform current density and carries total current I_0 in the –*y*-direction.

Figure 1



Part A

i. Point P is located inside of Wire S at horizontal position $x = a$. **Derive** an expression for the magnitude of the magnetic field at Point P due only to the current in Wire S in terms of *C*, *a*, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

A(i) S 内部

$$r = a$$

处的磁场安培定律需要的是环路内包围的电流，不能把整根导线的I_0 放进半径a的环路：

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}.$$

圆柱对称性使半径a的环路上磁场大小相同且沿切线，左侧为

$$B(2\pi a)。$$

把截面分成半径r、厚

$$dr$$

的薄环，其面积为

$$dA = 2\pi r\, dr;$$

流过薄环的电流为

$$dI = J(r)dA = Cr(2\pi r\, dr) = 2\pi C r^2\, dr.$$

因此

$$I_{\text{enc}} = \int_0^a 2\pi C r^2\, dr = \frac{2\pi C a^3}{3},$$

$$B(2\pi a) = \mu_0 \frac{2\pi C a^3}{3}, \qquad \boxed{B = \frac{\mu_0 C a^2}{3}}.$$

J是每单位面积电流，不是线电荷密度；漏掉薄环的

$$2\pi r$$

会使积分与单位都错误。

两长导线场：先定方向再相加

导线外部磁场大小μ_0I/(2πr)，r是观察点到那根导线中心的距离。拇指沿电流，弯曲四指给绕线磁场方向。

两根导线贡献同向才相加，反向才相减。先分别标位置、距离和方向，再求净磁场，不能把电流数值先相加后当作同一根线。

$$B = \frac{\mu_0 I}{2\pi r}.$$

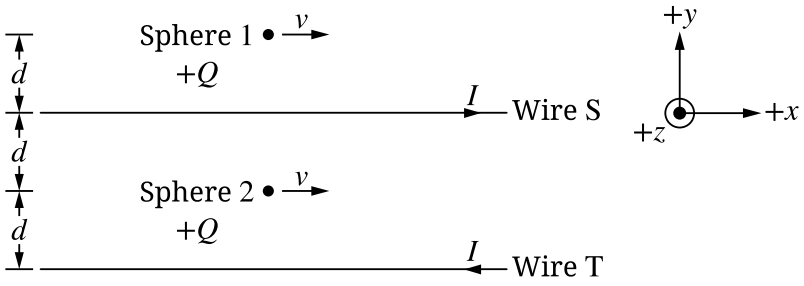
例题 2025 Q4 (b)

Question 4

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4. Long, parallel wires S and T are a distance $2d$ apart. Both wires carry equal currents I, but the currents are in opposite directions. Both wires are parallel to the x-axis. At the instant shown in Figure 1, Sphere 1 is a distance d above Wire S, Sphere 2 is a distance d below Wire S, and both spheres are moving with speed v in the $+x$-direction. Each sphere has positive charge $+Q$. Gravitational effects are negligible.

Figure 1



B. Derive an expression for the magnitude B_{tot} of the magnetic field at the location of Sphere 2 due to the currents in wires S and T in terms of d, I, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

Sphere 2位于Wire S下方、Wire T上方，离两根线的中心都为d。Wire S电流沿+x，用右手定则得它在球处产生−z方向的场；Wire T电流沿−x，它在上方球处也产生−z方向的场。两项同向，故大小相加。

对其中任一根长导线，选半径d的同心安培圆，磁场沿圆切线且大小相同。μ_0为真空磁导率；沿闭合圆把B乘每一小段弧长相加，得到B(2πd)。

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I, \qquad B_S = B_T = \frac{\mu_0 I}{2\pi d}.$$

$$B_{\text{tot}} = B_S + B_T = \frac{\mu_0 I}{\pi d}.$$

本问要求大小；净场方向为−z，可用来检查同向相加的判断。

两导线之间的场图：距离改变的两项相加

对反向电流，两导线之间的磁场贡献同向。沿x移动时，离一根线更近会增强它的贡献，同时离另一根线更远会减弱另一项。

把两距离写成x和10a−x，合场图是两项倒数的和。在两端附近较大，中间有最小值；不是穿过零的相消图。

$$B(x) = \frac{\mu_0 I_0}{2\pi}\left(\frac{1}{x} + \frac{1}{10a - x}\right).$$

例题 2026 Q1 (a)(ii)

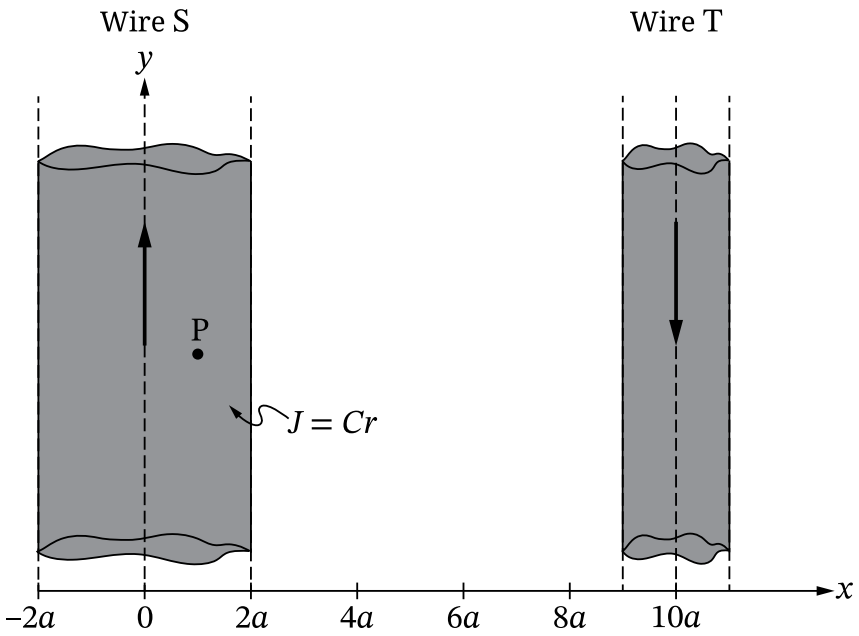
Question 1: Version J

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1. Two very long, cylindrical, current-carrying Wires S and T are oriented parallel to the *y*-axis, as shown in Figure 1. The centers of the wires are in the *xy*-plane. The wires are described as follows.

- Wire S has radius $2a$ and is centered at $x = 0$. Wire S has nonuniform current density and carries total current I_0 in the +*y*-direction. The magnitude of the current density in Wire S as a function of the radial distance *r* from the center of the wire is described by $J = Cr$, where *C* is a positive constant.
- Wire T has radius a and is centered at $x = 10a$. Wire T has uniform current density and carries total current I_0 in the –*y*-direction.

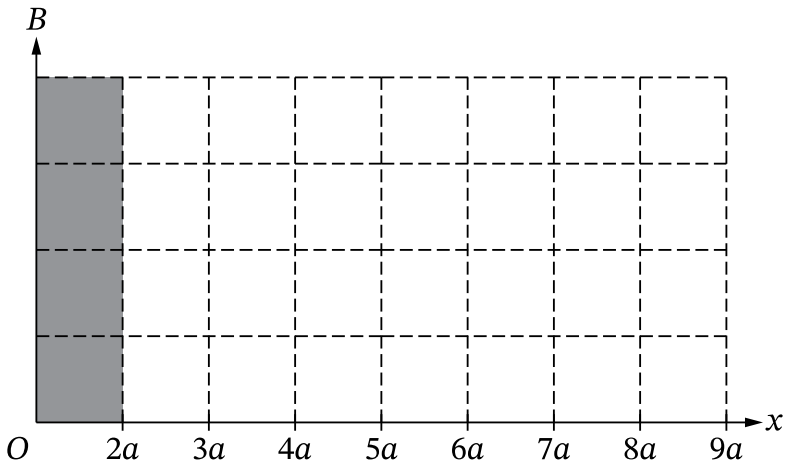
Figure 1



Part A

ii. On the axes shown in Figure 2, **sketch** a graph of the magnitude B of the magnetic field due to the currents in both wires as a function of x along the x-axis in the region $2a \leq x \leq 9a$. Sketches made in the shaded region will not be graded.

Figure 2



解：

A(ii) 两导线之间的场大小图区间

$$2a \le x \le 9a$$

位于两导线外部；每根线都被相应安培环路完全包围。S 在右侧的场向

$$-z,$$

T 在左侧的场也向

$$-z,$$

所以

$$B(x) = \frac{\mu_0 I_0}{2\pi}\left(\frac{1}{x} + \frac{1}{10a - x}\right).$$

令

$$u = x/a,$$

纵轴以

$$\mu_0 I_0/(2\pi a)$$

为单位，形状就是

$$f(u) = 1/u + 1/(10-u)。$$

$$f'(u) = -1/u^2 + 1/(10-u)^2,$$

在

$$u = 5$$

为零；左侧递减、右侧递增，故最低点在

$$x = 5a。$$

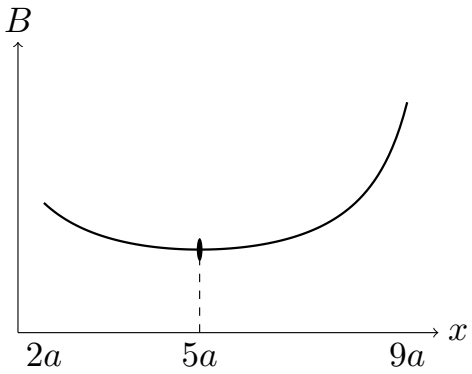
$$f''(u) = 2/u^3 + 2/(10-u)^3 > 0,$$

曲线始终向上弯曲，斜率随x增大而增大。端点

$$f(2) = 0.625,$$

$$f(9) = 1.111\ldots,$$

故右端更高。



带电粒子磁力：叠加场后再乘qv

磁力大小|q|vBsinα，α是速度与净磁场夹角；本题速度在平面内、磁场垂直平面，因此sinα=1。

同样q、v的两球只需比较各处净场大小。改变一根导线电流后，须重做两项场的方向与叠加，不能只说电流增大则力增大。

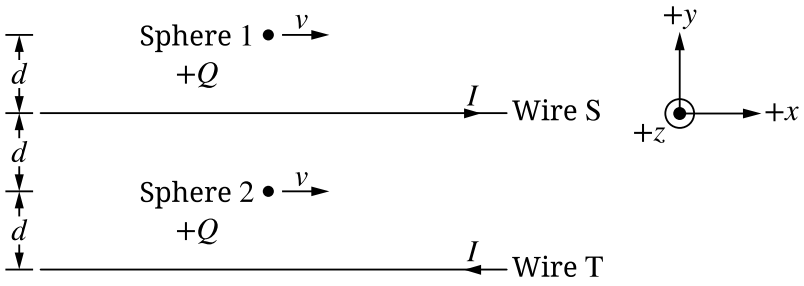
$$F = |q|vB\sin\alpha.$$

例题 2025 Q4 (a)

Question 4

4. Long, parallel wires S and T are a distance $2d$ apart. Both wires carry equal currents I, but the currents are in opposite directions. Both wires are parallel to the x-axis. At the instant shown in Figure 1, Sphere 1 is a distance d above Wire S, Sphere 2 is a distance d below Wire S, and both spheres are moving with speed v in the $+x$-direction. Each sphere has positive charge $+Q$. Gravitational effects are negligible.

Figure 1



A. F_1 is the magnitude of the magnetic force exerted on Sphere 1 due to the currents in wires S and T. F_2 is the magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Indicate whether F_2 is greater than, less than, or equal to F_1 by writing one of the following.

- $F_2 > F_1$
- $F_2 < F_1$
- $F_2 = F_1$

Justify your answer.

解：

A 比较磁力长直导线的场沿绕线的圆周，方向先用右手定则，大小随距离按

$$1/r$$

减小。记

$$b \ = \mu_0 I/(2\pi d)。$$

在Sphere 1：S 的场为

$$+bz,$$

$$\hat{}\mathrm{T}$$

距离是

$$3d,$$

场为

$$-(b/3)z\,,$$

$$\hat{}$$

相互抵消一部分，净场大小为

$$2b/3。$$

在Sphere 2：两条导线的距离都是d，S 的场为

$$-bz,$$

$$\hat{}\mathrm{T}$$

的场也为

$$-bz,$$

$$\hat{}$$

净场大小为

$$2b。$$

两球速度都与场垂直，

$$F = QvB;$$

所以

$$\boxed{F_2 > F_1}$$ （具体为 $F_2 = 3F_1$）.

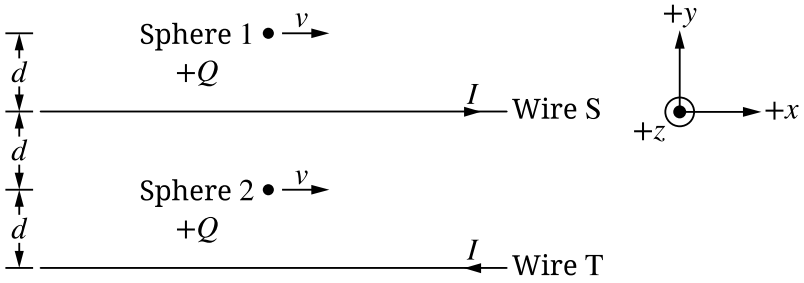
练习 2025 Q4 (c)

本问承接 2025 Q4 (b)。

Question 4

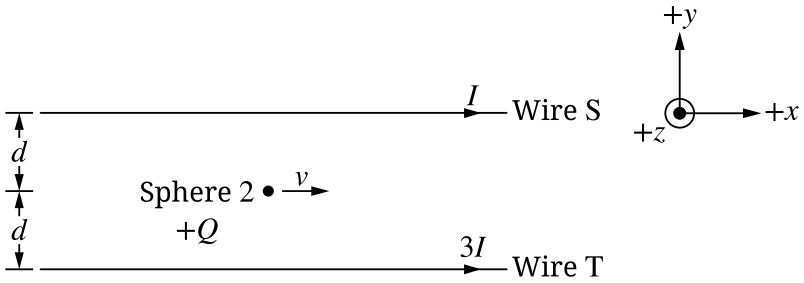
4. Long, parallel wires S and T are a distance $2d$ apart. Both wires carry equal currents I, but the currents are in opposite directions. Both wires are parallel to the x-axis. At the instant shown in Figure 1, Sphere 1 is a distance d above Wire S, Sphere 2 is a distance d below Wire S, and both spheres are moving with speed v in the $+x$-direction. Each sphere has positive charge $+Q$. Gravitational effects are negligible.

Figure 1



C. Later, Wire T carries current $3I$ in the $+x$-direction. At the instant shown in Figure 2, Sphere 2 is a distance d below Wire S and is moving with speed v in the $+x$-direction. F_{new} is the new magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Figure 2



Indicate whether F_{new} is greater than, less than, or equal to F_2 by writing one of the following.

- $F_{\text{new}} > F_2$
- $F_{\text{new}} < F_2$
- $F_{\text{new}} = F_2$

Briefly **justify** your answer by referencing your derivation in part B.

练习 2026 Q1 (b)

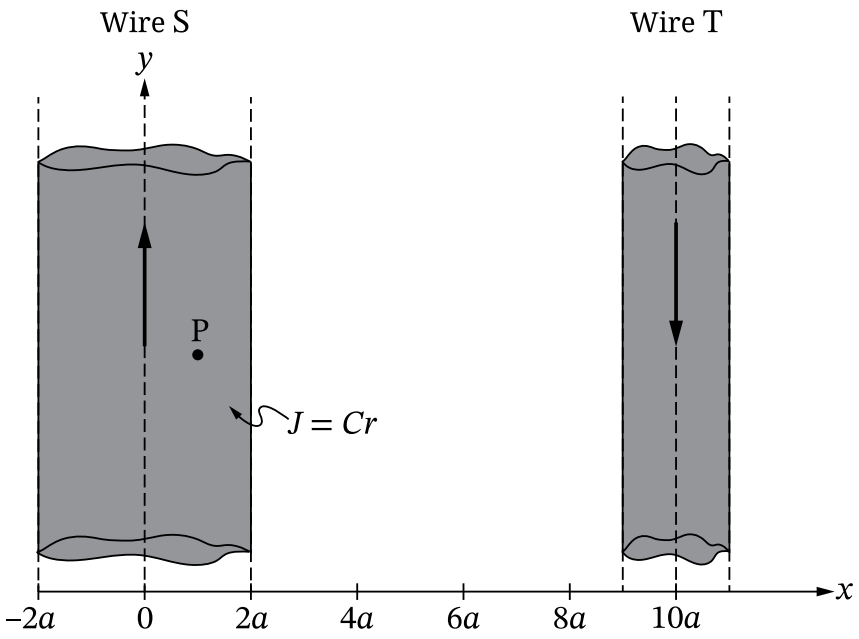
本问承接 2026 Q1 (a)(ii)。

Question 1: Version J

1. Two very long, cylindrical, current-carrying Wires S and T are oriented parallel to the *y*-axis, as shown in Figure 1. The centers of the wires are in the *xy*-plane. The wires are described as follows.

- Wire S has radius $2a$ and is centered at $x = 0$. Wire S has nonuniform current density and carries total current I_0 in the +*y*-direction. The magnitude of the current density in Wire S as a function of the radial distance *r* from the center of the wire is described by $J = Cr$, where *C* is a positive constant.
- Wire T has radius *a* and is centered at $x = 10a$. Wire T has uniform current density and carries total current I_0 in the –*y*-direction.

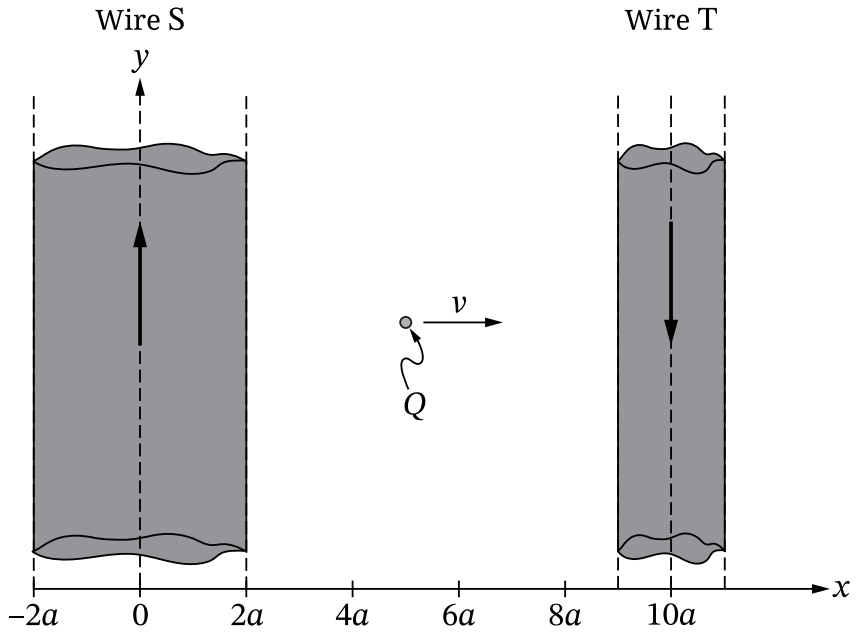
Figure 1



Part B

At the instant shown in Figure 3, a small sphere with charge Q is at horizontal position $x = 5a$ and moving in the xy-plane with speed v in the $+x$-direction. At this instant, the net magnetic force exerted on the sphere due to the currents in Wires S and T has magnitude F.

Figure 3



Derive an expression for F in terms of a, Q, I_0, v, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

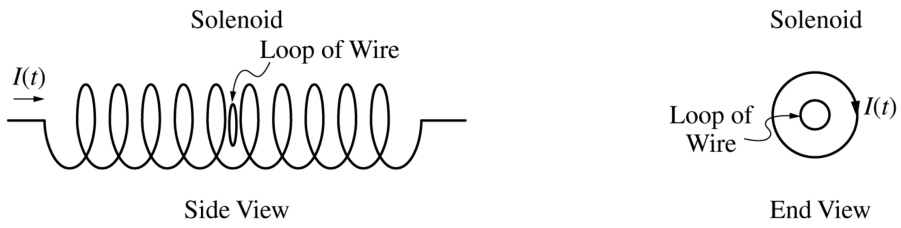
磁通变化与运动导体

楞次定律判电流：反对的是磁通变化

先说外磁通是入纸面还是出纸面，并判断它在所给时段增加还是减少。感应磁场反对这项变化：原磁通减弱时维持原方向，增强时产生反方向。

反求线圈电流时，让右手拇指朝所需的线圈内磁场，四指卷曲的方向就是电流方向。等价地，四指沿线圈电流卷曲，拇指指线圈内磁场；这与长直导线的‘拇指沿电流’用法不同。即使外场尚未换向，它的变化趋势也可能已经改变；不要用外场方向直接断定电流方向。

例题 2022 Set 2 Q3 (a)



Note: Figures not drawn to scale.

