AP Physics C · 力学

教师讲义

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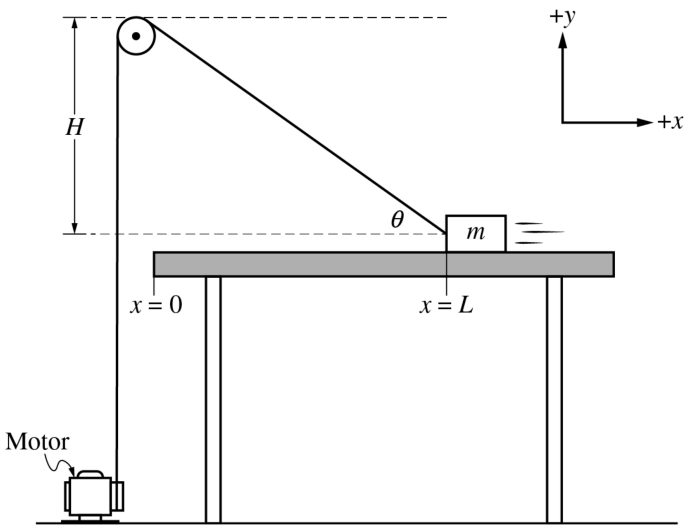
画出实际力，再选方向列式

滑动物体：实际力画一次，摩擦反对相对滑动

先确定画的是哪个物体，再逐个找与它接触或隔空作用的物体。桌面提供支持力，地球提供重力，绳提供张力；物块相对桌面滑动时，摩擦力反对这个相对滑动。

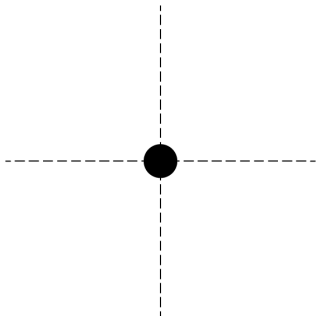
图中的F_T是张力、F_N是支持力、F_f是摩擦力、F_g是重力。箭头从题目指定的点出发。张力沿绳朝另一端；水平、竖直分量是同一支张力的投影，不能作为两支额外的力再画一次。

例题 2022 Set 1 Q1 (a)



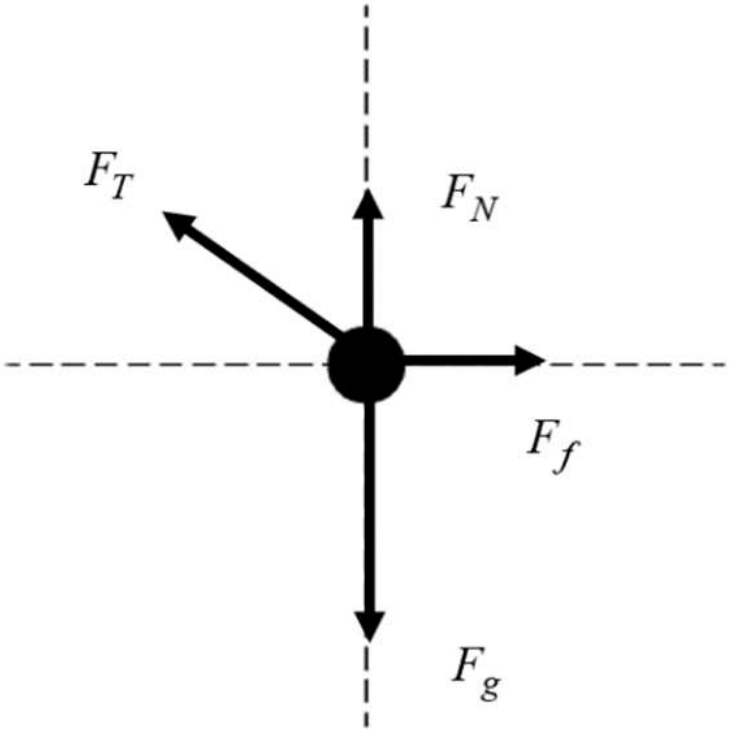
1. A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

(a) On the dot below that represents the block, draw and label the forces (not components) that act on the block when the block is at $x = L$. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