3. A single loop of wire with resistance $3.0\,\Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0\,\text{A/s}$. The direction of the current in the solenoid is clockwise, as shown in the end view.

(a) At time $t = 2.0\,\text{s}$, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

___ Clockwise ___ Counterclockwise ___ Zero

Justify your answer.

解：

$$\text{(a) } t = 2.0\text{ s}$$

的感应电流方向右手定则给顺时针的螺线管电流产生入纸面的场；

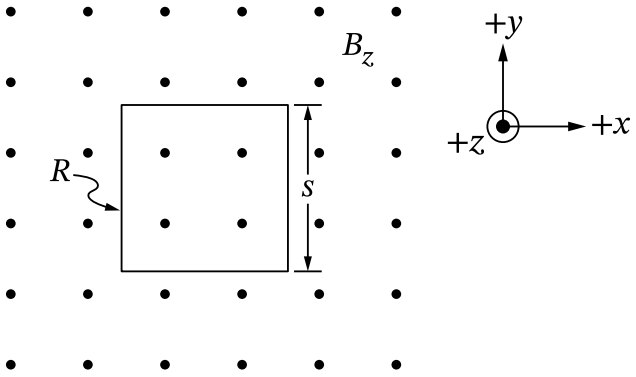
$$I_s = bt$$

随时间增大，入纸面的磁通也在增大。感应回路要抵抗这个增量，因此它的磁场应出纸面。对应的回路电流为Counterclockwise（逆时针）。这里抵抗的是磁通变化，而不是一定与原磁场同向或一定与管中电流反向。

练习 2026 Q4 (a)

Question 4

4. In Scenario 1, a square conducting loop with side length s has total resistance R. The loop is held fixed in the xy-plane in an external, uniform, time-varying magnetic field that is initially directed in the $+z$-direction, as shown. The z-component of the magnetic field changes as a function of time t as described by $B_z = B_0 \cos\left(2\pi \frac{t}{t_1}\right)$, where B_0 and t_1 are positive constants.



Part A

Indicate whether the induced current I in the loop is clockwise, counterclockwise, or zero during the time interval $0 < t < \frac{t_1}{4}$ by writing one of the following.

- I is clockwise.
- I is counterclockwise.
- I is zero.

Justify your answer without only manipulating equations.

回路入场时B没有改变，改变的是落在磁场区里的回路面积。外磁通Φ表示穿过回路的带符号磁场总量；均匀垂直场的一块重叠面积A贡献BA。本题取出纸面为正，场入纸面，所以该项为−BA。

矩形高2L，前缘由x=L起进入：此时覆盖宽度为x−L，面积为2L(x−L)。x随时间增大，使入纸面通量的绝对值增大。随后才能用楞次定律确定维持哪一种感应场、再用右手判断电流方向。

$$\Phi = -B\,[2L(x-L)] \quad (L < x < 2L).$$

例题 2024 Set 1 Q3 (b)(i)

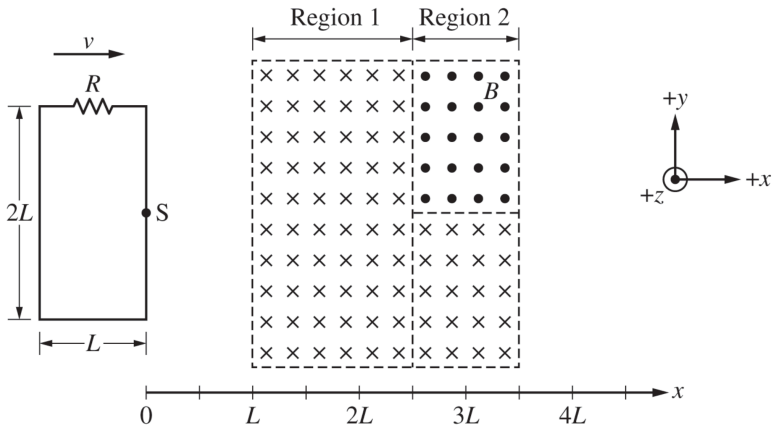


Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, that are parallel but are directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

(b) Consider the instant when Point S reaches $x = 1.5L$.

i. **Indicate** whether the current I_R that is induced in the rectangular loop when Point S reaches $x = 1.5L$ is clockwise, counterclockwise, or zero.

_____ Clockwise　　_____ Counterclockwise　　_____ Zero

Briefly **justify** your answer.

解：

(b)(i)　在

$$x=1.5L$$

的电流方向这时环正在进入指向纸内的场，向纸内的磁通量绝对值增加。楞次定律要求感应电流产生向纸外的磁场来抵抗这项增加。右手定则给出

逆时针.

感应场抵抗的是通量的改变，不是要求在所有阶段都与外场相反。

这次方环边长D，场沿+z。仍先表示覆盖面积：前缘从x=D开始进入，面积为D(x−D)，而外场强度B保持不变。用已示范的面积变化判断外磁通的变化，再确定感应电流方向。

练习 2024 Set 2 Q3 (b)(i)

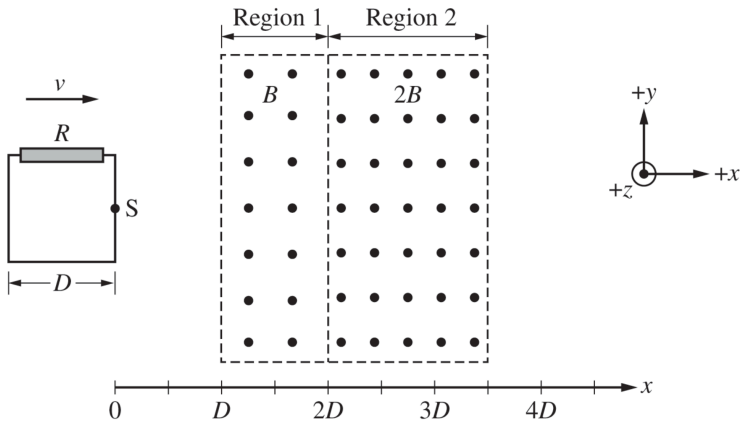


Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $+z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

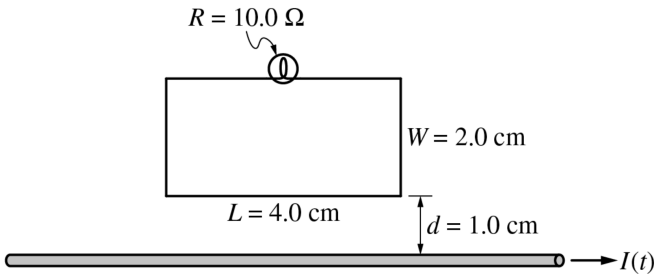
(b) Consider the instant when Point S reaches $x = 1.5D$.

i. **Indicate** whether the current I_S that is induced in the square loop when Point S reaches $x = 1.5D$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

例题 2022 Set 1 Q3 (a)



3. A lightbulb of resistance $R = 10.0\ \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0\ \text{cm}$ and a width $W = 2.0\ \text{cm}$ and is positioned so one of the longer sides of the loop is a distance $d = 1.0\ \text{cm}$ above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0\,\text{A}$ and $D = 2.0\ \text{A/s}$. At time $t = 5.0\ \text{s}$, the current in the long wire is instantaneously zero as the current changes direction.

(a) What is the direction, if any, of the magnetic field produced by the induced current in the rectangular loop as the current in the long wire changes direction?

___ Into the page ___ Out of the page ___ No direction, because the field is zero

Justify your answer.

解：

(a) 导线电流换向时，感应磁场的方向向右的导线电流在回路位置产生出纸面的场。电流以恒定斜率

$$dI_w/dt = -D$$

减少：先是出纸面磁通减小，过

$$t = 5.0\ \text{s}$$

后变成入纸面磁通增加。这两个表述都意味着有符号磁通一直在减小。楞次定律要求回路产生出纸面磁场来抵抗这一变化，故选Out of the page（出纸面），对应感应电流逆时针。

在换向瞬间

$$I_w = 0,$$

长导线此刻的磁场为零，但磁通变化率不为零；感应磁场因此不是零。感应由变化率决定，而不是由瞬时磁通值决定。

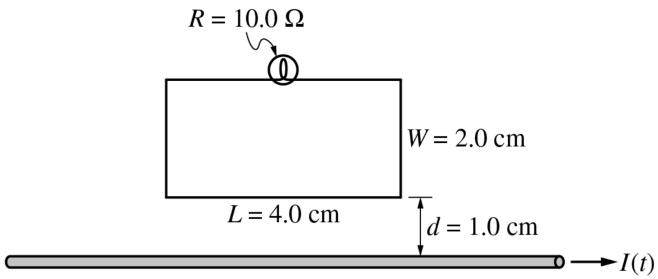
均匀场或非均匀场：算一个时刻的磁通

均匀场垂直回路时，磁通是BA；倾斜时是BAcosθ，θ是场与回路法线的夹角。

长直导线的场随距线r变化，矩形回路各条窄片磁通不同。取沿导线方向的片长ℓ、径向宽dr，dΦ=B(r)ℓdr，从近边到远边积分。

$$\Phi = BA\cos\theta, \qquad \Phi = \int B(r)\,\ell\,dr.$$

例题 2022 Set 1 Q3 (b)



3. A lightbulb of resistance $R = 10.0\ \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0\ \text{cm}$ and a width $W = 2.0\ \text{cm}$ and is positioned so one of the longer sides of the loop is a distance $d = 1.0\ \text{cm}$ above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0\ \text{A}$ and $D = 2.0\ \text{A/s}$. At time $t = 5.0\ \text{s}$, the current in the long wire is instantaneously zero as the current changes direction.

(b) Calculate the magnetic flux through the loop due to only the long wire at time $t = 3.0\ \text{s}$.

解：

$$\text{(b) } t = 3.0\ \text{s}$$

的磁通此时导线电流为

$$I_w = 10.0 - 2.0(3.0) = 4.0\ \text{A}。$$

长导线磁场随距离变化，不能把近边磁场乘整个面积。距离导线r、宽度为

$$dr$$

的水平窄条面积是

$$dA = L\,dr,$$

该条上磁场大小相同：

$$B_w(r) = \frac{\mu_0 I_w}{2\pi r}, \qquad \Phi_B = \int_d^{d+W} B_w L\,dr = \frac{\mu_0 I_w L}{2\pi}\ln\frac{d+W}{d}.$$

把厘米换为米后，积分上下限为0.010 与0.030 m：

$$\Phi_B = (2.0\times10^{-7})(4.0)(0.040)\ln 3 = \boxed{3.52\times10^{-8}\ \text{Wb}}.$$

按出纸面为正的约定，磁通为正。

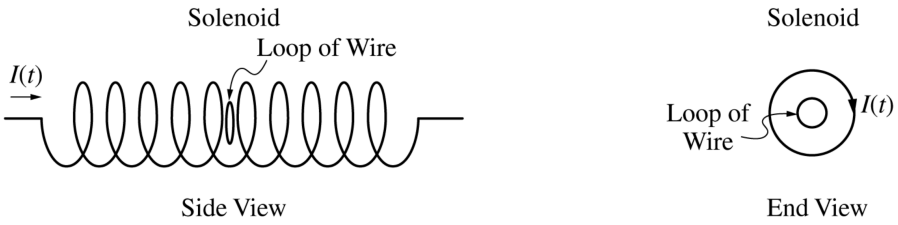
固定回路、场随时间变：先求磁通变化率

法拉第定律求的是磁通每秒改变多少，不是磁通本身。固定几何时，面积或几何积分系数不随时间变，可提出导数外。

均匀场用Φ=BA；长线场用已经求出的空间积分；螺线管内B=μ_0nI，其中n为每单位长度的匝数。最后用闭合回路总电阻把电动势变为电流。

$$|\mathcal{E}| = \left|\frac{d\Phi}{dt}\right|, \qquad I = \frac{|\mathcal{E}|}{R_{\text{total}}}.$$

例题 2022 Set 2 Q3 (b)



Note: Figures not drawn to scale.

3. A single loop of wire with resistance $3.0\,\Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. The direction of the current in the solenoid is clockwise, as shown in the end view.

(b) Calculate the current in the loop of wire at time $t = 2.0$ s.

解：

(b) 求感应电流回路法向平行于内部场且场近似均匀，故磁通大小为

$$\Phi = B_s A,$$

其中

$$A = \pi a^2$$

是单匝回路的面积，不是螺线管外表面积。法拉第定律与欧姆定律给

$$|\mathcal{E}| = A\left|\frac{dB_s}{dt}\right| = \mu_0 \frac{N}{\ell} A \frac{dI_s}{dt}, \qquad I_1 = \frac{|\mathcal{E}|}{R_1}.$$

由于

$$dI_s/dt = b$$

恒定，感应电流也恒定，不需要把

$$t = 2$$

时的

$$I_s = 10\ \text{A}$$

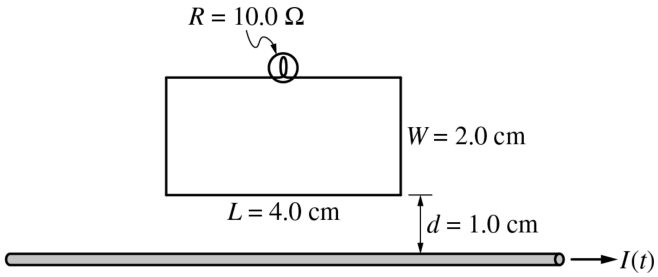
直接代入“感应电流公式”。代入数值：

$$I_1 = \frac{(4\pi \times 10^{-7})(500/0.25)[\pi(0.10)^2](5.0)}{3.0} = \boxed{1.32 \times 10^{-4}\ \text{A} \approx 1.3 \times 10^{-4}\ \text{A}}.$$

方向由(a)给出，为逆时针。

例题 2022 Set 1 Q3 (c)

本问承接 2022 Set 1 Q3 (b)。



3. A lightbulb of resistance $R = 10.0\ \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0\ \text{cm}$ and a width $W = 2.0\ \text{cm}$ and is positioned so one of the longer sides of the loop is a distance $d = 1.0\ \text{cm}$ above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0\,\text{A}$ and $D = 2.0\ \text{A/s}$. At time $t = 5.0\ \text{s}$, the current in the long wire is instantaneously zero as the current changes direction.

(c) Calculate the current through the lightbulb at time $t = 3.0\,\text{s}$.

解：

(c) 灯泡电流几何位置不动，时间变化仅来自导线电流。因此先保留

$$I_w(t)$$

再求导：

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 L}{2\pi}\ln\frac{d+W}{d}\frac{dI_w}{dt} = \frac{\mu_0 LD}{2\pi}\ln 3.$$

正的环路方向是与出纸面法向配套的逆时针，因而符号与(a)相符。欧姆定律给

$$I_1 = \frac{\mathcal{E}}{R} = \frac{(2.0\times10^{-7})(0.040)(2.0)}{10.0}\ln 3 = \boxed{1.76\times10^{-9}\,\text{A} \approx 1.8\times10^{-9}\,\text{A}}.$$

由于

$$dI_w/dt$$

恒定，此理想模型中感应电流在换向前后都保持同一大小和方向。

例题 2025 Q2 (b)

Question 2: Version J

2. A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA\cos(\omega t)$.

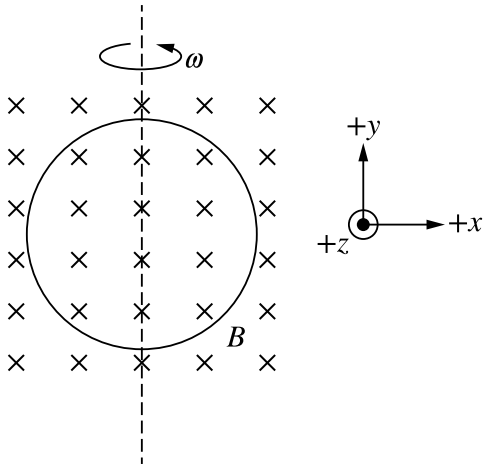


Figure 1

B. **Derive** an expression for the maximum induced current in the loop in terms of A, R, B, ω, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

B 最大感应电流先用法拉第定律求电动势，再用欧姆定律把电动势变为电流：

$$\mathcal{E} = -\frac{d}{dt}[BA\cos(\omega t)] = BA\omega\sin(\omega t),$$

$$I(t) = \frac{\mathcal{E}}{R} = \frac{BA\omega}{R}\sin(\omega t).$$

正弦绝对值最大为1，因此

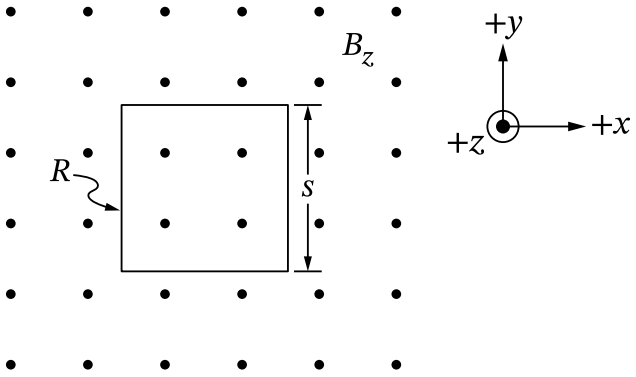
$$\boxed{I_{\max} = \frac{BA\omega}{R}}.$$

这里的最大值是电流的最大绝对值，不是某个指定方向的有符号电流。

练习 2026 Q4 (b)

Question 4

4. In Scenario 1, a square conducting loop with side length s has total resistance R. The loop is held fixed in the xy-plane in an external, uniform, time-varying magnetic field that is initially directed in the $+z$-direction, as shown. The z-component of the magnetic field changes as a function of time t as described by $B_z = B_0 \cos\left(2\pi \frac{t}{t_1}\right)$, where B_0 and t_1 are positive constants.



Part B

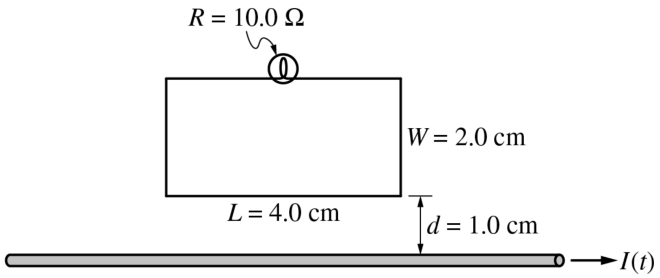
Derive an expression for I as a function of t in terms of s, R, B_0, t_1, t, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

测到感应电流不同：检查变化率与投影

电流公式包含磁通变化率和总电阻。电流比计算大时，可来自实际外场变化率更大；电流偏小时，可来自回路法线未平行场，实际投影面积较小。

判断实验原因须把题给选项代回对应因子。位置、场方向或变化速度不是同一个量，不能泛泛说“误差”。

例题 2022 Set 1 Q3 (d)



3. A lightbulb of resistance $R = 10.0\ \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0\ \text{cm}$ and a width $W = 2.0\ \text{cm}$ and is positioned so one of the longer sides of the loop is a distance $d = 1.0\ \text{cm}$ above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0\,\text{A}$ and $D = 2.0\ \text{A/s}$. At time $t = 5.0\ \text{s}$, the current in the long wire is instantaneously zero as the current changes direction.

(d) A group of students attempts to experimentally verify whether the current through the lightbulb is consistent with the current calculation from part (c). The current in the rectangular loop is measured to be greater than the current calculated in part (c). Which of the following could explain this discrepancy? Select one answer.

___ The students did not account for Earth's magnetic field.
___ The rectangular loop is tilted and is not in the same plane as the wire.
___ The resistance of the lightbulb is greater than the recorded value.
___ The long side of the rectangular loop is shorter than the recorded value.
___ The current in the long wire changes at a faster rate than expected.

Briefly justify your answer.

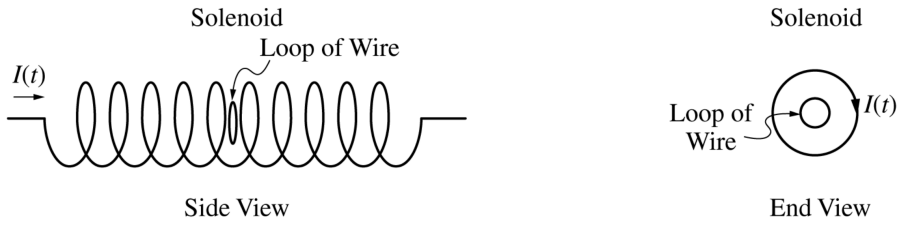
解：

(d) 实测电流大于计算值选择导线电流变化得比预期更快。公式显示I_1 正比于

$$|dI_w/dt|;$$

该变化率增大，磁通变化率、电动势和感应电流均增大。稳定的地磁场不增加磁通变化率；回路倾斜、灯泡电阻增大或沿导线的边长缩短，都不能解释本题的电流偏大。

练习 2022 Set 2 Q3 (d)



Note: Figures not drawn to scale.

3. A single loop of wire with resistance $3.0\,\Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0\,\text{A/s}$. The direction of the current in the solenoid is clockwise, as shown in the end view.

(d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0\,\text{s}$ is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

___ The experiment did not account for Earth's magnetic field.
___ The plane of the loop is not perpendicular to the axis of the solenoid.
___ The center of the loop is not on the axis of the solenoid.
___ The resistance of the loop is less than the given value.
___ The radius of the loop is actually larger than $0.10\,\text{m}$.

Justify your answer.

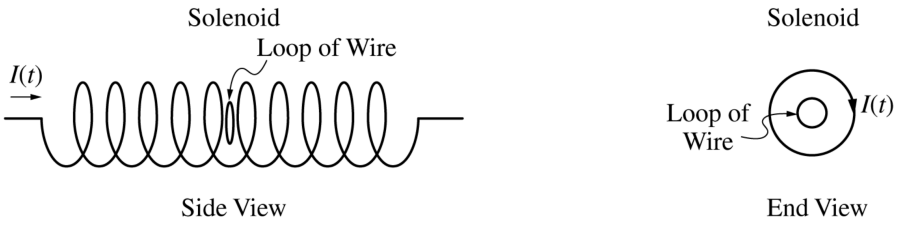
面积与回路电阻同时变化：逐因子比较

同材质、同粗细导线的电阻正比于导线长度。回路尺寸加倍时，面积变成四倍，周长及电阻通常变成两倍；感应电流按面积除以电阻比较。

矩形换朝向时沿导线方向的长度和跨越非均匀场的范围都会改变，须回到磁通积分。题目直接指定新电阻时，以题给值为准。

$$I_{\max} \propto \frac{A}{R} \quad \text{（同一均匀变化场）}.$$

例题 2022 Set 2 Q3 (e)



Note: Figures not drawn to scale.

3. A single loop of wire with resistance $3.0\,\Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0\,\text{A/s}$. The direction of the current in the solenoid is clockwise, as shown in the end view.

(e) The power supply is now turned off. The original loop of wire is then replaced with a second loop made from wire that has the same thickness and is made from the same material as the original loop of wire. The second loop has radius 0.20 m, is placed in the same orientation as the original loop, and fits completely inside the solenoid. The power supply is turned on, and the current I in the solenoid as a function of t is again given by the equation $I(t) = \beta t$, where $\beta = 5.0\,\text{A/s}$. Which of the following expressions correctly indicates the ratio $\frac{I_2}{I_1}$ where I_1 represents the current induced in the original loop of wire in part (b) and I_2 represents the current induced in the second loop of wire?

$___\frac{I_2}{I_1} = 1$ $\quad$ $___1 < \frac{I_2}{I_1} < 2$ $\quad$ $___\frac{I_2}{I_1} = 2$ $\quad$ $___\frac{I_2}{I_1} > 2$

Justify your answer.

解：

(e) 回路半径加倍第二个回路半径为

$$a_2 = 0.20\text{ m} = 2a_1,$$

仍完全在管内且朝向相同。面积增加为四倍，所以同一管中电流变化率对应的感应电动势满足

$$\mathcal{E}_2 = 4\mathcal{E}_1。$$

但题目还给了相同材料和相同线粗，不能把回路电阻当作不变。导线电阻为

$$R = \rho \ell_w / A_w;$$

材料电阻率ρ和导线截面积A_w不变，圆周长

$$\ell_w = 2\pi a$$

随半径加倍，故

$$R_2 = 2R_1。$$

于是

$$\boxed{\frac{I_2}{I_1} = \frac{\mathcal{E}_2/R_2}{\mathcal{E}_1/R_1} = \frac{4}{2} = 2.}$$

电流变为原来的两倍；只考虑面积会错得四倍，只考虑电阻会错得减半，必须把两个变化同时放进

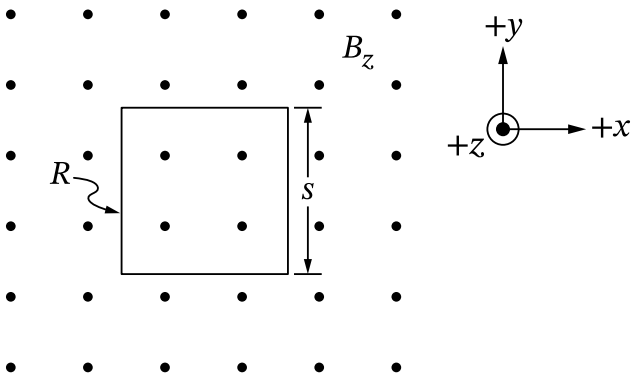
$$I = \mathcal{E}/R。$$

练习 2026 Q4 (c)

本问承接 2026 Q4 (b)。

Question 4

4. In Scenario 1, a square conducting loop with side length s has total resistance R. The loop is held fixed in the xy-plane in an external, uniform, time-varying magnetic field that is initially directed in the +z-direction, as shown. The z-component of the magnetic field changes as a function of time t as described by $B_z = B_0 \cos\left(2\pi \frac{t}{t_1}\right)$, where B_0 and t_1 are positive constants.



Part C

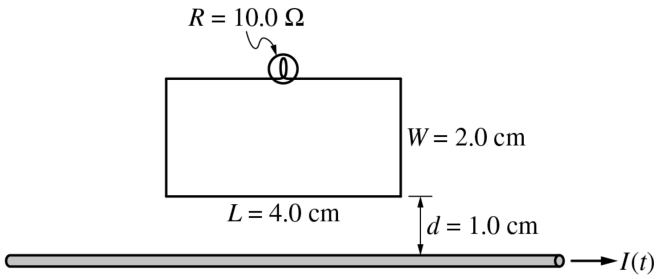
In Scenario 2, a new square conducting loop with side length $2s$ and total resistance $2R$ is placed in the same external, uniform, time-varying magnetic field as in Scenario 1.

Indicate whether the maximum induced current I_2 in Scenario 2 is greater than, less than, or equal to the maximum induced current I_1 in Scenario 1 by writing one of the following.

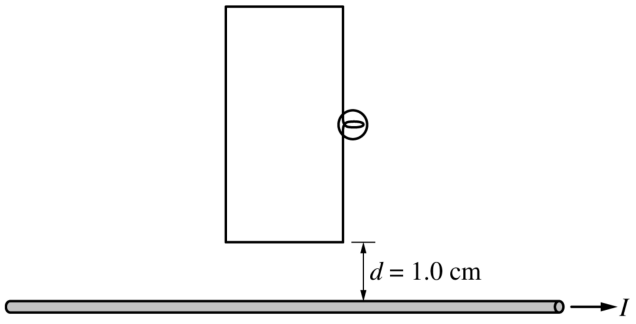
- $I_2 > I_1$
- $I_2 < I_1$
- $I_2 = I_1$

Briefly **justify** your answer by referencing the expression you derived in part B.

例题 2022 Set 1 Q3 (e)



3. A lightbulb of resistance $R = 10.0\ \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0\ \text{cm}$ and a width $W = 2.0\ \text{cm}$ and is positioned so one of the longer sides of the loop is a distance $d = 1.0\ \text{cm}$ above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0\ \text{A}$ and $D = 2.0\ \text{A/s}$. At time $t = 5.0\ \text{s}$, the current in the long wire is instantaneously zero as the current changes direction.



(e) Later, the same rectangular loop with lightbulb is rotated such that a short side of the loop is 1.0 cm above and parallel to the long current-carrying wire, as shown. The current in the wire is again initially flowing from left to right and given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. The current through the lightbulb in the loop's new orientation at time $t = 3.0$ s is I_2. Which of the following correctly relates the current I_2 to I_1, the current through the lightbulb in part (c)?

___$I_2 < I_1$ ___$I_2 = I_1$ ___ $I_2 > I_1$

Justify your answer.

解：

(e) 旋转矩形后比较I_2与I_1旋转后短边W平行于导线，近边仍距导线d，而离导线的径向跨度变成L。积分条带面积改为

$$W\,dr、$$

积分上限改为

$$d + L:$$

$$I_2 = \frac{\mu_0 DW}{2\pi R}\ln\frac{d+L}{d}, \qquad \frac{I_2}{I_1} = \frac{W\ln[(d+L)/d]}{L\ln[(d+W)/d]} = \frac{0.020\ln 5}{0.040\ln 3} \approx 0.732.$$

所以$\boxed{I_2 < I_1}$。矩形面积虽然不变，旋转后较多面积移到了离导线较远、磁场较弱的位置；单位导线电流对应的磁通减少，同一变化率产生的电动势也更小。必须比较积分系数，不能仅凭面积相同判断电流相同。

两场区中的回路：重叠面积给磁通图

场在每个区域均匀，但场的方向或大小可不同。把回路在各区的重叠面积分别记A_1、A_2，带符号磁通为B_1A_1+B_2A_2。

方形或矩形匀速移动时，边界扫入面积常随位置线性变化；在每个边刚越过边界的点分段计算。相反方向要相减，同方向而大小不同仍是相加。

$$\Phi = B_1A_1 + B_2A_2.$$

例题 2024 Set 1 Q3 (a)

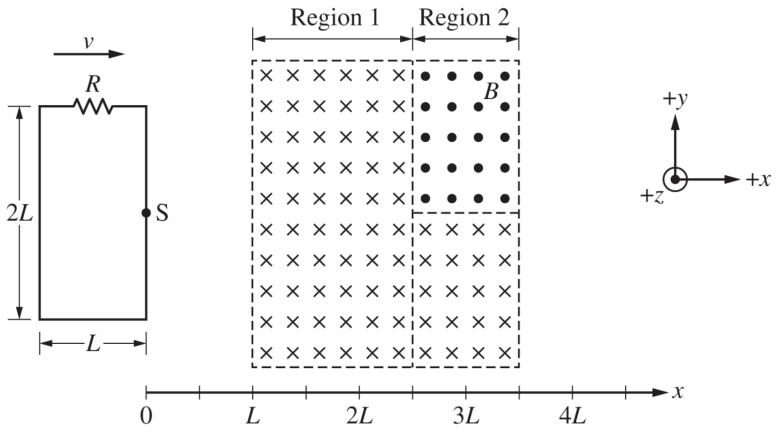
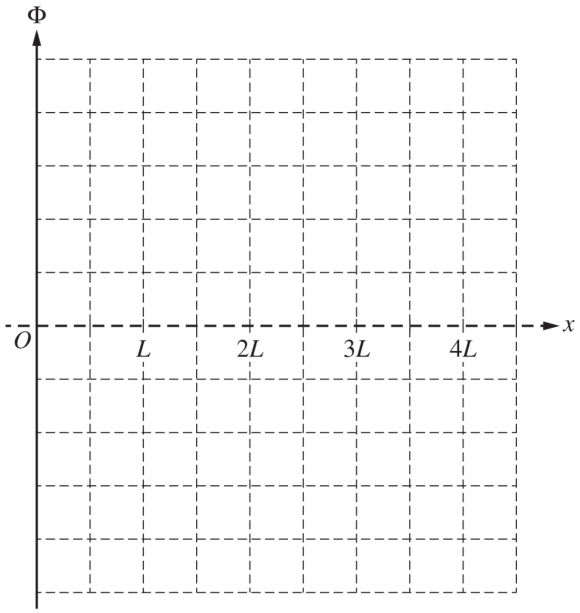


Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, that are parallel but are directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the rectangular loop as a function of the position x of Point S from $x = 0$ to $x = 4.5L$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.



解:

(a) 磁通量随S 位置的图均匀场穿过面积A的通量为

$$B_z A。$$

Region 2 中，环在分界线上下覆盖的面积相等，两个方向的通量在每个位置都抵消。因此只需计算环覆盖Region 1 的宽度，再乘高

$$2L$$

与带符号的

$$-B。$$

.

$$0 \le x \le L:$$

环尚未进入磁场,

$$\Phi = 0。$$

.

$$L < x < 2L:$$

覆盖宽度为

$$x - L,$$

负通量的绝对值线性增加。·

$$2L \le x \le 2.5L:$$

整个环都在Region 1，通量保持

$$-2BL^2。$$

.

$$2.5L < x < 3.5L:$$

留在Region 1 的宽度为

$$3.5L - x,$$

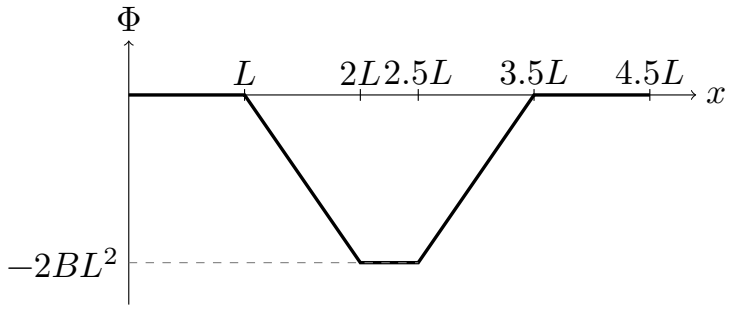
负通量的绝对值线性减少。·

$$3.5L \le x \le 4.5L:$$

Region 1 已没有环的面积，Region 2 正负抵消，通量为零。

因此

$$\Phi(x) = \begin{cases} 0, & 0 \le x \le L, \\ -2BL(x-L), & L < x < 2L, \\ -2BL^2, & 2L \le x \le 2.5L, \\ -2BL(3.5L-x), & 2.5L < x < 3.5L, \\ 0, & 3.5L \le x \le 4.5L. \end{cases}$$



不能因环已进入Region 2 就把该区某一半的通量单独作为总通量；另一半面积同时存在。

例题 2024 Set 2 Q3 (a)

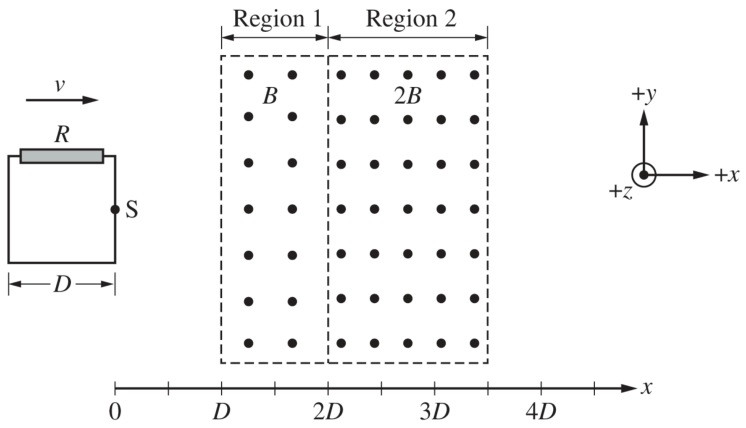
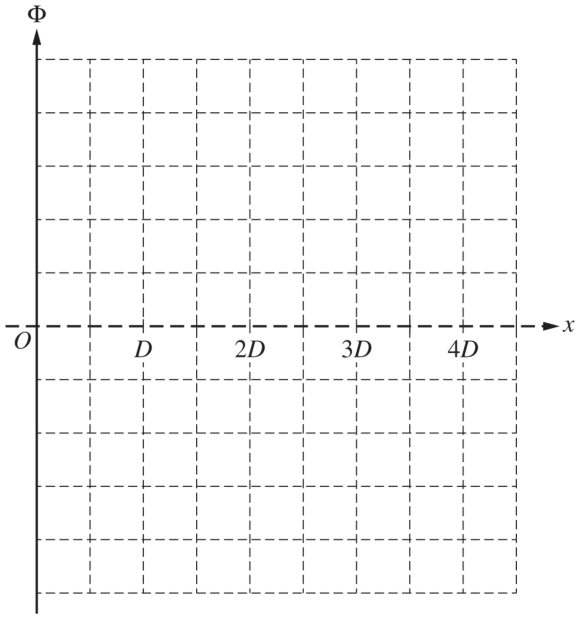


Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $+z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the square loop as a function of the position x of Point S from $x = 0$ to $x = 4.5D$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.



解：

(a) 磁通量图先分别计算在两区中的面积，再把正通量相加。

$$0 \le x \le D$$

时尚未进入，

$$\Phi = 0。$$

在

$$D < x < 2D,$$

进入第一区的宽度为

$$x - D、$$

高为D，所以

$$\Phi = BD(x - D)。$$

在

$$2D < x < 3D,$$

环仍有一部分在第一区，宽度为

$$3D - x;$$

另有一部分在第二区，宽度为

$$x - 2D。$$

于是

$$\Phi = BD(3D - x) + 2BD(x - 2D) = BD(x - D).$$

每向右移动一小段，第一区减少的通量与第二区新增的通量抵消一部分，净增量仍为

$$BD\,dx,$$

因此在

$$x = 2D$$

不改变斜率。

在

$$3D \le x \le 3.5D,$$

整环位于

$$2B$$

区，

$$\Phi = 2BD^2。$$

再向右，覆盖磁场的宽度为

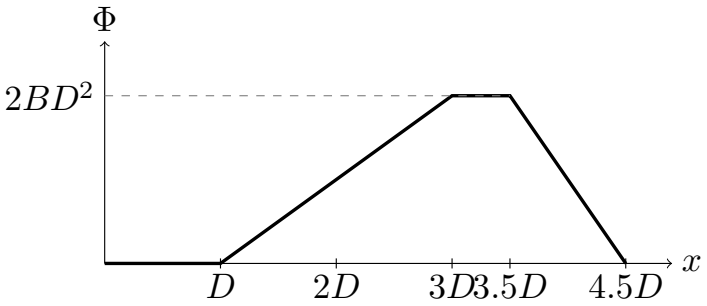
$$4.5D - x,$$

最后在

$$x = 4.5D$$

全部离开。

$$\Phi(x) = \begin{cases} 0, & 0 \le x \le D, \\ BD(x - D), & D < x < 3D, \\ 2BD^2, & 3D \le x \le 3.5D, \\ 2BD(4.5D - x), & 3.5D < x < 4.5D, \\ 0, & x = 4.5D. \end{cases}$$



退出段的场为

$$2B,$$

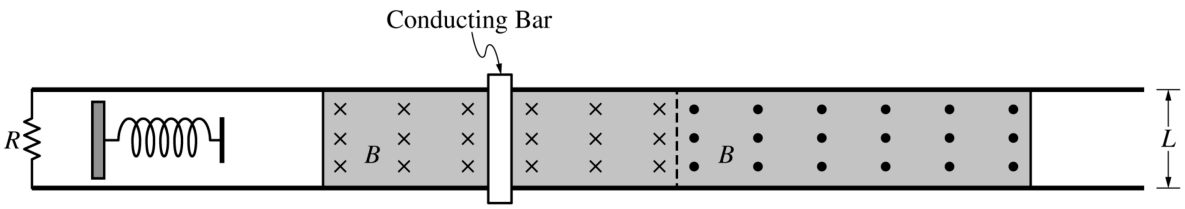
面积减少速度与最初进入相同，所以退出段斜率绝对值是最初进入段的两倍。

移动棒的磁阻力方向：阻碍运动

棒扫过区域使回路磁通改变，楞次定律决定感应电流；电流在棒中受磁力。

没有电池供能时，感应磁力使棒机械能转成电阻热，沿导轨的分量应阻碍运动。画合力箭头前仍要检查重力和支持力是否相抵，以及磁场是否垂直回路面。

例题 2023 Set 1 Q2 (a)



Top View (at time t_B)

2. Two horizontal, parallel, conducting rails are separated by distance $L = 0.40\,\text{m}$. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23\,\text{kg}$ is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
- At time t_1, the bar loses contact with the spring and slides to the right.
- At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50\,\text{T}$ that is directed into the page, as shown.
- At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50\,\text{T}$ but is directed out of the page.
- At time t_4, the bar enters a region with no magnetic field.

Consider time t_B such that $t_2 < t_B < t_3$.

(a) On the following diagram of the bar, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.