解：

物块向左滑，摩擦力向右；绳子把物块拉向滑轮，张力向左上。竖直另外有桌面的支持力向上、重力向下。把这四支实际力从物块画出，不再把张力的两个分量画成额外的力。

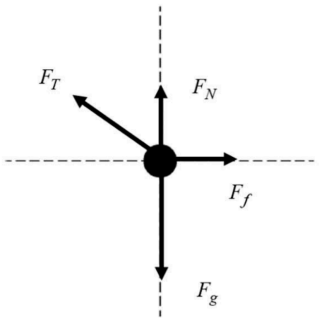


2022 Set 1 Question 1(a)：四个实际力。

官方评分

(a)	For correctly drawing and labeling the force of gravity and the normal force on the block of mass m	**1 point**
	For correctly drawing and labeling the force of friction on the block of mass m	**1 point**
	For correctly drawing and labeling the force of tension on the block of mass m	**1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth" $F_{Earth\ on\ block}$, $F_{E,Block}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force. F_{string}, F_s, F_T, $F_{Tension}$, T, "string force," "tension force," or similar labels may be used for the tension force exerted by the string.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) 3 points

练习 2022 Set 2 Q1 (a)

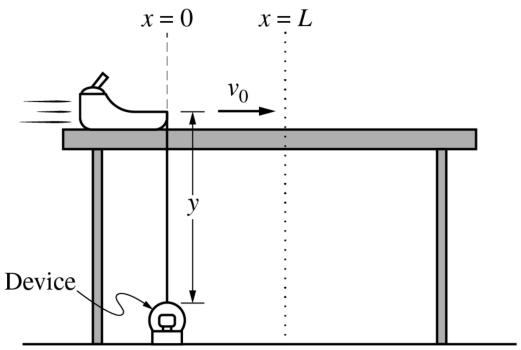


Figure 1

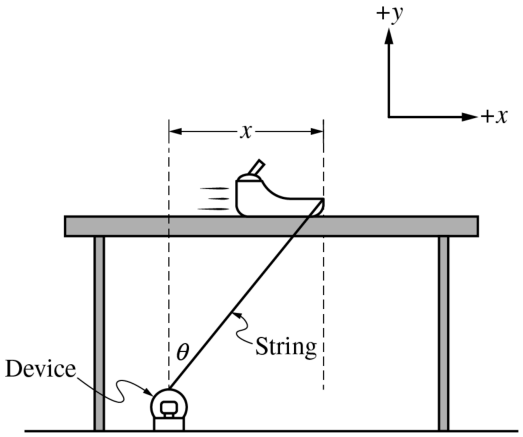
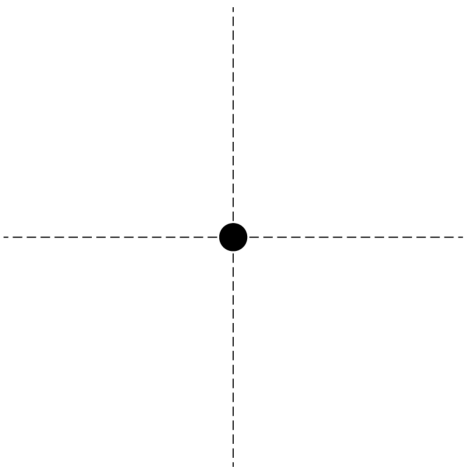


Figure 2

Note: Figures not drawn to scale.

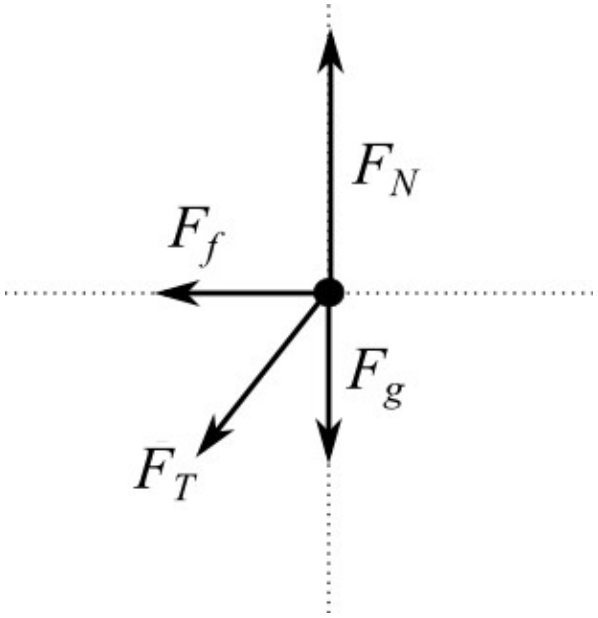
1. A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.