解：

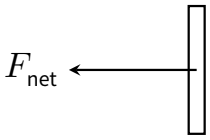
(a) t_B时的合力方向棒已脱离弹簧，水平力只来自磁制动。向右移动使回路内入纸面的磁通增加，感应电流产生出纸面的场来抵抗增量。入纸面场中，棒内的正电荷受

$$q\mathrm{v} \times \mathrm{B}$$

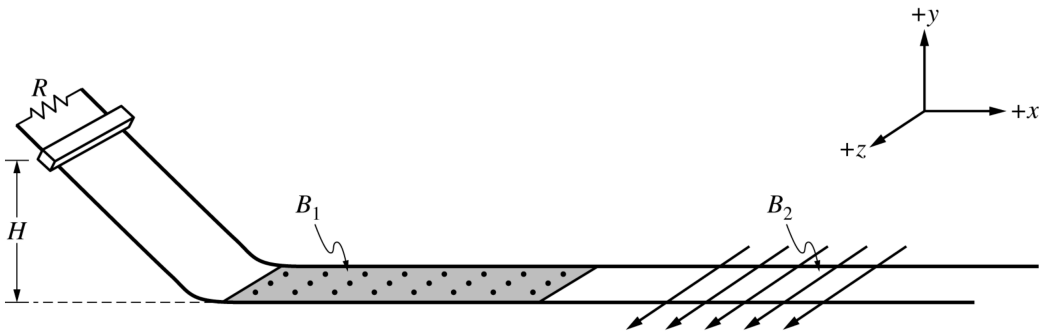
向上的作用，感应电流沿棒向上；于是载流棒的

$$I\ell \times \mathrm{B}$$

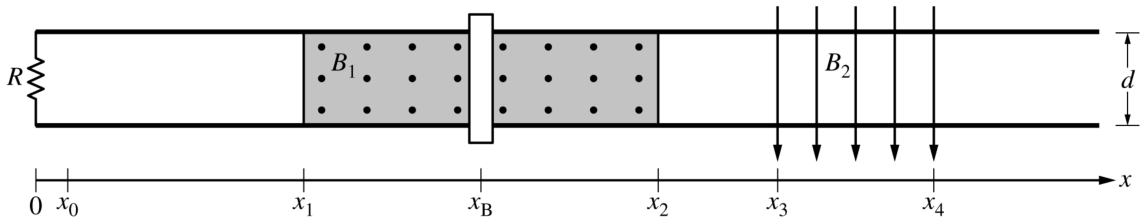
指向左。合力箭头应向左。



练习 2023 Set 2 Q2 (a)



Perspective View (at time $t = 0$)



Top View (at time t_B)

2. Two parallel conducting rails are separated by distance $d = 0.30\,\text{m}$. A resistor of resistance $R = 0.20\,\Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

   - At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
   - At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\,\text{T}$ that is directed in the $+y$-direction.
   - At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
   - At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\,\text{T}$ that is directed in the $+z$-direction.
   - At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

The bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

(a) On the following diagram of the bar, as observed from the Top View, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.



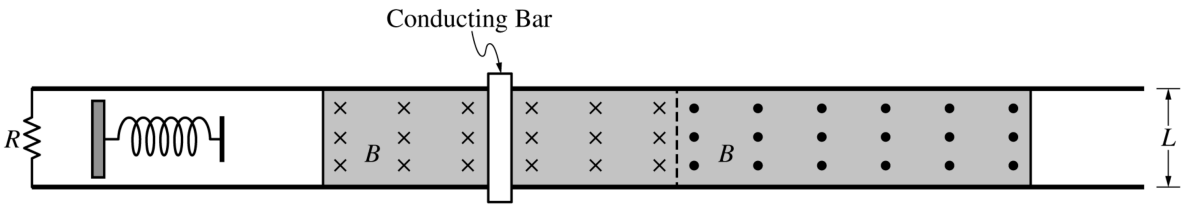
扫入面积速率：求运动感应电流

棒长ℓ以垂直棒的速度v运动时，每秒扫入面积ℓv；均匀垂直场使电动势大小Bℓv。矩形环进入场区时，ℓ是正在切过场边界的横向边长。

若场与回路法线不平行，只取穿过回路的场分量。用总电阻除电动势，不能在有棒内阻时只用外电阻。

$$|\mathcal{E}| = B_{\perp}\ell v, \qquad I = \frac{B_{\perp}\ell v}{R_{\text{total}}}.$$

例题 2023 Set 1 Q2 (b)(i)



Top View (at time t_B)

2. Two horizontal, parallel, conducting rails are separated by distance $L = 0.40\,\text{m}$. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23\,\text{kg}$ is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

   - At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
   - At time t_1, the bar loses contact with the spring and slides to the right.
   - At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50\,\text{T}$ that is directed into the page, as shown.
   - At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50\,\text{T}$ but is directed out of the page.
   - At time t_4, the bar enters a region with no magnetic field.

Consider time t_B such that $t_2 < t_B < t_3$.

(b) At time t_B, the speed of the bar is $v = 2.5\,\text{m/s}$.

i. Calculate the magnitude of the current in the bar at time t_B.

解：

(b)(i) 电流大小棒扫过面积的速率为

$$Lv,$$

因此

$$|\mathcal{E}| = BLv。$$

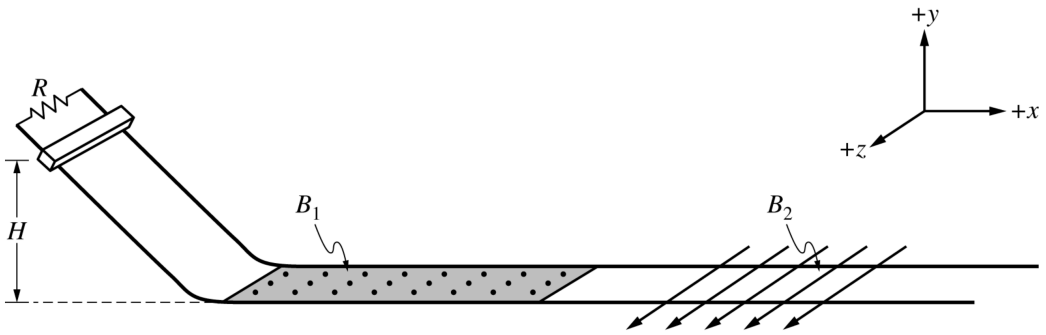
$$v = 2.5\ \mathrm{m/s}$$

时

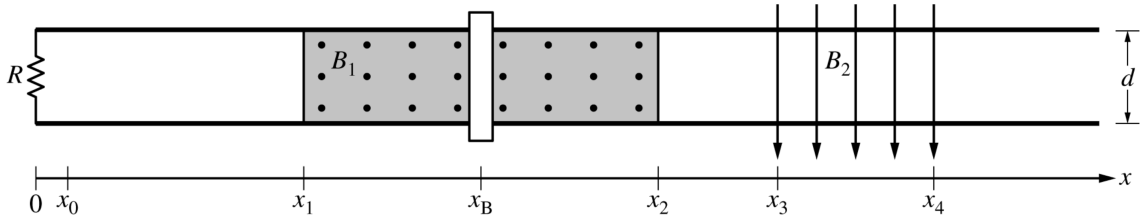
$$|\mathcal{E}| = (0.50)(0.40)(2.5) = 0.50\,\mathsf{V}, \qquad \boxed{I = \frac{BLv}{R} = \frac{0.50}{0.30} = 1.7\,\mathsf{A}}.$$

这里求大小；电流方向已由(a)确定，不用把负的有符号电动势直接当成负的电流大小。

练习 2023 Set 2 Q2 (b)(i)



Perspective View (at time $t = 0$)



Top View (at time t_B)

2. Two parallel conducting rails are separated by distance $d = 0.30\,\text{m}$. A resistor of resistance $R = 0.20\,\Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

   - At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
   - At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\,\text{T}$ that is directed in the $+y$-direction.
   - At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
   - At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\,\text{T}$ that is directed in the $+z$-direction.
   - At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

   The bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

(b) At time t_B, the speed of the bar is $v = 2.5\,\text{m/s}$.

i. Calculate the magnitude of the current in the bar at time t_B.

练习 2024 Set 1 Q3 (b)(ii)

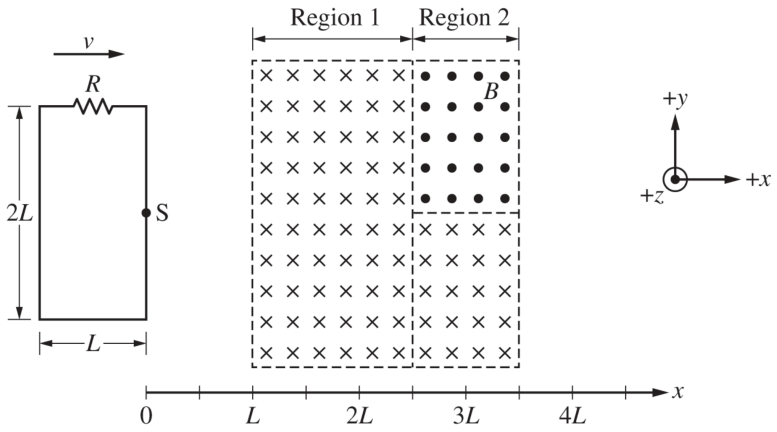


Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, that are parallel but are directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

(b) Consider the instant when Point S reaches $x = 1.5L$.

ii. **Derive** an expression for I_R when Point S reaches $x = 1.5L$. If $I_R = 0$, indicate how the derived expression shows that $I_R = 0$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

练习 2024 Set 2 Q3 (b)(ii)

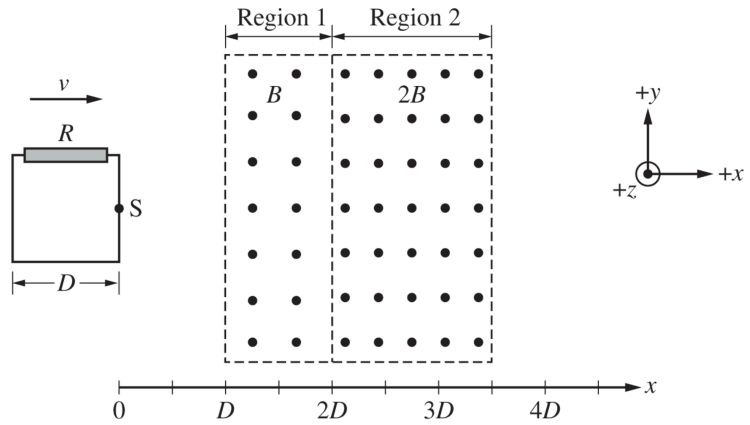


Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $+z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

(b) Consider the instant when Point S reaches $x = 1.5D$.

ii. **Derive** an expression for I_S when Point S reaches $x = 1.5D$. If $I_S = 0$, indicate how the derived expression shows that $I_S = 0$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

有电流的棒：求沿轨合力大小

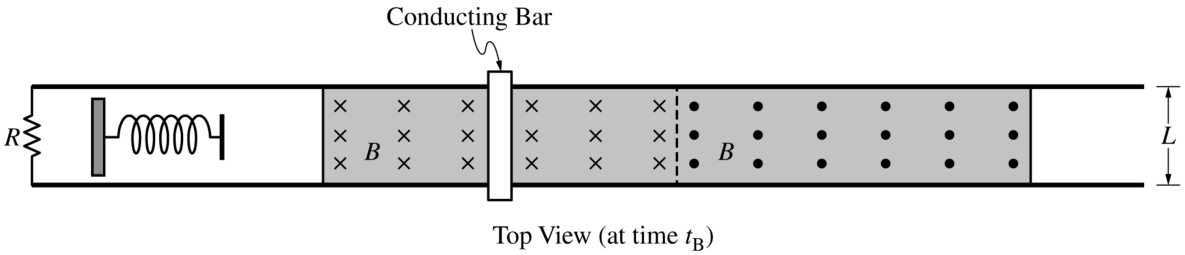
直棒与磁场垂直时，磁力大小IBℓ。水平导轨上重力与支持力相抵，离开弹簧后没有其他水平力，故磁力就是所求合力。

先用此前求出的棒电流，再乘实际场强和棒长；不要把电动势直接当作力。

$$F_B = IB\ell \sin\alpha.$$

例题 2023 Set 1 Q2 (b)(ii)

本问承接 2023 Set 1 Q2 (b)(i)。



Top View (at time t_B)

2. Two horizontal, parallel, conducting rails are separated by distance $L = 0.40\,\text{m}$. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23\,\text{kg}$ is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

   - At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
   - At time t_1, the bar loses contact with the spring and slides to the right.
   - At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50\,\text{T}$ that is directed into the page, as shown.
   - At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50\,\text{T}$ but is directed out of the page.
   - At time t_4, the bar enters a region with no magnetic field.

Consider time t_B such that $t_2 < t_B < t_3$.

(b) At time t_B, the speed of the bar is $v = 2.5\,\text{m/s}$.

ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

解：

(b)(ii) 合力大小棒与磁场垂直，磁力为

$$F = ILB。$$

代入未过早舍入的电流：

$$F = \frac{BLv}{R}LB = \frac{B^2L^2v}{R} = \frac{(0.50)^2(0.40)^2(2.5)}{0.30} = \boxed{0.33\,\text{N}}.$$

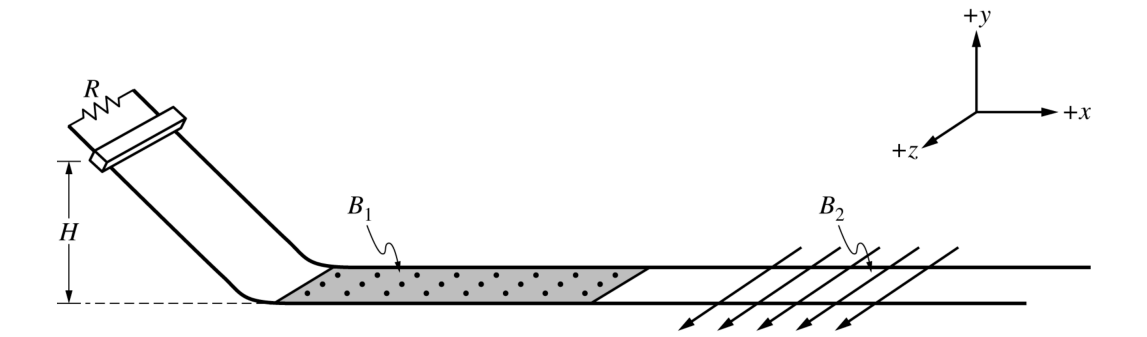
方向为负x。由

$$Fv = I^2R$$

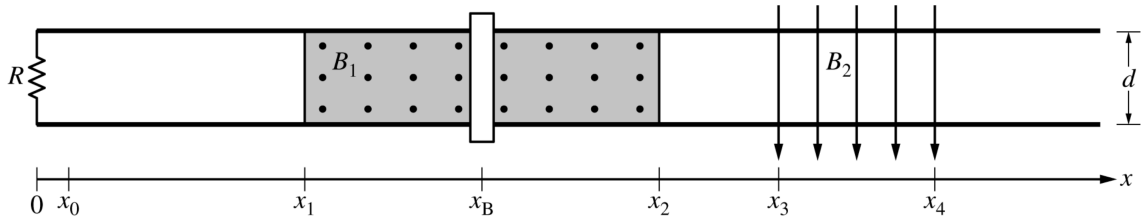
可检查：两边均约为$0.83\ \text{W}$，棒损失的机械功率与电阻产热一致。

练习 2023 Set 2 Q2 (b)(ii)

本问承接 2023 Set 2 Q2 (b)(i)。



Perspective View (at time $t = 0$)



Top View (at time t_B)

2. Two parallel conducting rails are separated by distance $d = 0.30\,\text{m}$. A resistor of resistance $R = 0.20\,\Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
- At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\,\text{T}$ that is directed in the $+y$-direction.
- At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
- At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\,\text{T}$ that is directed in the $+z$-direction.
- At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

The bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

(b) At time t_B, the speed of the bar is $v = 2.5\,\text{m/s}$.

ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

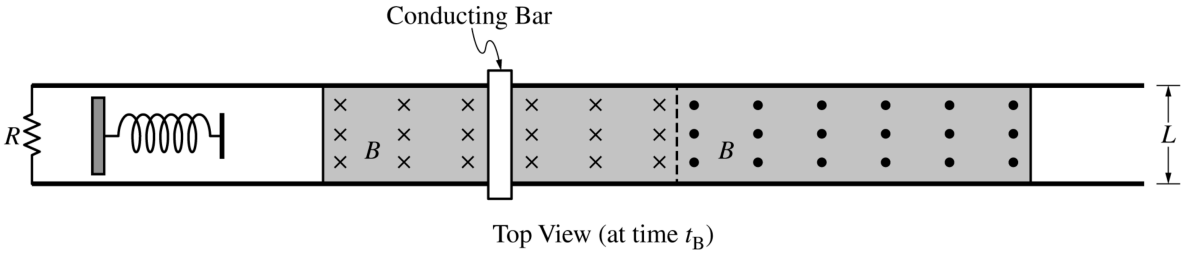
分段速度图：按每段实际力判断斜率

v–t图的斜率是加速度。弹簧推动或重力加速阶段速度增加；进入磁制动区后磁力反向且与v成正比，速度下降但下降越来越慢；离开磁场后无沿轨力，速度水平。

磁场平行于回路面时穿过回路的磁通为零，不继续画磁制动下降。每个指定时刻对应的场区必须从原图追踪。

$$\frac{dv}{dt} = -\frac{B_{\perp}^2 \ell^2}{mR_{\text{total}}} v \quad \text{（纯磁制动段）}.$$

例题 2023 Set 1 Q2 (c)



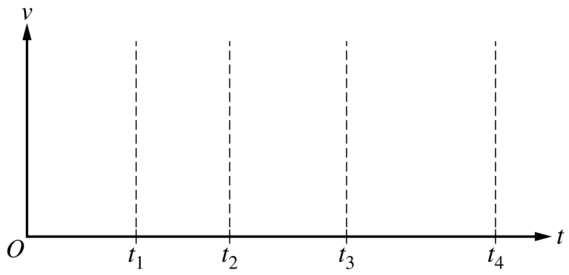
Top View (at time t_B)

2. Two horizontal, parallel, conducting rails are separated by distance $L = 0.40\,\text{m}$. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23\,\text{kg}$ is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

   - At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
   - At time t_1, the bar loses contact with the spring and slides to the right.
   - At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50\,\text{T}$ that is directed into the page, as shown.
   - At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50\,\text{T}$ but is directed out of the page.
   - At time t_4, the bar enters a region with no magnetic field.

Consider time t_B such that $t_2 < t_B < t_3$.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



解：

(c) 从释放到离场的速度图

$$0 < t < t_1:$$

压缩弹簧推动棒，速度从零上升；压缩量逐渐变小，弹力与加速度减小，所以曲线向下弯曲，离开自然长度的弹簧时斜率为零。

$$t_1 < t < t_2:$$

无水平力，速度保持常数。

$$t_2 < t < t_4:$$

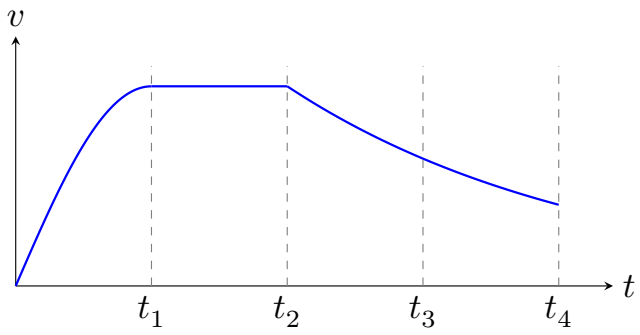
磁制动力与速度成正比，

$$m\frac{dv}{dt} = -\frac{B^2L^2}{R}v.$$

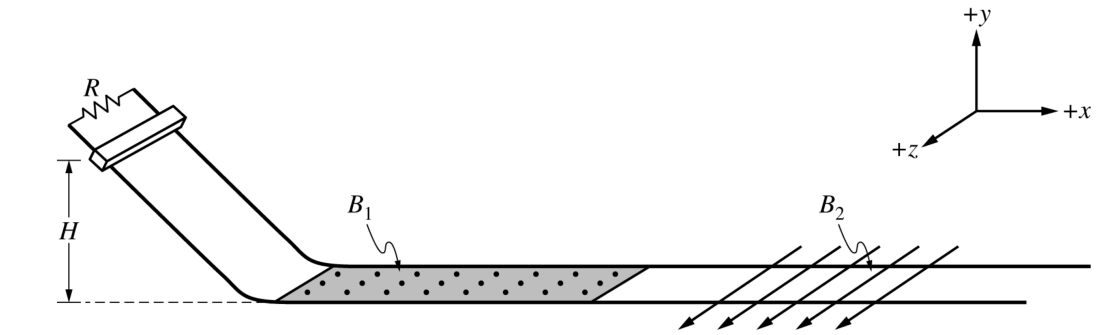
因此速度下降，而随速度下降制动力减小，负斜率逐渐变小，曲线向上弯曲。t_3 处场反向使电流也反向；两个反向共同保持磁力向左，而且

$$B^2$$

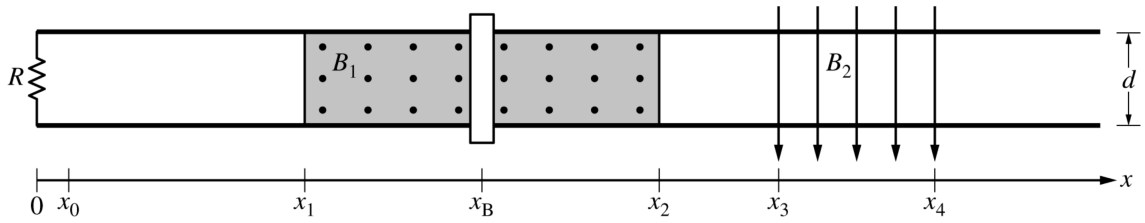
不变，故速度图在该处连续且斜率连续，仍在下降。下图仅表示时间次序与形状，不表示四个时间间隔相等。



练习 2023 Set 2 Q2 (c)



Perspective View (at time $t = 0$)



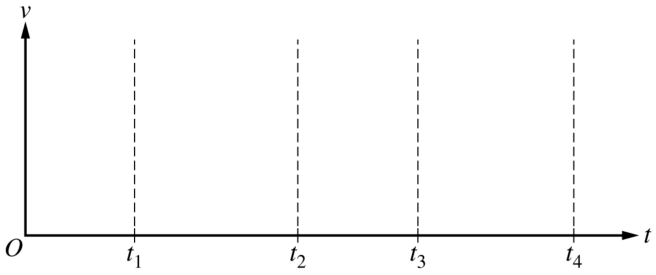
Top View (at time t_B)

2. Two parallel conducting rails are separated by distance $d = 0.30\,\text{m}$. A resistor of resistance $R = 0.20\,\Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

   - At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
   - At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\,\text{T}$ that is directed in the $+y$-direction.
   - At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
   - At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\,\text{T}$ that is directed in the $+z$-direction.
   - At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

The bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



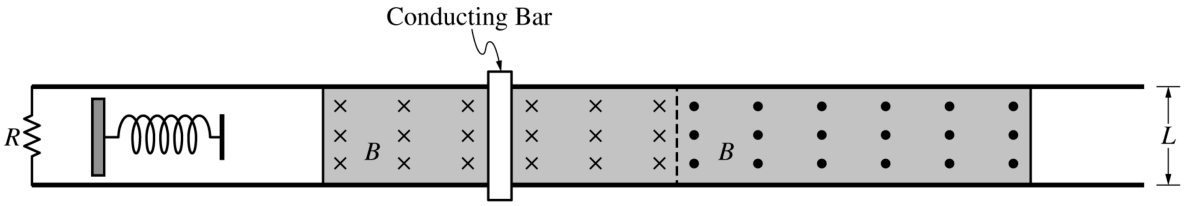
新电阻的位置：先辨并联或串联

同一对节点之间的两电阻并联，电流有分支；唯一不分支路径上的电阻串联。同样两只电阻，连接不同会使总电阻降低或提高。

先在原图追踪节点，写出总电阻，不凭器件数目判断。

$$R_{\text{series}} = R_1 + R_2, \qquad \frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

例题 2023 Set 1 Q2 (d)(i)



Top View (at time t_B)

2. Two horizontal, parallel, conducting rails are separated by distance $L = 0.40\,\text{m}$. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23\,\text{kg}$ is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

    - At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
    - At time t_1, the bar loses contact with the spring and slides to the right.
    - At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50\,\text{T}$ that is directed into the page, as shown.
    - At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50\,\text{T}$ but is directed out of the page.
    - At time t_4, the bar enters a region with no magnetic field.

Consider time t_B such that $t_2 < t_B < t_3$.

(d) The scenario is repeated but an additional resistor of resistance $R = 0.30\,\Omega$ is connected, as shown.

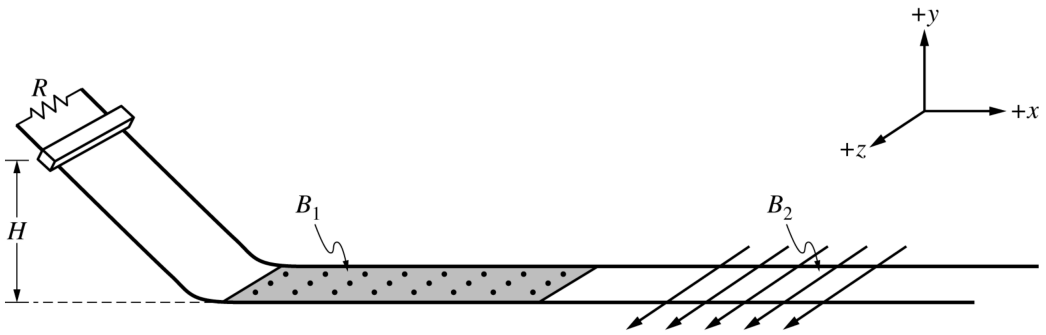
i. Determine the total resistance R_{total} of the closed circuit for the new scenario.

解：

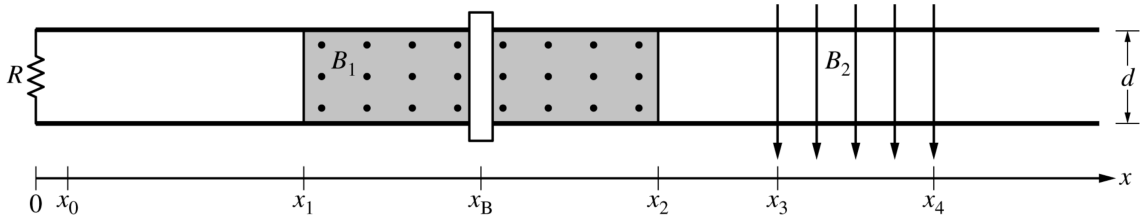
新电阻的两端接到原电阻的同一对节点，因此两只电阻并联。它们各为0.30 Ω；并联时电导相加。

$$\frac{1}{R_{\text{total}}} = \frac{1}{0.30\,\Omega} + \frac{1}{0.30\,\Omega}, \qquad R_{\text{total}} = 0.15\,\Omega.$$

练习 2023 Set 2 Q2 (d)(i)



Perspective View (at time $t = 0$)



Top View (at time t_B)

2. Two parallel conducting rails are separated by distance $d = 0.30\,\text{m}$. A resistor of resistance $R = 0.20\,\Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

   - At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
   - At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\,\text{T}$ that is directed in the $+y$-direction.
   - At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
   - At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\,\text{T}$ that is directed in the $+z$-direction.
   - At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

   The bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

(d) The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance $R = 0.20\,\Omega$. The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.

i. Determine the total resistance of the closed circuit.

总电阻改变：比较刚入场加速度

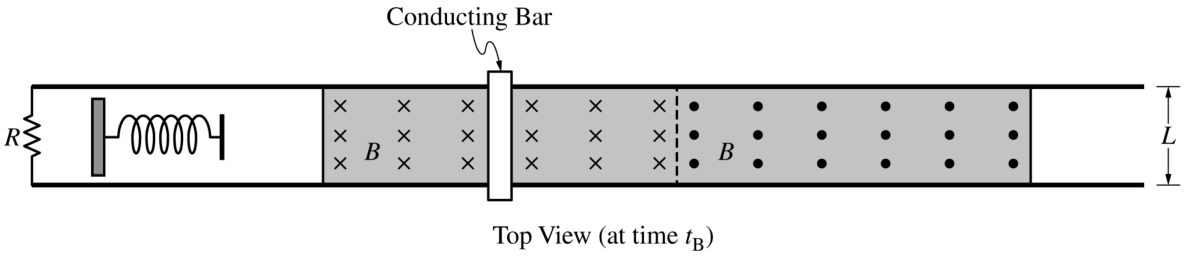
初始棒速、场强、棒长和质量不变时，动生电动势相同；电流与总电阻成反比，磁力和加速度也相应改变。

这里比较的是进入瞬间，不是进入后很久的速度。先用前问求出的新总电阻，再追踪电流→磁力→加速度。

$$|a| = \frac{B_\perp^2 \ell^2 v}{m R_{\text{total}}}.$$

例题 2023 Set 1 Q2 (d)(ii)

本问承接 2023 Set 1 Q2 (d)(i)。



Top View (at time t_B)

2. Two horizontal, parallel, conducting rails are separated by distance $L = 0.40\,\text{m}$. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23\,\text{kg}$ is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

   - At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
   - At time t_1, the bar loses contact with the spring and slides to the right.
   - At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50\,\text{T}$ that is directed into the page, as shown.
   - At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50\,\text{T}$ but is directed out of the page.
   - At time t_4, the bar enters a region with no magnetic field.

Consider time t_B such that $t_2 < t_B < t_3$.

(d) The scenario is repeated but an additional resistor of resistance $R = 0.30\,\Omega$ is connected, as shown.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{original}. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to a_{original} ? Justify your answer.

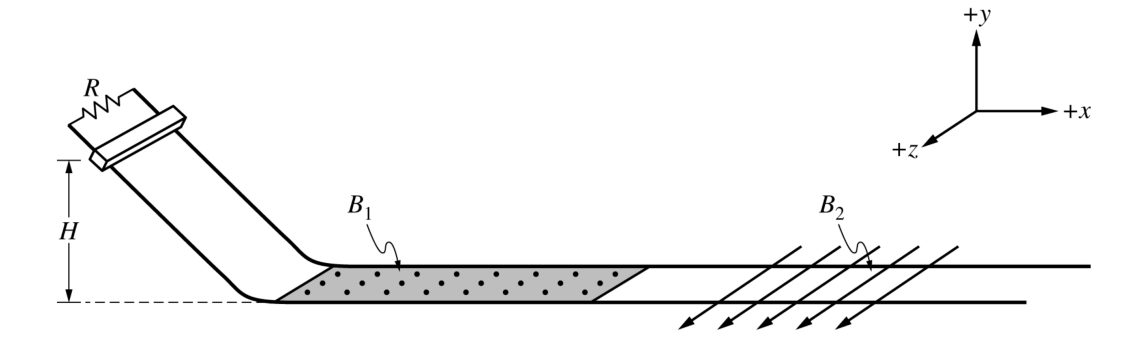
解：

两次进入磁场的瞬间，棒的质量、速度、磁场与长度相同。由前面求出的动生电动势，电动势均为 BLv。新回路的等效电阻由0.30 Ω降为0.15 Ω，故棒中的总电流加倍。磁阻力和加速度大小因此加倍，选择$a_{\text{new}} > a_{\text{original}}$。

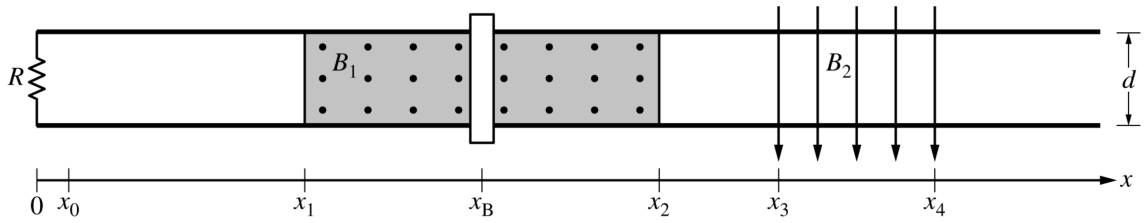
$$I = \frac{BLv}{R_{\text{total}}}, \qquad |a| = \frac{BIL}{m} = \frac{B^2L^2v}{mR_{\text{total}}}.$$

练习 2023 Set 2 Q2 (d)(ii)

本问承接 2023 Set 2 Q2 (d)(i)。



Perspective View (at time $t = 0$)



Top View (at time t_B)

2. Two parallel conducting rails are separated by distance $d = 0.30\,\text{m}$. A resistor of resistance $R = 0.20\,\Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

   - At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
   - At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\,\text{T}$ that is directed in the $+y$-direction.
   - At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
   - At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\,\text{T}$ that is directed in the $+z$-direction.
   - At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

   The bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

(d) The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance $R = 0.20\,\Omega$. The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{original}. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to a_{original} ? Justify your answer.

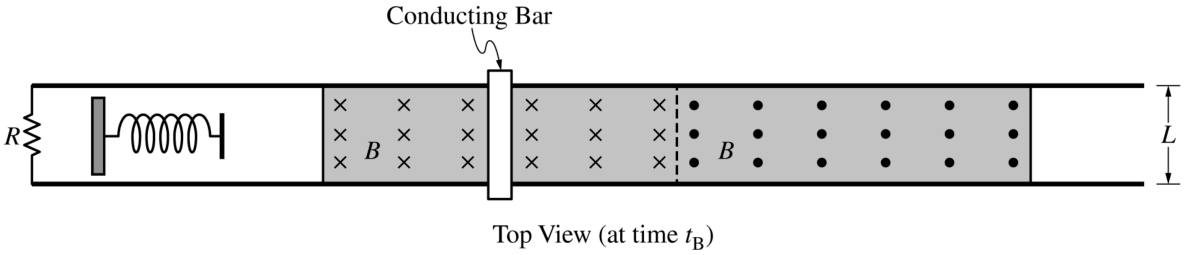
指定增大或减小：追踪真正的相关因子

动生电动势B⊥ℓv与棒进入磁场时的速度有关；电流还除以总电阻。选修改时保持题目未允许改变的条件。

由无摩擦斜轨释放时，速度由mgH=mv^2/2得到v=√(2gH)；改变高度会改变入场速度。弹簧给能量时，改变磁场并不改变入场前由弹簧得到的速度。

$$|\mathcal{E}| = B_\perp \ell v, \qquad I = \frac{|\mathcal{E}|}{R_{\text{total}}}, \qquad v = \sqrt{2gH}$$ （无摩擦落差H）.

例题 2023 Set 1 Q2 (e)



Top View (at time t_B)

2. Two horizontal, parallel, conducting rails are separated by distance $L = 0.40$ m. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23$ kg is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
- At time t_1, the bar loses contact with the spring and slides to the right.
- At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50$ T that is directed into the page, as shown.
- At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50$ T but is directed out of the page.
- At time t_4, the bar enters a region with no magnetic field.

Consider time t_B such that $t_2 < t_B < t_3$.

(e) Describe a modification to m, B, or L that will result in a smaller induced potential difference across the original resistor immediately after the bar enters the first uniform magnetic field. Justify your answer.

解：

(e) 使原电阻上的初始感应电压更小可把磁场大小减小，其他条件不变。棒在进入磁场前的运动未改变，入场瞬间v相同，而原回路

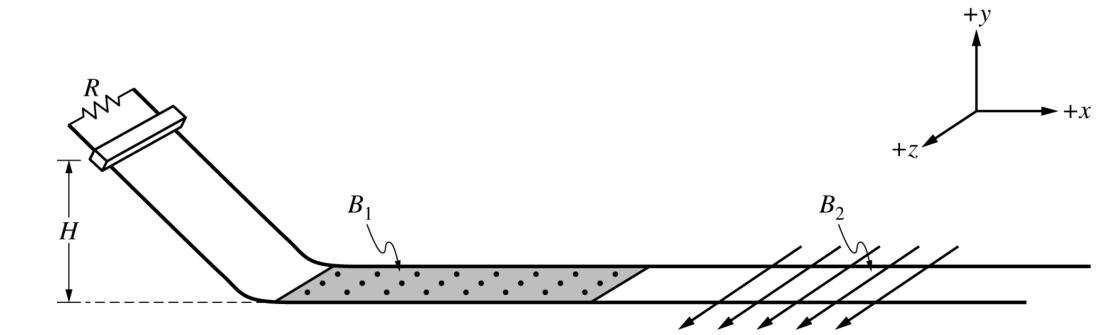
$$|\Delta V_R| = BLv$$

正比于B，所以电压变小。另两个可行变化是减小导轨间距L，或在弹簧初始压缩量不变时增大棒质量：相同弹簧能量给

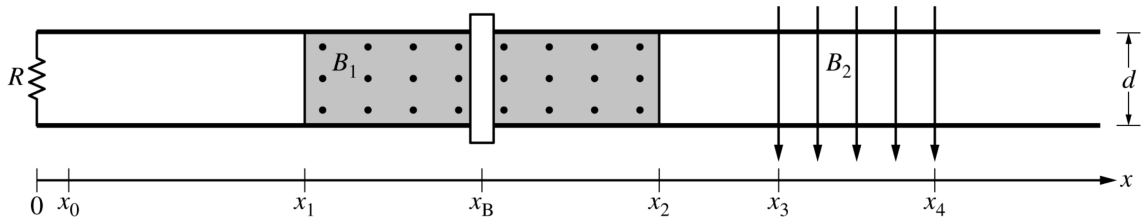
$$\tfrac{1}{2}mv^2,$$

质量较大时入场速度较小。题目只需选择其中一个并说明这条因果关系。

练习 2023 Set 2 Q2 (e)



Perspective View (at time $t = 0$)



Top View (at time t_B)

2. Two parallel conducting rails are separated by distance $d = 0.30\,\text{m}$. A resistor of resistance $R = 0.20\,\Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

   - At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
   - At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\,\text{T}$ that is directed in the $+y$-direction.
   - At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
   - At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\,\text{T}$ that is directed in the $+z$-direction.
   - At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

   The bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

(d) The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance $R = 0.20\,\Omega$. The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.

(e) Describe a modification to H, B_1, or d that will result in a larger induced current in the new bar immediately after the bar enters the first uniform magnetic field. Justify your answer.

已经求出电流：求瞬时耗散功率

电阻把电能转成热，瞬时功率I^2R。回路只含题给电阻时，用感应电流和该电阻；如果有多个电阻，求哪个器件的功率就用该器件的电流和电阻。

$$P = I^2R = \frac{\mathcal{E}^2}{R}.$$

例题 2024 Set 1 Q3 (b)(iii)

本问承接 2024 Set 1 Q3 (b)(ii)。

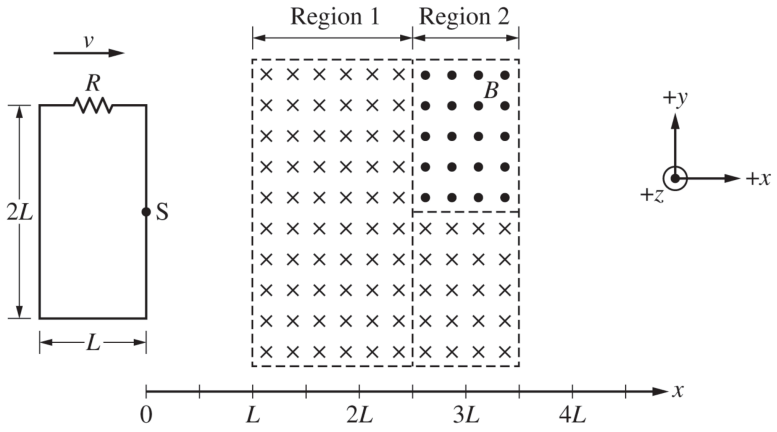


Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, that are parallel but are directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

(b) Consider the instant when Point S reaches $x = 1.5L$.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5L$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

解：

(b)(iii)　电阻耗散功率电阻发热功率是

$$P = I_R^2 R,$$

无论电流方向如何，功率都非负：

$$P = \left(\frac{2BLv}{R}\right)^2 R = \boxed{\frac{4B^2L^2v^2}{R}}.$$

$$BLv$$

的单位为伏特，所以

$$(BLv)^2/R$$

的单位为瓦特。

练习 2024 Set 2 Q3 (b)(iii)

本问承接 2024 Set 2 Q3 (b)(ii)。

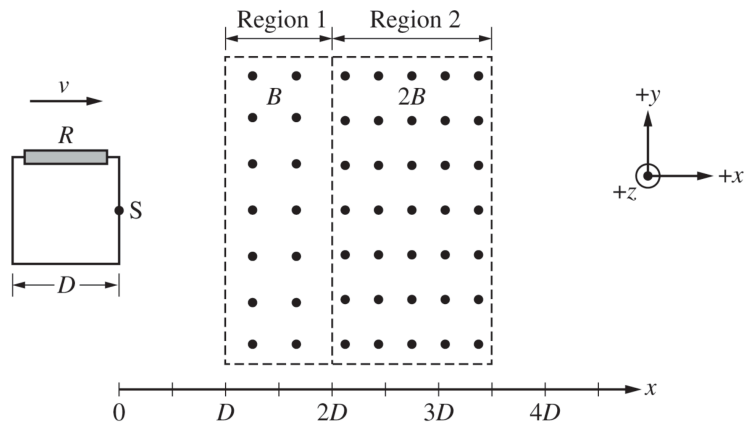


Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $+z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

(b) Consider the instant when Point S reaches $x = 1.5D$.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5D$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

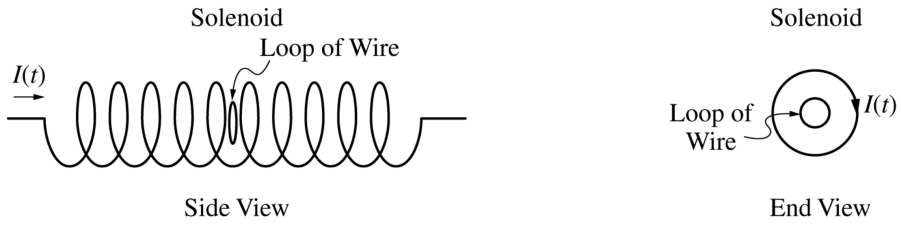
总耗热：把各阶段功率乘持续时间或积分

功率是每秒耗散的能量。若某阶段电流恒定，热量为I^2RΔt；电流随时间改变，则各小段热量相加为∫I^2Rdt。

移动场区分界后，要重新列每段的电流和持续时间，不只比较总通过时间。不能漏掉跨过两种场的过渡阶段。

$$E_R = \int I^2 R\,dt, \qquad E_j = I_j^2 R \Delta t_j \ \text{（恒流段）}.$$

例题 2022 Set 2 Q3 (c)



Note: Figures not drawn to scale.