(a) On the dot below that represents the sled, draw and label the forces (not components) that are exerted on the sled a short time after $t = 0$ but before the sled has come to rest. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



解：

雪橇向右滑，摩擦向左；绳子连接右上方的雪橇与地面的装置，故张力向左下。支持力向上、重力向下。

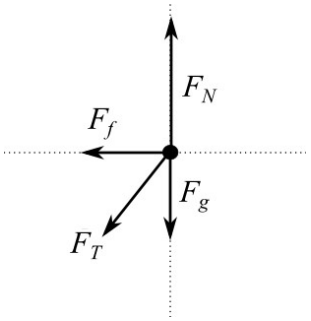


2022 Set 2 Question 1(a)：张力向下压，支持力增大。

官方评分

(a)	For correctly drawing and labeling the force of gravity and the normal force on the sled	**1 point**
	For correctly drawing and labeling the force of friction on the on the sled	**1 point**
	For correctly drawing and labeling the force of tension on the on the sled	**1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{E,Block}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force. F_{string}, F_s, F_T, $F_{Tension}$, T, "string force," "tension force," or similar labels may be used for the tension force exerted by the string.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) 3 points

比较力箭头：先读两条力—形变关系

题目要求箭头长度表示力的相对大小时，除了方向，还要用当前压缩量求两次的弹力。先把同一个压缩量分别代入两条原题关系，再按比值画水平箭头；质量没有变且竖直没有加速度，两次的重力和支持力相同。

例题 2023 Set 1 Q1 (a)

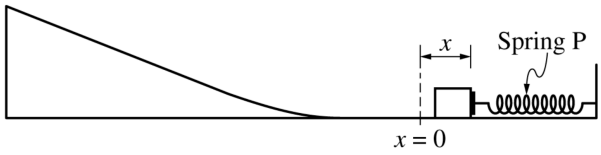


Figure 1

1. A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the maximum height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_{\text{P}}(x) = kx$, where $k = 100.0\,\text{N/m}$, and for Spring Q, $F_{\text{Q}}(x) = Cx^{1/2}$, where $C = 20.0\,\text{N/m}^{1/2}$.

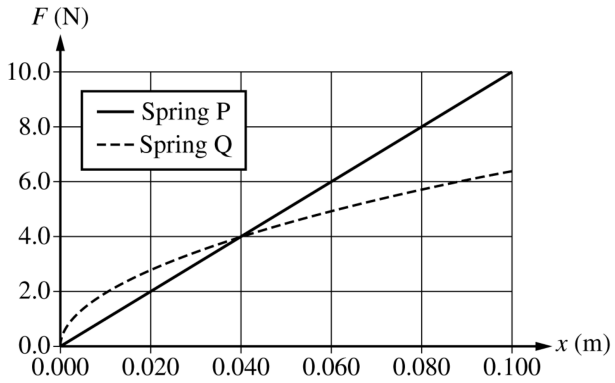
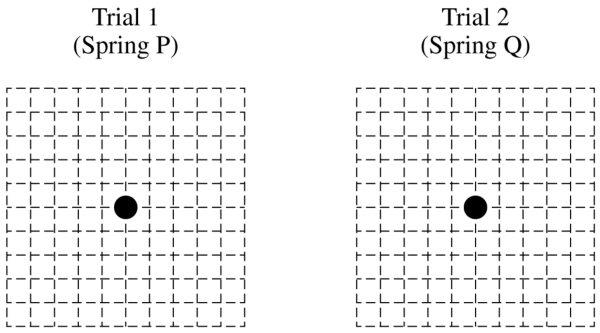


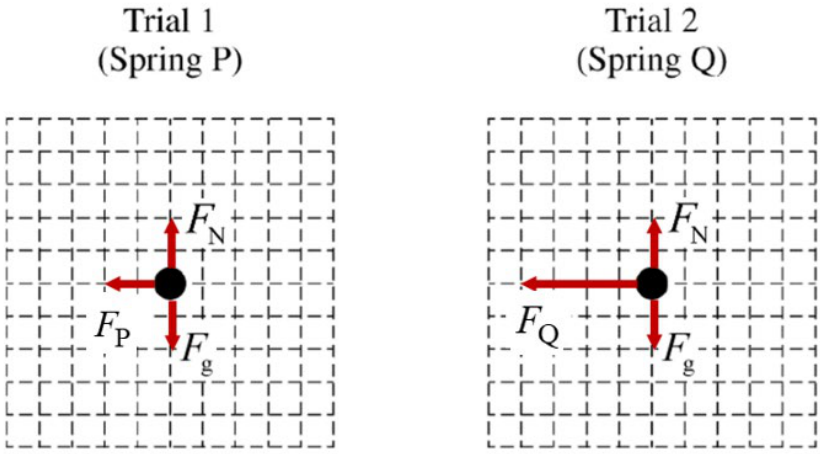
Figure 2

(a) For the experiment, the block is pushed against one of the springs, compressing the spring a distance $x = 0.010\,\text{m}$. The block is then released from rest. In Trial 1, Spring P is used, and in Trial 2, Spring Q is used. On the following dots, representing the block in Trials 1 and 2, draw and label the forces (not components) that are exerted on the block at the instant the block is released. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. The lengths of the horizontal vectors should represent the relative magnitude of the horizontal forces and the lengths of the vertical vectors should represent the relative magnitude of the vertical forces.



解：

两次都有重力向下、支持力向上、弹力向左。竖直两力长度相同。压缩0.010米时，P弹力为1牛，Q弹力为2牛，所以Q的水平箭头画成P的两倍。

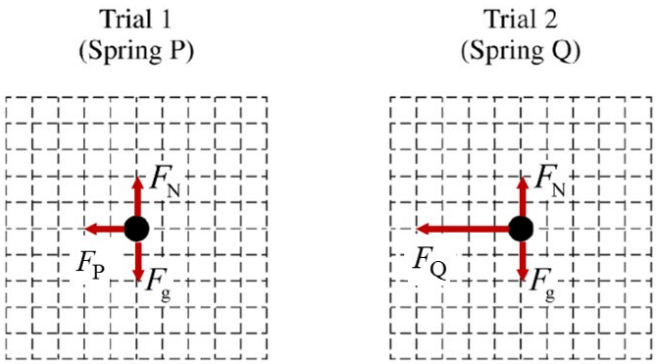


两种弹簧对同一物块的受力：Q弹力为P的两倍。

官方评分

(a)	For correctly drawing and labeling the force of gravity and the normal force on both dots	**1 point**
	For drawing and labeling the spring force to the left on both dots, where the force for Spring Q (Trial 2) is twice as long as the force for Spring P (Trial 1)	**1 point**

Example Solution



Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force", "F Earth on block", "F on block by Earth", $F_{\text{Earth on block}}$, $F_{E,Block}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force", "ground force", or similar labels may be used for the normal force.

Scoring Note: A response that includes extraneous vectors can earn a maximum of 1 point.

Total for part (a)　2 points

刚体受力：每个外力画在真实作用点

物体会转动时，箭头放在哪里也影响力矩。重力画在质心，绳力画在连接点，支点力画在支点。把挂块对杆的拉力和杆自己的重力分开；在已经松手的情境，不再画手的作用力。

例题 2024 Set 1 Q3 (a)