3. A single loop of wire with resistance $3.0\,\Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. The direction of the current in the solenoid is clockwise, as shown in the end view.

(c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0$ s.

解：

(c) 前2.0 s 耗散的总能量回路中的电能以焦耳热形式耗散，功率

$$P = I_1^2 R_1。$$

本题感应电流恒定，故功率恒定，可以直接乘时间：

$$P = (1.32 \times 10^{-4})^2 (3.0) \approx 5.20 \times 10^{-8}\ \text{W},$$

$$E_{\text{diss}} = \int_0^{2.0} I_1^2 R_1 \, dt = P(2.0) = \boxed{1.04 \times 10^{-7}\ \text{J} \approx 1.0 \times 10^{-7}\ \text{J}}.$$

电流在螺线管中持续增大，却只会令回路磁通以恒定速率变化；这是本题能把积分简化为

$$P\Delta t$$

的原因。

例题 2024 Set 1 Q3 (c)

本问承接 2024 Set 1 Q3 (b)(iii)。

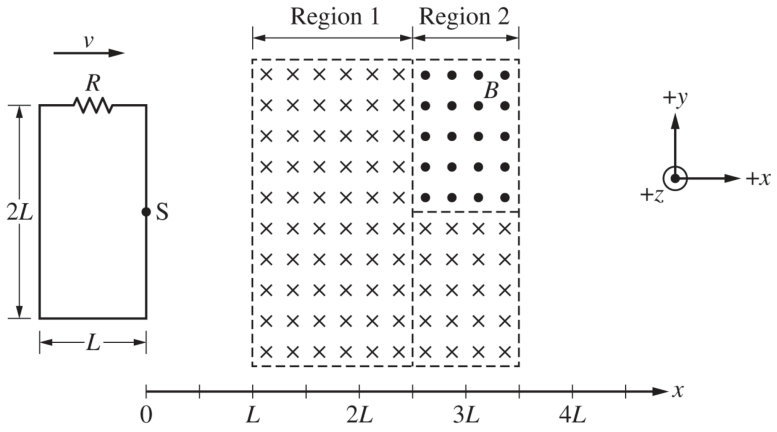


Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, that are parallel but are directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

The total energy dissipated by the resistor in the rectangular loop as Point S moves from $x = 0$ to $x = 4.5L$ is E_{original}.

The vertical boundary between regions 1 and 2 is now shifted to $x = 1.5L$. After the boundary is shifted, the rectangular loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5L$ is E_{new}.

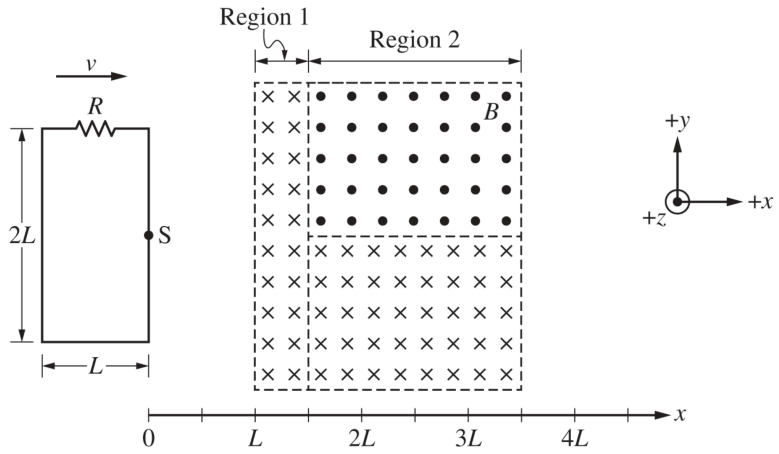


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to E_{original}.

_____ $E_{\text{new}} > E_{\text{original}}$ _____ $E_{\text{new}} < E_{\text{original}}$ _____ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

解：

(c) 移动边界后，总耗散能量的比较Region 1 的右边界改到

$$1.5L,$$

其宽度由

$$1.5L$$

变为

$$0.5L。$$

Region 2 的上下抵消仍成立。新情形进入Region 1 时的非零电流只持续

$$0.5L/v,$$

离开Region 1 时也只持续

$$0.5L/v;$$

两段之间通量不变，电流为零。每个非零阶段的电流大小仍为

$$2BLv/R。$$

原情形的两个非零阶段各持续

$$L/v。$$

功率相同而非零功率的总时间减半，因此

$$\boxed{E_{\text{new}} < E_{\text{original}}}.$$

还可计算验证：

$$E_{\text{original}} = \frac{4B^2L^2v^2}{R}\frac{2L}{v} = \frac{8B^2L^3v}{R},$$

$$E_{\text{new}} = \frac{4B^2L^2v^2}{R}\frac{L}{v} = \frac{4B^2L^3v}{R} = \frac{1}{2}E_{\text{original}}.$$

只比较起止通量不够：两次起止通量都为零，但发热取决于电流平方对时间的累计。

练习 2024 Set 2 Q3 (c)

本问承接 2024 Set 2 Q3 (b)(iii)。

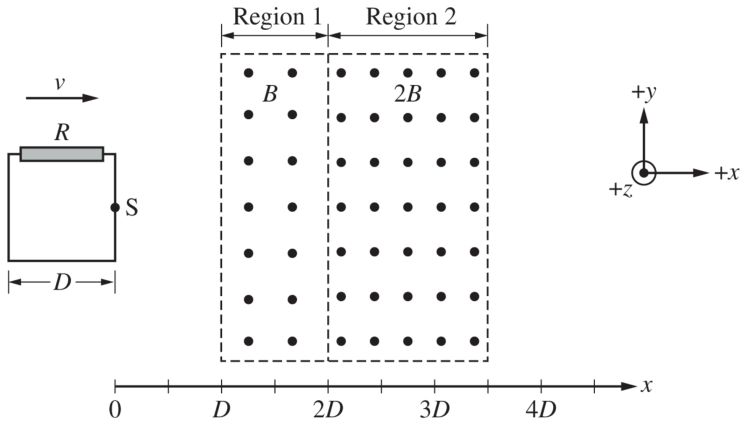


Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $+z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

The total energy dissipated by the resistor in the square loop as Point S moves from $x = 0$ to $x = 4.5D$ is E_{original}.

The vertical boundary between regions 1 and 2 is now shifted to $x = 2.5D$. After the boundary is shifted, the square loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5D$ is E_{new}.

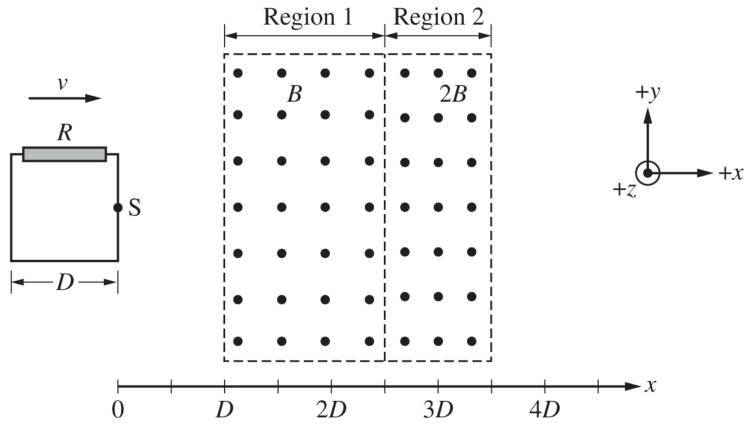


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to E_{original}.

_____ $E_{\text{new}} > E_{\text{original}}$ _____ $E_{\text{new}} < E_{\text{original}}$ _____ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

三角环面积变化：宽度不是常数

三角环进入场区时，切过边界的横向宽度随深入程度线性改变，不能照用矩形恒定宽度。

先用相似三角形表示边界处宽度，再写dA/dt=宽度×v，得到I=(B/R)dA/dt。只有开始退出场区后才会因面积减少而反向；本题所画区间还没有退出。原题没有指定顺时针或逆时针哪一种为正，画图前应声明自己的正方向，并在全图保持一致；把整幅图关于横轴反射也可表示同一物理过程。

$$I = \frac{B}{R}\frac{dA}{dt}, \qquad \frac{dA}{dt} = w(x)v.$$

例题 2024 Set 1 Q3 (d)

The original magnetic fields are modified so that the region $L < x < 3.5L$ contains an external uniform magnetic field of magnitude B that is directed in the $-z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length L and height $2L$. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the lower-leading corner of the loop.

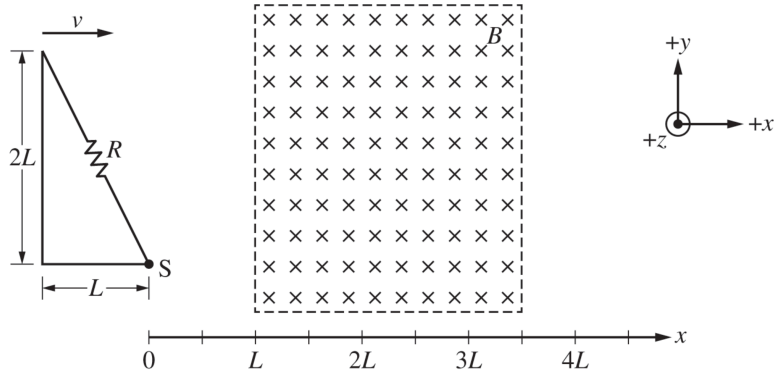
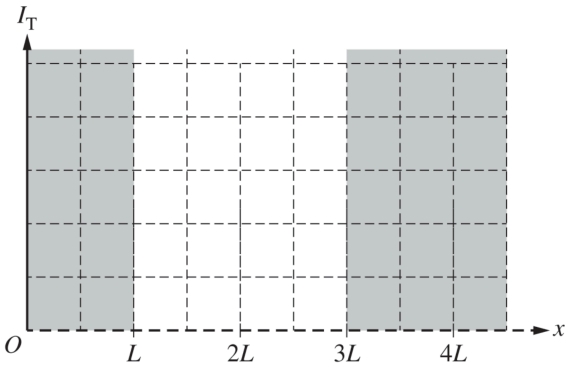


Figure 3

(d) On the following axes, **sketch** a graph of the induced current I_T in the triangular loop as Point S moves from $x = L$ to $x = 3L$.



解:

(d) 三角形环的电流图这里改成单一的入纸面磁场，范围

$$L < x < 3.5L。$$

三角环水平底边长L、竖直高度

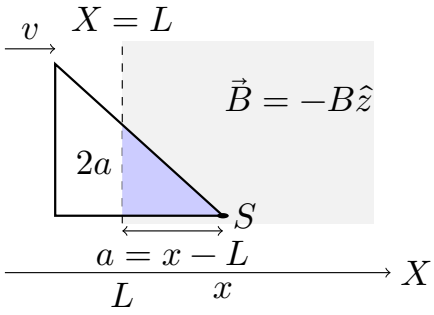
$$2L;$$

S 是右下方领先的尖角。取逆时针电流为正。

示意位置

$$L < x < 2L:$$

着色部分为已经进入磁场的三角形，a是尖角超过左边界的水平距离。图只表示几何关系。



先看

$$L < x < 2L。$$

尖角进入的水平深度为

$$a = x - L。$$

由相似三角形，磁场边界在环内截出的竖直长度为

$$2a。$$

已经进入的部分是底a、高

$$2a$$

的三角形，所以

$$A_{\text{in}} = \frac{1}{2}a(2a) = a^2 = (x-L)^2.$$

这一步解释了面积不是与进入距离成正比。恒速移动使

$$dx/dt = v,$$

于是

$$\mathcal{E}_{\text{ind}} = -\frac{d(-BA_{\text{in}})}{dt} = B[2(x-L)]v,$$

$$\boxed{I_T(x) = \frac{2Bv}{R}(x-L) \qquad (L < x < 2L)}.$$

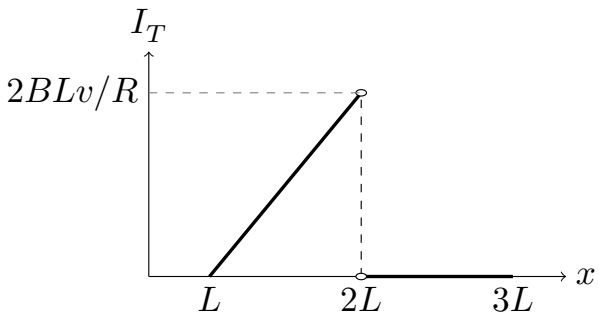
电流从零线性增加，始终逆时针。到

$$x > 2L,$$

整个三角形已在同一均匀场中；在本题画图范围

$$2L < x \leq 3L$$

内，它尚未离开右边界，通量不变，因此电流为零。



$$x = 2L$$

两侧的理想模型发生突变：虚线仅标出这个跳变位置，不表示一个持续经过全部中间电流值的过程。

练习 2024 Set 2 Q3 (d)

The original magnetic fields are modified so that the region $D < x < 3.5D$ contains an external uniform magnetic field of magnitude B that is directed in the $+z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length D and height D. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the upper-leading corner of the loop.

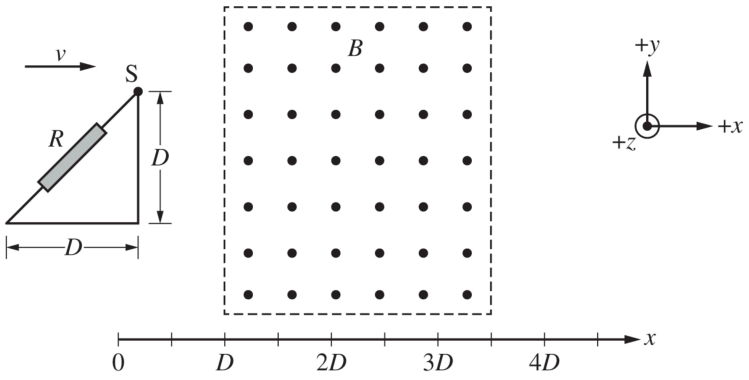
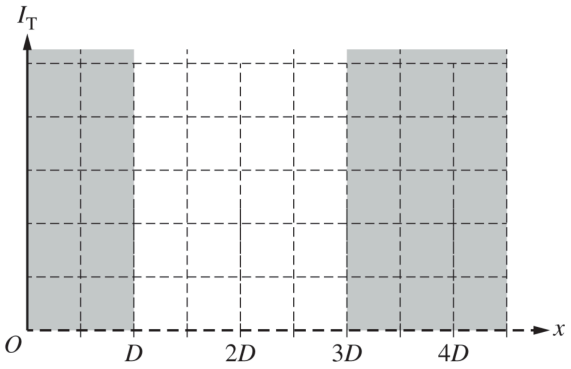


Figure 3

(d) On the following axes, **sketch** a graph of the induced current I_T in the loop as Point S moves from $x = D$ to $x = 3D$.



正弦磁通：用导数而非通量高度画电动势

转动回路磁通Φ=BAcosωt，ω是角速度。电动势取其导数的绝对值；余弦最大处变化率为零，过零处变化率最大。

把指定时刻代入|sinωt|再按同一尺度画柱，别照搬cosωt的高度。

$$|\mathcal{E}| = BA\omega|\sin\omega t|.$$

例题 2025 Q2 (a)

Question 2: Version J

2. A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA\cos(\omega t)$.

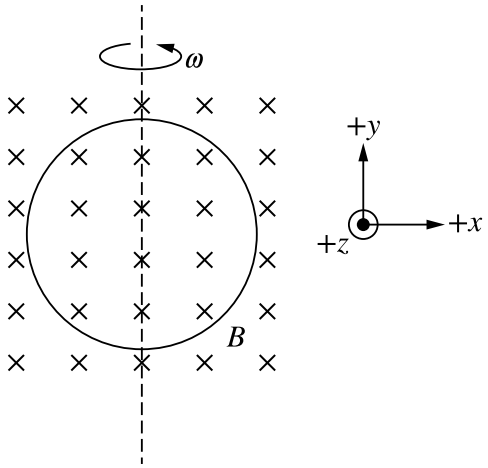


Figure 1

A. The absolute value of the induced emf in the loop is $|\varepsilon|$. The partially completed bar chart in Figure 2 shows a bar that represents $|\varepsilon|$ at $t = \frac{3}{4}T$. In Figure 2, **draw** bars to represent $|\varepsilon|$ at times $t = 0$, $\frac{1}{4}T$, and $\frac{1}{2}T$ relative to $|\varepsilon|$ shown at $\frac{3}{4}T$. If $|\varepsilon| = 0$, **write** a "0" in that column.

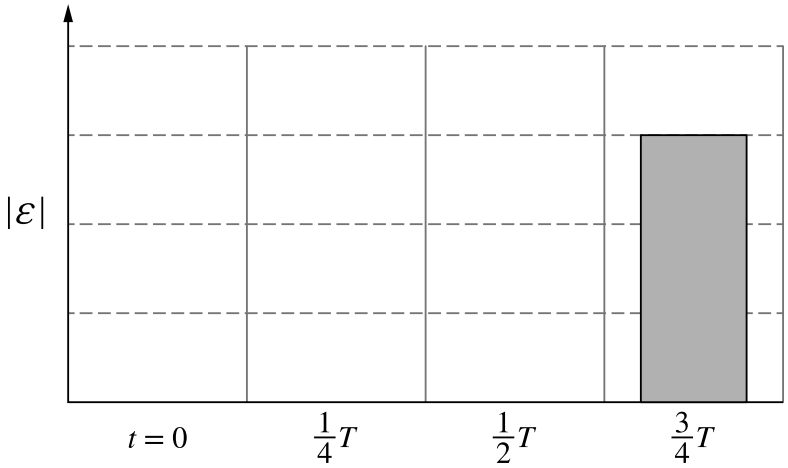


Figure 2

解：

A 补全电动势柱状图感应电动势取决于磁通量的变化速度，不取决于磁通量本身的大小。法拉第定律给出

$$\mathcal{E} = -\frac{d\Phi}{dt} = BA\omega \sin(\omega t), \qquad \varepsilon = |\mathcal{E}|.$$

因此

$$t = 0, T/2$$

时为零；

$$t = T/4, 3T/4$$

时都达到

$$BA\omega。$$

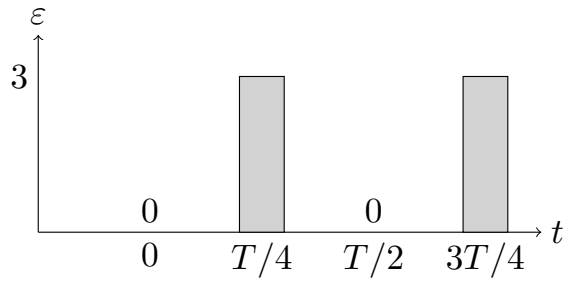
原图

$$3T/4$$

的柱高为3 个相对单位，故

$$T/4$$

也画高3，其余两列写“0”。



功率曲线：电动势平方改变周期与符号

电阻功率P=$\mathcal{E}^2$/R，所以正负电动势都给非负功率。正弦电动势平方后，在一圈里有两个相同峰，最大电动势对应最大功率，电动势为零对应零功率。

画出全部指定时段后，再用P与$\mathcal{E}$的平方关系检查功率图和柱状图一致。

$$P = \frac{(BA\omega)^2}{R}\sin^2\omega t.$$

例题 2025 Q2 (c)

Question 2: Version J

---

2. A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA\cos(\omega t)$.

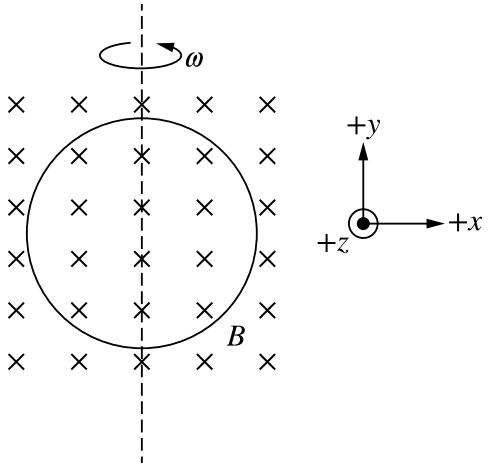


Figure 1

C. On the axes shown in Figure 3, **sketch** a graph of the instantaneous power P dissipated by the loop as a function of t during the time interval $0 \le t \le T$.