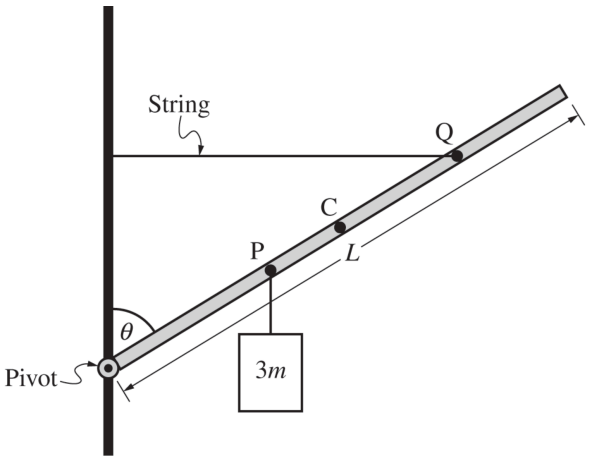
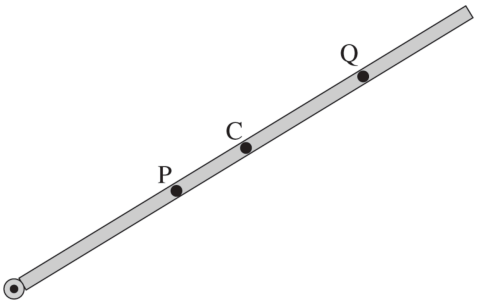


Figure 1

Note: Figure not drawn to scale.

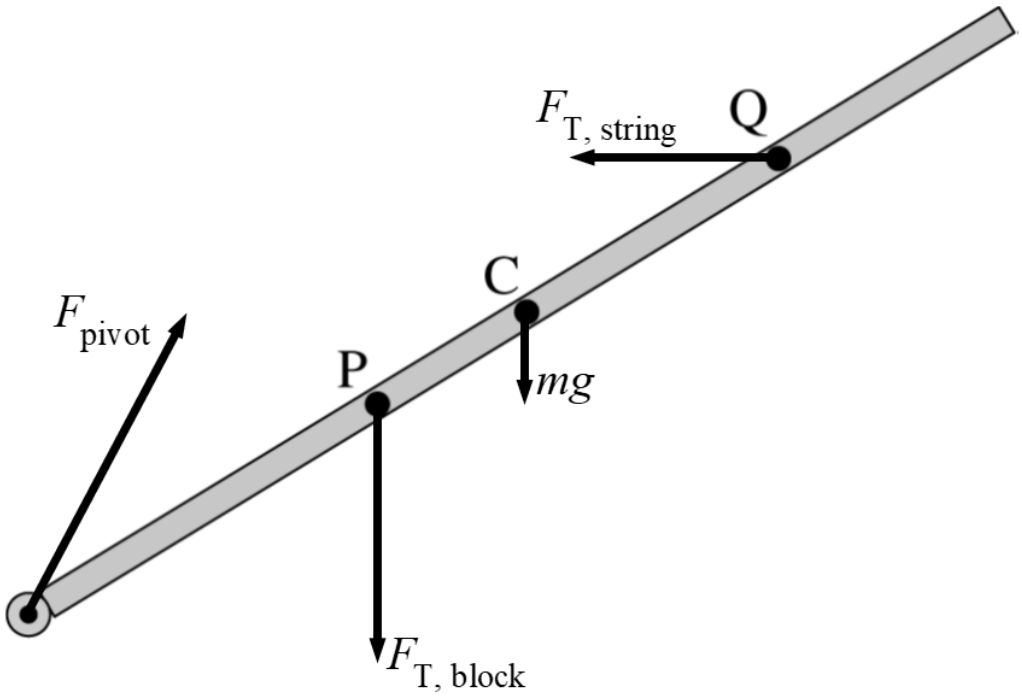
3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(a) On the following representation of the rod, **draw** and **label** the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.



解：

只画作用在杆上的外力：P点向下的块的拉力3mg，C点向下的杆重mg，Q点向左的绳张力，以及支点向右上的合支持力。支点力要从支点画起，不从质心画起。

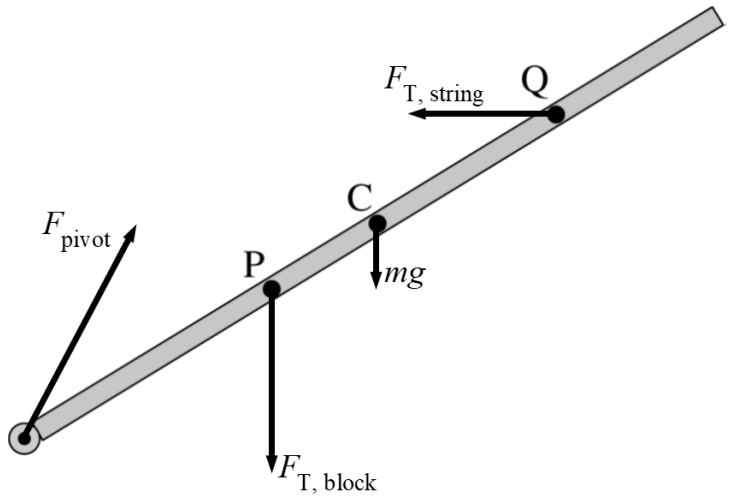


支点、P、C、Q保持原题的位置关系。

官方评分

(a)	For drawing and appropriately labeling the downward forces that are exerted on the rod at points P and C **Scoring Note:** Labeling the downward force of tension as F_{block}, $3mg$, or similar, may earn this point.	**1 point**
	For drawing and appropriately labeling a leftward force that is exerted on the rod at Point Q	**1 point**
	For drawing and appropriately labeling a force that is directed up and to the right that is exerted on the rod at the pivot, and no extraneous forces are present	**1 point**

Example Response



Scoring Note: Examples of appropriate labels for the force due to gravity include: F_{G}, F_{g}, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity.

Scoring Note: Examples of appropriate labels for the force from the pivot include: F_p, F_{pivot}, F_{n}, F_{N}, N, "normal force," "pivot force."

Scoring Note: Examples of appropriate labels for the tension force include F_t, F_{T}, T, F_{string}, and "Force from string."

Total for part (a) 3 points

练习 2022 Set 2 Q3 (a)

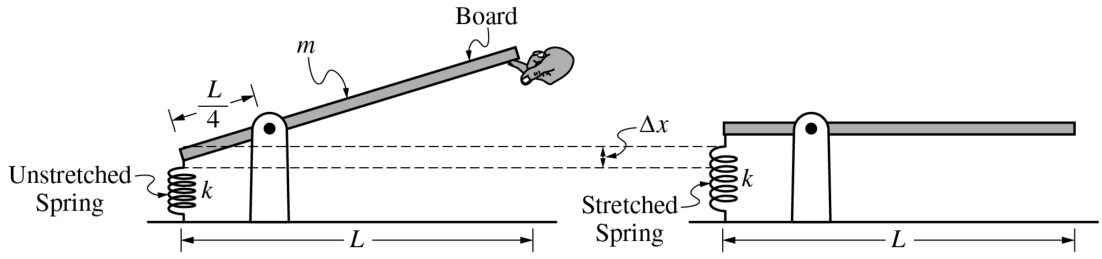
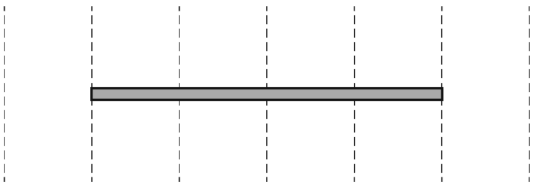


Figure 1 Figure 2

Note: Figures not drawn to scale.

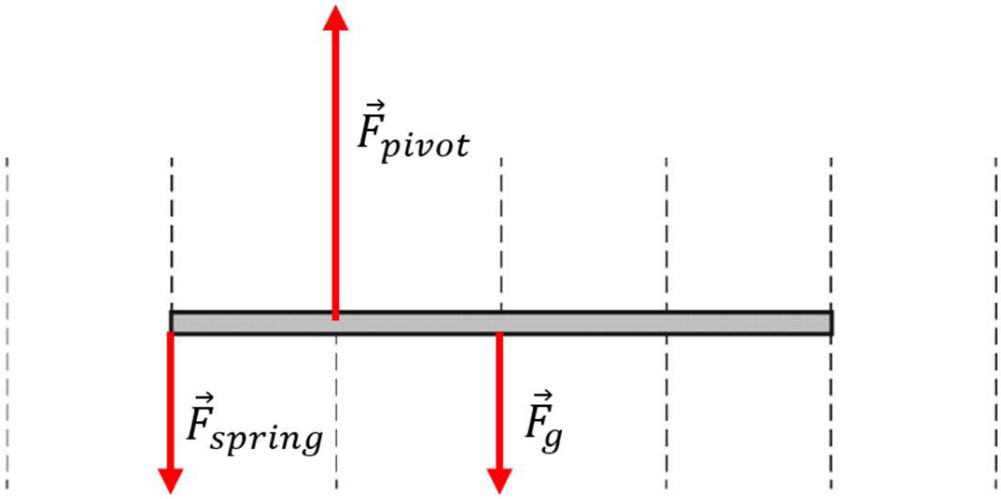
3. A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I.