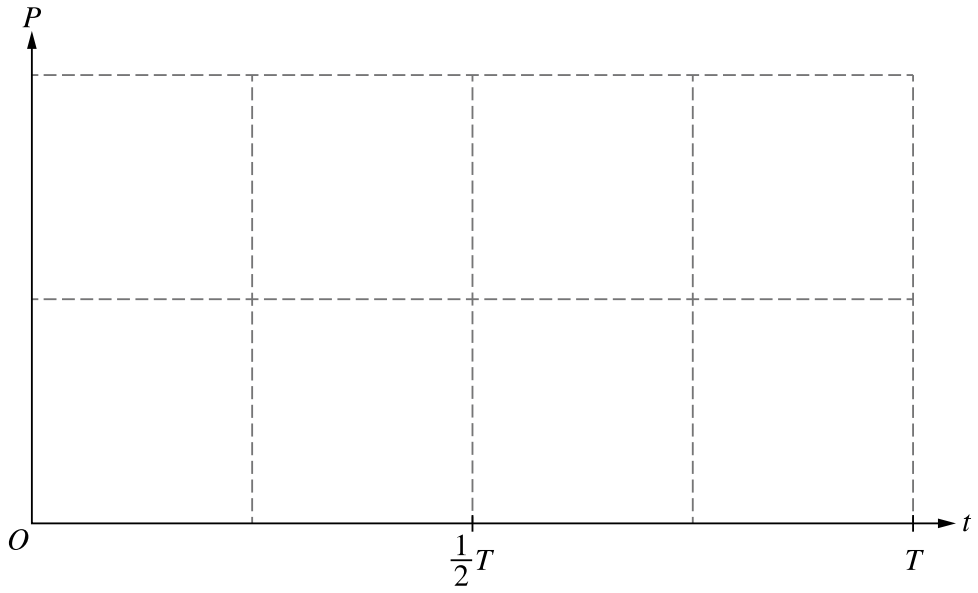


Figure 3

解：

C　功率图电阻发热功率为

$$P = I^2 R。$$

平方使正、负电流都发热，因此功率不应有负半周：

$$\boxed{P(t) = \frac{B^2 A^2 \omega^2}{R} \sin^2(\omega t)}.$$

在一个旋转周期内，功率有两座等高峰。

$$0, T/2, T$$

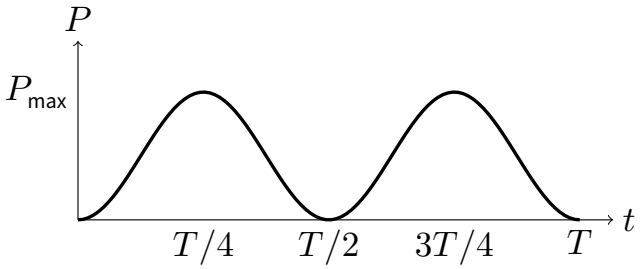
为零；

$$T/4, 3T/4$$

为最大值

$$P_{\text{max}} = B^2A^2\omega^2/R。$$

这些零点处曲线斜率也为零。



练习 2025 Q2 (d)

本问承接 2025 Q2 (a)、2025 Q2 (c)。

Question 2: Version J

---

2. A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA\cos(\omega t)$.

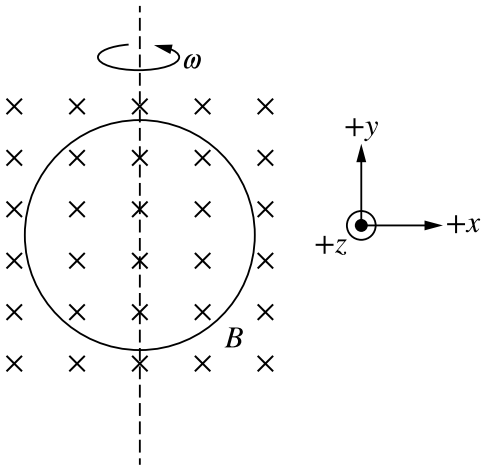


Figure 1

D. Indicate whether the sketch you drew in part C is or is not consistent with the bars that you drew in part A. Briefly **justify** your answer by referencing the functional dependence between P and $|\varepsilon|$.

电感瞬态与LC振荡实验

电压表测指定器件：跨接其两端

电压表接到两个不同电势节点之间，测那一器件的端电压，应并联而不是切断主回路后串入。

测电感器就跨接电感器端点；测指定电阻就跨接其端点。正、负端按电池驱动电流沿路径的电势降低安排。

例题 2024 Set 1 Q2 (a)(i)

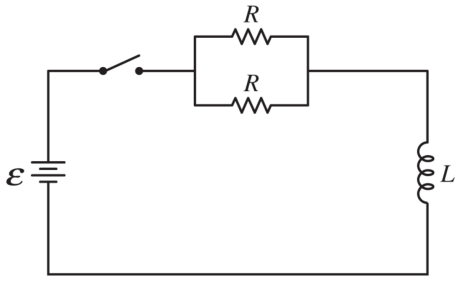


Figure 1

2. Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that decreases with time.



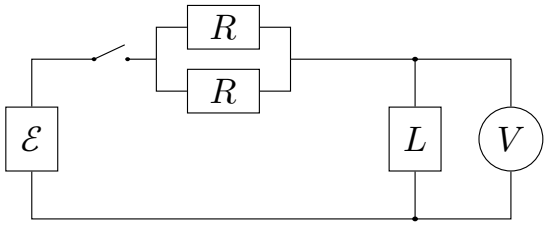
Voltmeter Symbol

解：

(a)(i) 电压表的连接将电压表并联在电感器两端。电压表两根导线接到电感器的两个端点，测量端电压V_L；不能把表串入回路。

刚闭合开关，电感器阻碍电流从零突然改变，电阻电压为零，电池电压主要落在电感器上。电流逐渐增加后，电阻电压增加，电感器电压下降，正好满足题目要求。

下图用元件名称方框表示元件，两个R仍是并联；圆圈V 跨接L 的两端。



练习 2024 Set 2 Q2 (a)(i)

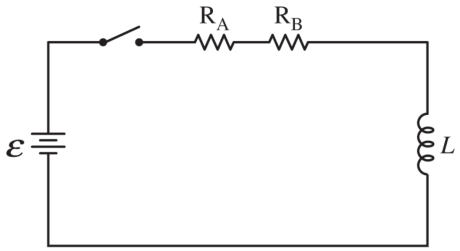


Figure 1

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(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.



Voltmeter Symbol

记录瞬态：同一初始状态重复采样

先断开开关等待电流恢复为零，记录型电压表跨接指定器件。闭合开关同时取t=0，连续记录足够密的电压读数直至接近稳态。

重复试验以降低偶然误差；每次先恢复相同初始状态，不能在尚有电流时把下一次读数当作从零开始。

例题 2024 Set 1 Q2 (a)(ii)

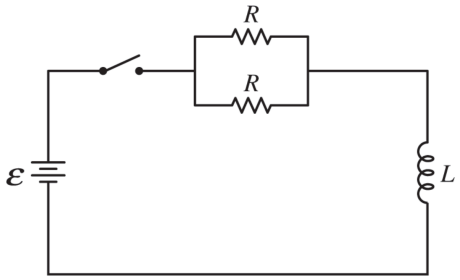


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(a)

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using the measured quantity that decreases with time. Provide enough detail so that another student could replicate the experiment.

解：

(a)(ii) 可重复的数据收集程序先断开开关，等待电路电流恢复为零；将记录电压随时间的电压表跨接电感器，并设置足够快的采样。闭合开关的瞬间设为

$$t = 0,$$

从刚闭合开始连续记录多组

$$(t, V_L),$$

直到电压接近零，或至少覆盖能确定时间常量的下降段。保持电池、电感器和电阻不变，重新断开、等待后再重复，以减少偶然误差。

只记录稳态的零电压，无法判断下降有多快，也就无法求R。

练习 2024 Set 2 Q2 (a)(ii)

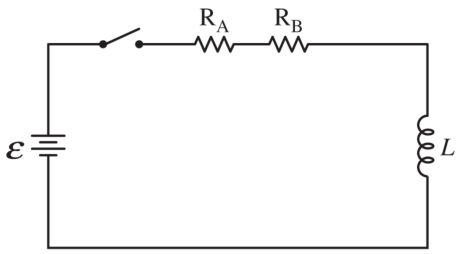


Figure 1

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(a)

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

RL电压曲线：电感下降、电阻上升

理想电感阻止电流突然改变。初始电流为零，电阻电压为零，电感承担电池电压；很久后电流稳定，电感电压L dI/dt为零。

时间常数τ=L/R总。电感电压指数下降；电阻电压随电流指数上升至分压终值。先求两电阻是并联还是串联，R总和终值由原图决定。

$$I = \frac{\mathcal{E}}{R_{\text{total}}}(1 - e^{-t/\tau}), \quad V_L = \mathcal{E}e^{-t/\tau}, \quad \tau = \frac{L}{R_{\text{total}}}.$$

例题 2024 Set 1 Q2 (b)(i)

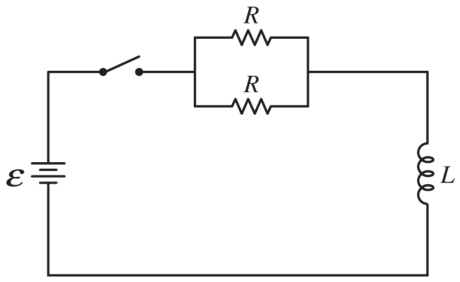


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(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.



Figure 2

解：

(b)(i) 测量曲线两个R并联的等效电阻为

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R}, \qquad R_{\text{eq}} = \frac{R}{2}.$$

理想LR 电路的时间常量是电感除以串联的等效电阻：

$$\tau = \frac{L}{R/2} = \frac{2L}{R}.$$

电感器两端电压随时间按指数下降：

$$\boxed{V_L(t) = \mathcal{E}\, e^{-t/\tau} = \mathcal{E}\, e^{-Rt/(2L)}}.$$

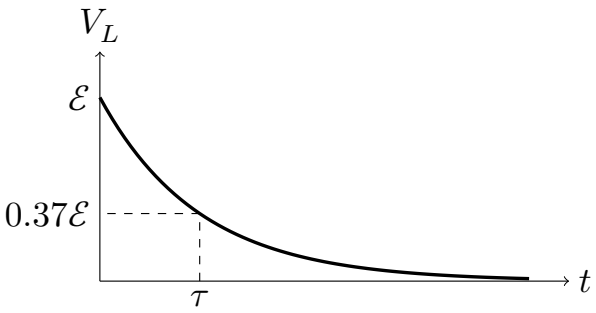
横轴标t，纵轴标V_L。曲线从

$$(0, \mathcal{E})$$

开始，下降且向上凹，逐渐接近

$$V_L = 0;$$

不是匀速下降到零后停止的直线。



例题 2024 Set 2 Q2 (b)(i)

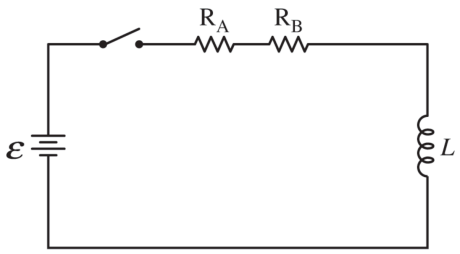


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- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.



Figure 2

解：

(b)(i) 画V_A 随t 的曲线两个电阻串联，总电阻

$$2R,$$

因此

$$\tau = \frac{L}{2R}.$$

刚合闸

$$V_A = 0。$$

达到稳态后，理想电感器没有端电压，总电流为

$$\mathcal{E}/(2R),$$

所以单个电阻上的终值为

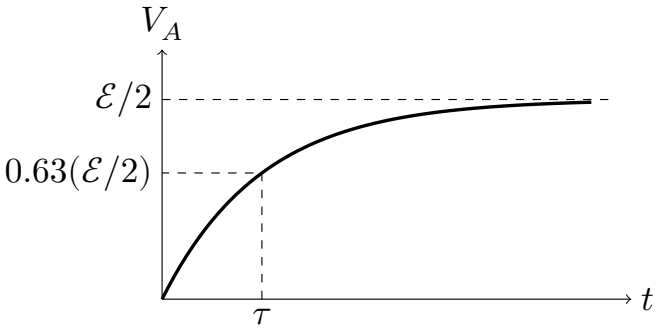
$$\mathcal{E}/2。$$

瞬态曲线为

$$\boxed{V_A(t) = \frac{\mathcal{E}}{2}\left(1 - e^{-t/\tau}\right) = \frac{\mathcal{E}}{2}\left(1 - e^{-2Rt/L}\right)}.$$

横轴t、纵轴V_A；曲线从原点开始，增大且向下凹，水平渐近线标

$$\mathcal{E}/2。$$



若改选两电阻组合的总电压，曲线形状及τ相同，终值则是$\mathcal{E}$；本解答选单个R_A，因此必须用

$$\mathcal{E}/2。$$

曲线读时间常数：再换回单个电阻

指数下降图在初值的1/e处读τ；上升图在终值的1−1/e处读τ。两者得到的都是L/R总，不是直接L/R某一只。

求某一个电阻R时，先由原图写R总：两个相同电阻并联是R/2，串联是2R，再由τ关系解R。

$$\tau = \frac{L}{R_{\text{total}}}, \qquad 1/e \approx 0.368, \quad 1 - 1/e \approx 0.632.$$

例题 2024 Set 1 Q2 (b)(ii)

本问承接 2024 Set 1 Q2 (b)(i)。

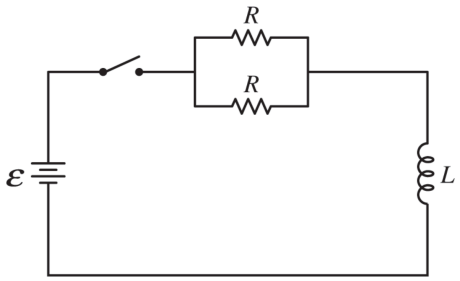


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(b)

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

解：

(b)(ii) 由图求一个电阻的R在曲线上找电压降到初值

$$1/e \approx 0.37$$

倍的时刻。因为

$$e^{-t/\tau} = e^{-1}$$

对应

$$t = \tau,$$

读出的横坐标就是时间常量，故

$$\boxed{R = \frac{2L}{\tau}}.$$

也可把全部数据拟合成

$$V_L = Ae^{-bt};$$

理想情况下

$$A = \mathcal{E},$$

指数系数

$$b = R/(2L),$$

所以

$$R = 2Lb。$$

b的单位为s^{-1},

$$L/\tau$$

的单位为欧姆。

注意

$$L/\tau$$

给出的是

$$R/2,$$

不是题中单个电阻的R；并联关系决定了最后的系数2。

练习 2024 Set 2 Q2 (b)(ii)

本问承接 2024 Set 2 Q2 (b)(i)。

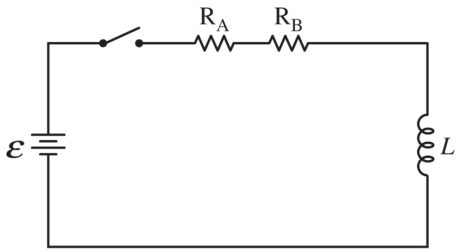


Figure 1

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(b)

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

RL回路方程：电感压降含电流变化率

沿电流方向绕回路：电池提供$\mathcal{E}$升压，电阻降压IR总，电感压降L dI/dt。dI/dt表示电流每秒的变化率；它与电流I是不同量。

先将回路升降压相加为零，再写微分方程。原问要求不要解时，不必写时间函数。

$$\mathcal{E} - IR_{\text{total}} - L\frac{dI}{dt} = 0.$$

例题 2024 Set 1 Q2 (c)

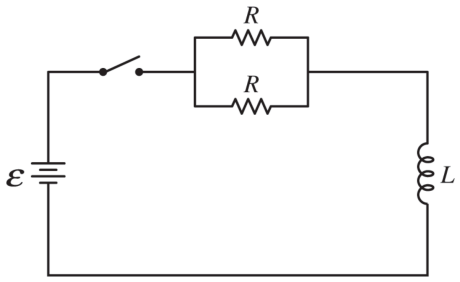


Figure 1

2. Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

解：

(c) 从回路定律建立电流微分方程沿电流方向绕闭合回路，电池提供电势升高，电阻组合和电感器提供电势降低：

$$\mathcal{E} - V_R - V_L = 0.$$

总电流I流过并联组合，因此

$$V_R = I(R/2)。$$

电感器在电流增加时产生反向电动势，其端电压降低量为

$$L\,dI/dt;$$

$$dI/dt$$

表示此刻电流每秒改变多少。代入得到

$$\boxed{\mathcal{E} - \frac{R}{2}I - L\frac{dI}{dt} = 0},$$

或

$$\boxed{\frac{dI}{dt} = \frac{\mathcal{E}}{L} - \frac{R}{2L}I}.$$

题目只要求建立方程，到这里停止，不再求

$$I(t)。$$

练习 2024 Set 2 Q2 (c)

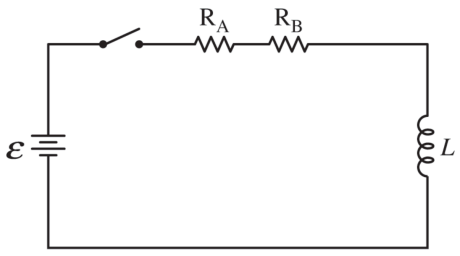


Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive,** but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

电感含内阻：稳态仍保留电阻压降

达到稳态时dI/dt=0，理想电感部分的电压为零，但线圈内阻r的Ir压降仍存在。

电压表跨整个线圈时测得内阻电压；跨别的电阻时，新增内阻降低总稳态电流，因此它的稳态压降降低。先明确电压表跨哪一对节点。

$$I_\infty = \frac{\mathcal{E}}{R_{\text{external}} + r}.$$

例题 2024 Set 1 Q2 (d)

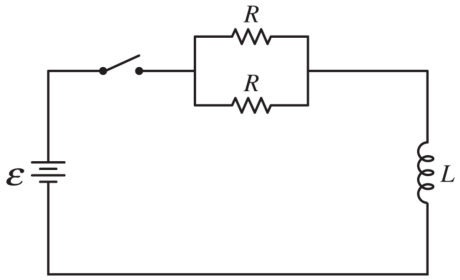


Figure 1

2. Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

After reaching steady state, the absolute value of the potential difference across the inductor is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across the new inductor is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

_____ $|\Delta V_2| > |\Delta V_1|$ _____ $|\Delta V_2| < |\Delta V_1|$ _____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

解：

(d) 有内阻的电感器：稳态电压比较原来的理想电感器达到稳态后，电流不再变化，

$$dI/dt = 0,$$

所以其端电压

$$\Delta V_1 = 0。$$

新电感器除了电感效应，还有内部电阻

$$r > 0。$$

稳态时电感部分的电压仍为零，但内部电阻的电压为

$$I_\infty r > 0:$$

$$I_\infty = \frac{\mathcal{E}}{R/2 + r}, \qquad \Delta V_2 = \frac{\mathcal{E} r}{R/2 + r} > 0.$$

所以

$$\boxed{\Delta V_2 > \Delta V_1}.$$

“稳态电感器相当于导线”只适用于理想零内阻的电感器，不能把新元件的电阻也抹去。

练习 2024 Set 2 Q2 (d)

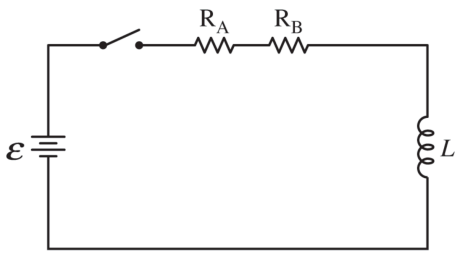


Figure 1

2. Students are asked to determine the resistance R of identical resistors $\mathrm{R_A}$ and $\mathrm{R_B}$. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

After reaching steady state, the absolute value of the potential difference across $\mathrm{R_A}$ is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across $\mathrm{R_A}$ is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

_____ $|\Delta V_2| > |\Delta V_1|$ _____ $|\Delta V_2| < |\Delta V_1|$ _____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

LC周期实验：测电压的重复间隔

理想LC回路中，电场储能和磁场储能来回交换。无电源回路中，电容电压q/C与电感压降相抵；结合电流是电荷变化率，得到Lq″+q/C=0，其中q″表示电荷的二阶时间导数。它写成q″=−q/(LC)时，回复作用正比于电荷且方向相反，与简谐振荡方程q″=−ω^2q对应，故ω=1/√(LC)，周期T=2π√(LC)。

电容电压q/C具有相同周期。记录电压随时间的波形，以相邻同方向波峰的间隔测T；对不同已知C重复，可反求未知电感L。

$$T = 2\pi\sqrt{LC}.$$

例题 2026 Q3 (a)(i)

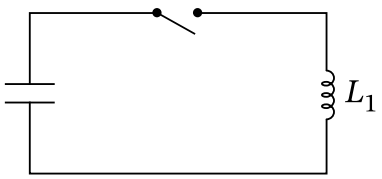
Question 3

The following information applies to parts A and B.

---

3. In Experiment 1, students are asked to collect data that can be graphed to determine the inductance L_1 of an inductor. The students have access to multiple charged capacitors, each of a different known capacitance, and a voltmeter that measures potential difference as a function of time. In each trial, the inductor is connected to a charged capacitor and an initially open switch, as shown in Figure 1.

Figure 1



Part A

i. **Indicate** quantities that could be measured by the students that would allow them to determine L_1 using a linear graph.

解：

A(i) 第一实验测什么把电压表并联在电容两端，闭合开关后记录

$$V_C(t),$$

由相邻同方向波峰的时间差求振荡周期T。对若干不同的已知电容重复，得到

$$(C, T)$$

数据。为什么周期能够给电感？能量在电容电场与电感磁场之间来回交换，LC 回路方程为

$$L_1 \frac{d^2q}{dt^2} + \frac{q}{C} = 0.$$

它与简谐运动方程

$$q'' + \omega^2 q = 0$$

对应，故

$$\omega^2 = 1/(L_1 C),$$

$$T = 2\pi\sqrt{L_1 C}.$$

因为

$$V_C = q/C,$$

电压的周期与电荷周期相同。

周期测量误差：多周期总时长取平均

单个周期的端点时间可能读偏。测连续n个完整周期的总时间Δt，以Δt/n得到周期；n越大，同样端点读数误差占单周期的比例越小。

同一电容多次重复、不同电容多组测量可以检验偶然误差与整体趋势。

$$T = \frac{\Delta t}{n}.$$

例题 2026 Q3 (a)(ii)

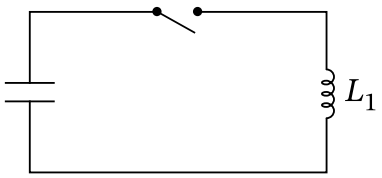
Question 3

The following information applies to parts A and B.

---

3. In Experiment 1, students are asked to collect data that can be graphed to determine the inductance L_1 of an inductor. The students have access to multiple charged capacitors, each of a different known capacitance, and a voltmeter that measures potential difference as a function of time. In each trial, the inductor is connected to a charged capacitor and an initially open switch, as shown in Figure 1.

Figure 1



Part A

ii. Briefly **describe** a method to reduce experimental uncertainty.

解：

A(ii)　减少不确定度连续测量n个完整周期的总时间

$$\Delta t,$$

以

$$T = \Delta t / n$$

求周期，降低选取单个波峰的时间读数影响；每个电容重复试验并平均，再使用不同电容取得覆盖较大范围的数据。

周期式线性化：选择平方后的坐标

T对C不是直线，因为T正比于$\sqrt{}$C。把周期式平方得到T^2=$4\pi^2$LC；因此横轴C、纵轴T^2形成直线，斜率$4\pi^2$L。

也可用$\sqrt{}$C作横轴、T作纵轴，但斜率的含义随之改变。选择一组后，读图和反求必须始终使用同一关系。

$$T^2 = 4\pi^2 LC.$$

例题 2026 Q3 (b)(i)

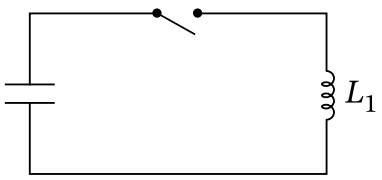
Question 3

The following information applies to parts A and B.

---

3. In Experiment 1, students are asked to collect data that can be graphed to determine the inductance L_1 of an inductor. The students have access to multiple charged capacitors, each of a different known capacitance, and a voltmeter that measures potential difference as a function of time. In each trial, the inductor is connected to a charged capacitor and an initially open switch, as shown in Figure 1.

Figure 1



Part B

i. **Indicate** what quantities the students could graph on the horizontal and vertical axes to create a linear graph that can be used to determine L_1.

解：

B(i) 线性图的坐标轴平方周期式：

$$T^2 = 4\pi^2 L_1 C.$$

横轴取C（F），纵轴取

$$T^2$$

（s^2），得到线性图。直接画T对C是平方根曲线；若想画T，横轴必须改用

$$\sqrt{C}。$$

平方周期图斜率：反求电感

斜率由拟合线上两点的纵坐标差除以横坐标差。T^2对C图斜率b=$4\pi^2$L，因此电感b/($4\pi^2$)。

原表或图没有固定选择时，需先说明与前问所选坐标一致；不能把另一组坐标的斜率公式混来使用。

$$L = \frac{b}{4\pi^2}.$$

例题 2026 Q3 (b)(ii)

本问承接 2026 Q3 (b)(i)。

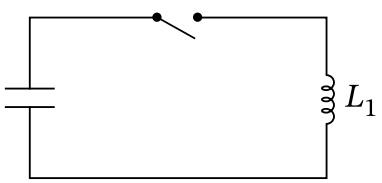
Question 3

The following information applies to parts A and B.

---

3. In Experiment 1, students are asked to collect data that can be graphed to determine the inductance L_1 of an inductor. The students have access to multiple charged capacitors, each of a different known capacitance, and a voltmeter that measures potential difference as a function of time. In each trial, the inductor is connected to a charged capacitor and an initially open switch, as shown in Figure 1.

Figure 1



Part B

ii. Briefly **describe** the relationship between L_1 and a feature of the graph from part B (i). Your answer may include an equation that relates L_1 and the chosen feature of the graph.

解：

B(ii) 用图求L_1对

$$T^2$$

-C图，拟合斜率

$$m_1 = 4\pi^2 L_1,$$

所以

$$\boxed{L_1 = \frac{m_1}{4\pi^2}}.$$

量纲验证：

$$\mathrm{s}^2/\mathrm{F} = \mathrm{H},$$

结果单位为亨利。

RL测量表：把电感电压与变化率配对

回路关系$\mathcal{E}$−IR−L dI/dt=0说明电感电压VL=$\mathcal{E}$−IR。表里同时给I和dI/dt时，可以计算VL，画VL对dI/dt，斜率就是L。

本题$\mathcal{E}$=12 V、R=10 Ω，因此每一行用12−10I计算纵坐标。dI/dt横轴单位A/s，VL纵轴单位V。

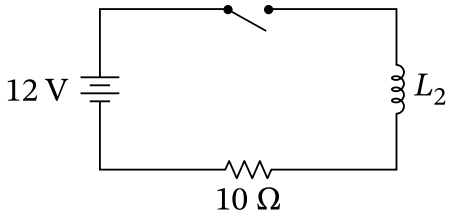
$$V_L = 12 - 10I = L\frac{dI}{dt}.$$

例题 2026 Q3 (c)(i)

The following information applies to parts C and D.

In Experiment 2, the students are asked to use a graph to determine the inductance L_2 of a new inductor. The inductor is connected to a $12\ \text{V}$ battery, a $10\ \Omega$ resistor, and an initially open switch, as shown in Figure 2.

Figure 2

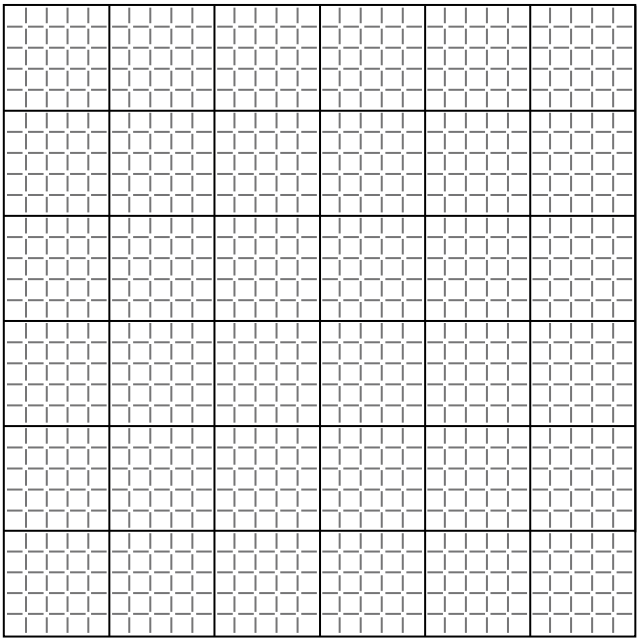


After the switch is closed, the current I in the circuit is measured at different times and the corresponding rate of change of the current, $\frac{dI}{dt}$, is determined. The collected data are shown in Table 1.

Table 1

I (A)	$\frac{dI}{dt}$ (A/s)
0.40	280
0.50	230
0.75	150
0.90	100
1.00	68

Part C



i. **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine L_2.

解：

C(i) 第二实验线性化沿电流方向绕回路，电源升压抵消电阻与电感压降：

$$12 - IR - L_2\frac{dI}{dt} = 0.$$

因此电感两端电压

$$V_L = 12 - 10I,$$

满足

$$V_L = L_2(dI/dt)。$$

取横轴

$$dI/dt$$

（A/s），纵轴

$$V_L = 12 - 10I$$

（V），斜率就是L_2。

计算后的数据：按一组坐标逐行描点

先沿前问确定的坐标计算每一行VL，再把(dI/dt,VL)画到同一幅图。横纵轴均匀标数值刻度和单位，范围覆盖所有点。

与原表保持逐行对应，不把某一行的I与另一行的变化率配对。

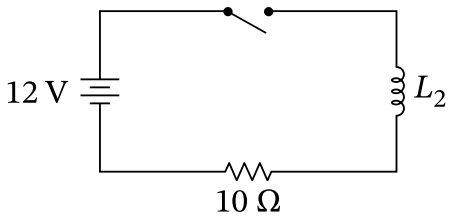
例题 2026 Q3 (c)(ii)

本问承接 2026 Q3 (c)(i)。

The following information applies to parts C and D.

In Experiment 2, the students are asked to use a graph to determine the inductance L_2 of a new inductor. The inductor is connected to a $12\ \text{V}$ battery, a $10\ \Omega$ resistor, and an initially open switch, as shown in Figure 2.

Figure 2



After the switch is closed, the current I in the circuit is measured at different times and the corresponding rate of change of the current, $\frac{dI}{dt}$, is determined. The collected data are shown in Table 1.

Table 1

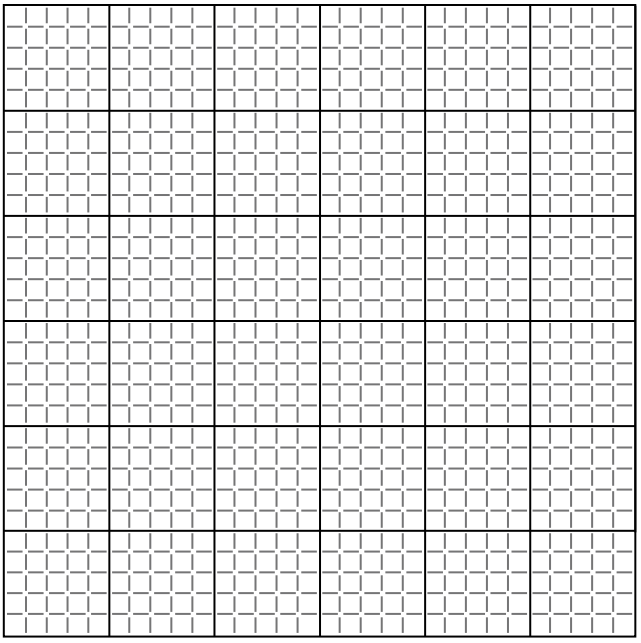
I (A)	$\frac{dI}{dt}$ (A/s)
0.40	280
0.50	230
0.75	150
0.90	100
1.00	68

Part C

to determine L_2.

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Clearly **label** each axis with a numerical scale.
- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work and will not be scored.



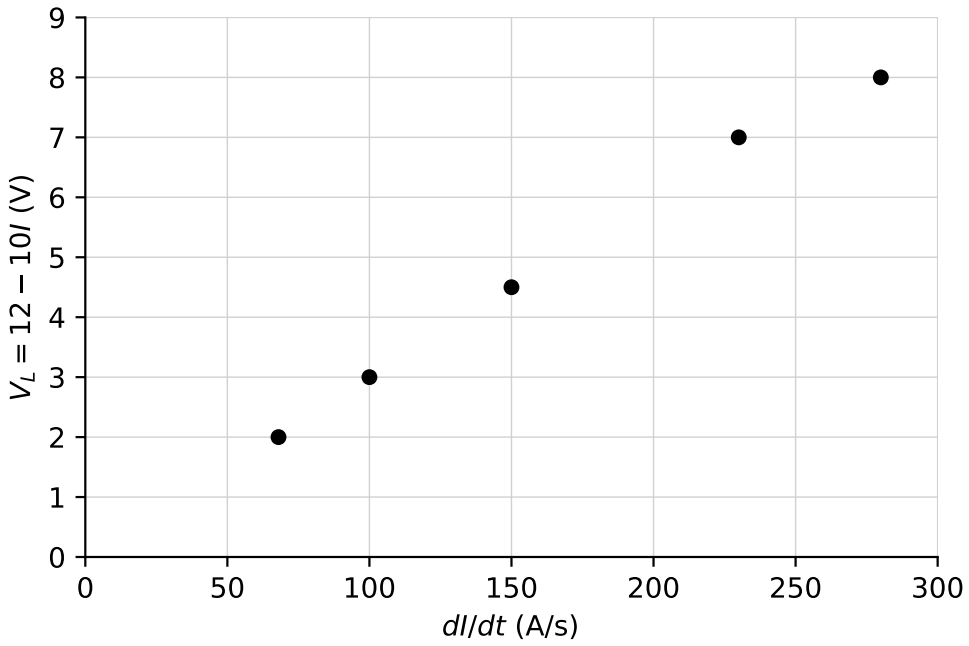
解：

C(ii) 计算与作图

I (A)	dI/dt (A/s)	$V_L = 12 - 10I$ (V)
0.40	280	8.0
0.50	230	7.0

I (A)	dI/dt (A/s)	$V_L = 12 - 10I$ (V)
0.75	150	4.5
0.90	100	3.0
1.00	68	2.0

横轴可标0-300 A/s，纵轴0-9 V；两轴均匀刻度，依次标出五点。



各点按原测量表逐行对应；此图只描点。

电感数据点：拟合整组线性趋势

电感电压与电流变化率的关系是直线。把所有点的共同趋势拟合为一条直线，点不必全在线上。不要逐点连折线；图上下一问用的是拟合线斜率，因此所画直线应覆盖足够大的横轴跨度。

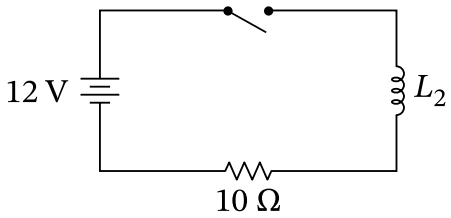
例题 2026 Q3 (c)(iii)

本问承接 2026 Q3 (c)(ii)。

The following information applies to parts C and D.

In Experiment 2, the students are asked to use a graph to determine the inductance L_2 of a new inductor. The inductor is connected to a $12\ \text{V}$ battery, a $10\ \Omega$ resistor, and an initially open switch, as shown in Figure 2.

Figure 2

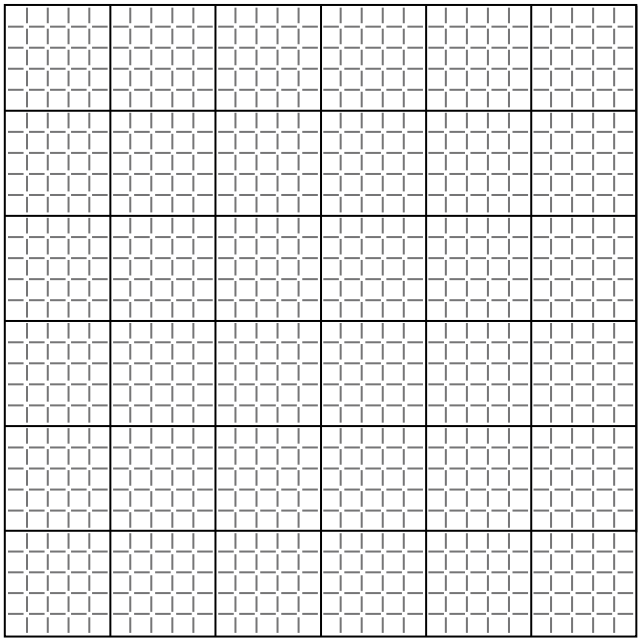


After the switch is closed, the current I in the circuit is measured at different times and the corresponding rate of change of the current, $\frac{dI}{dt}$, is determined. The collected data are shown in Table 1.

Table 1

I (A)	$\frac{dI}{dt}$ (A/s)
0.40	280
0.50	230
0.75	150
0.90	100
1.00	68

Part C



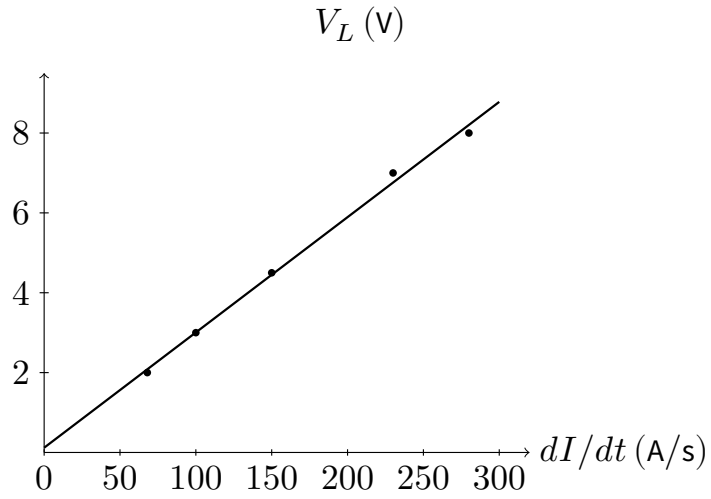
iii. **Draw** a best-fit line for the data plotted in part C (ii).

解：

C(iii) 最佳拟合线一条通过点群中间的直线即可；下图采用

$$V_L \approx 0.02887(dI/dt) + 0.119$$

的拟合。不要将数据点依次连成折线。



电压对变化率的斜率：计算电感数值

从拟合线上选相距较远的两点，纵轴差用伏特、横轴差用A/s，斜率单位V/(A/s)=H。

理论关系VL=L(dI/dt)使斜率直接等于电感L；本型无需像周期实验那样再除$4\pi^2$。

$$L = \frac{\Delta V_L}{\Delta(dI/dt)}.$$

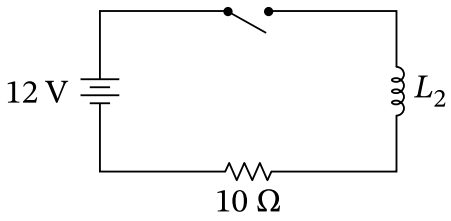
例题 2026 Q3 (d)

本问承接 2026 Q3 (c)(iii)。

The following information applies to parts C and D.

In Experiment 2, the students are asked to use a graph to determine the inductance L_2 of a new inductor. The inductor is connected to a $12\ \text{V}$ battery, a $10\ \Omega$ resistor, and an initially open switch, as shown in Figure 2.

Figure 2



After the switch is closed, the current I in the circuit is measured at different times and the corresponding rate of change of the current, $\frac{dI}{dt}$, is determined. The collected data are shown in Table 1.

Table 1

I (A)	$\frac{dI}{dt}$ (A/s)
0.40	280
0.50	230
0.75	150
0.90	100
1.00	68

Part D

Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for L_2.

解：

D　电感的数值斜率单位为

$$\mathrm{V}/(\mathrm{A}/\mathrm{s}) = \mathrm{H}。$$

在拟合线上用相距较远的两点

$$(100, 3.006)$$

与

$$(280, 8.203),$$

有

$$L_2 = \frac{8.203 - 3.006}{280 - 100}\,\mathrm{H} \approx \boxed{0.0289\,\mathrm{H} \approx 0.029\,\mathrm{H}}.$$

每点的

$$V_L/(dI/dt)$$

可用来检查数量级，却不能代替按题要求用最佳拟合线斜率计算。