(a) On the rectangle below, which represents the board, draw and label the forces (not components) that act on the board while the board-spring system is in equilibrium. Each force should be represented by an arrow that starts on, and points away from, the board, and should represent the location at which that force acts.



解：

木板左端弹簧把板向下拉，板重在质心向下，支点的支持力向上。三支力按实际作用点画；弹簧拉力不是向上托力。

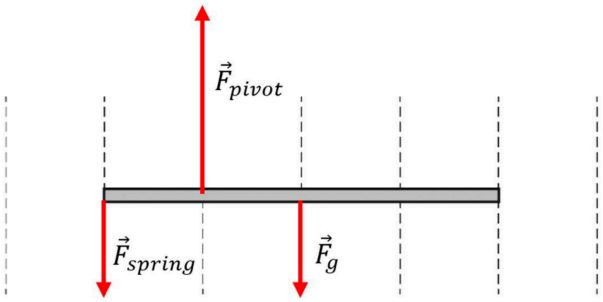


支点距左端四分之一板长，质心距支点也为四分之一板长。

官方评分

(a)	For correct location and label of the force exerted on the board by the spring	**1 point**
	For correct location and label of the force exerted on the board by gravity	**1 point**
	For correct location and label of the force exerted on the board by the pivot	**1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{E,Block}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force, which can be used instead of F_{pivot}. F_{spring}, F_S, "spring force," or similar labels may be used for the force exerted by the spring.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) 3 points

选定系统：内部相互作用不画成外力

盘与黏土一起作为系统时，二者相互接触的力成对留在系统内部，不出现在系统外力图上。画地球、绳和轴对这个系统的作用。若题目剪断一条绳，剪断侧的张力已经消失，要按剪断后的情境重新列力。

例题 2024 Set 2 Q3 (a)

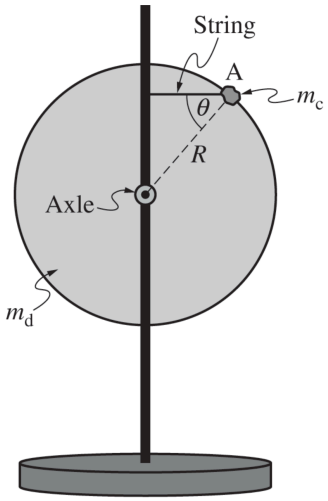
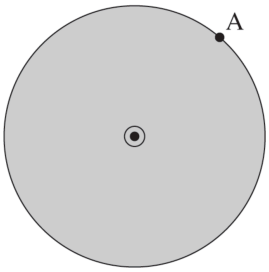


Figure 1

Note: Figure not drawn to scale.

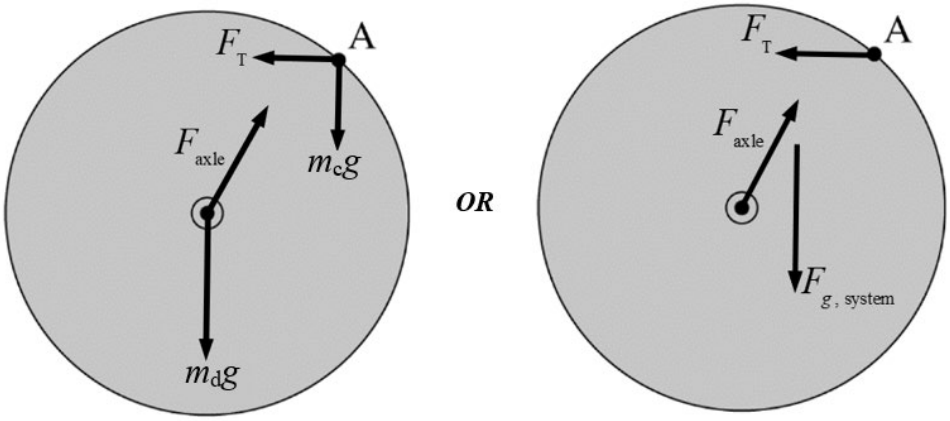
3. A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(a) On the following representation of the clay-disk system, **draw** and **label** the external forces (not components) exerted on the system. Each force must be represented by a distinct arrow that starts on, and points away from, the point at which the force is exerted on the system.



解：

盘与黏土作为整体，二者之间的力为内部力，不画。外力是中心的盘重向下、A处黏土重向下、A处张力向左，以及轴的支持力向右上。

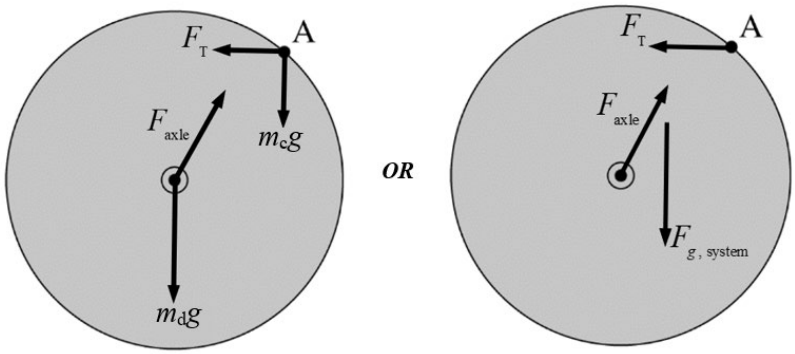


盘重过轴，黏土重的力臂取决于黏土位置。

官方评分

(a)	For drawing and appropriately labeling separate gravitational forces that are exerted on the disk and the lump of clay and exerted at the correct locations **Scoring Note:** Drawing a downward gravitational force exerted on the clay-disk system at the correct location can earn this point.	**1 point**
	For drawing and appropriately labeling a leftward force exerted on the system at Point A	**1 point**
	For drawing and appropriately labeling a force directed up and right that is exerted on the system at the axle, and no extraneous forces are present	**1 point**

Example Responses



Scoring Note: Examples of appropriate labels for the gravitational force include F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on disk," "F on disk by Earth," $F_{Earth\ on\ Disk}$, $F_{E,Disk}$, and $F_{Disk,E}$. The labels G or g are not appropriate labels for the gravitational force.

Scoring Note: Examples of appropriate labels for the normal force from the axle include F_N, F_{axle}, N, "normal force," and "axle force."

Scoring Note: Examples of appropriate labels for the tension force include F_t, F_T, T, F_{string}, and "Force from string."

Total for part (a)	**3 points**

练习 2022 Set 1 Q3 (b)

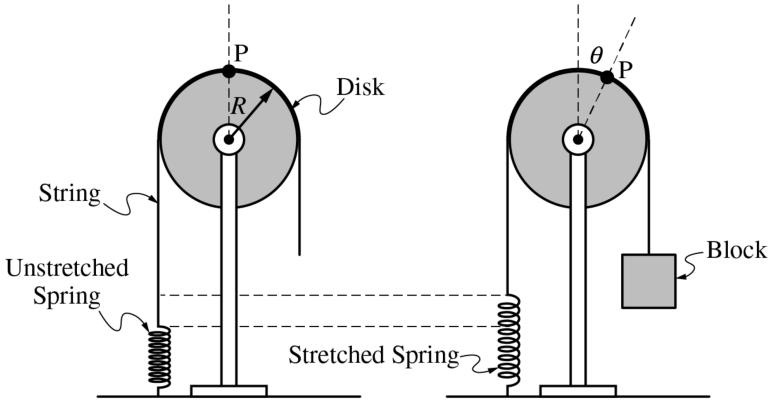


Figure 1 Figure 2

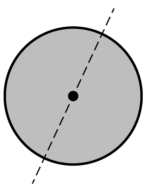
Note: Figures not drawn to scale.

3. A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2} M_d R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

   A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

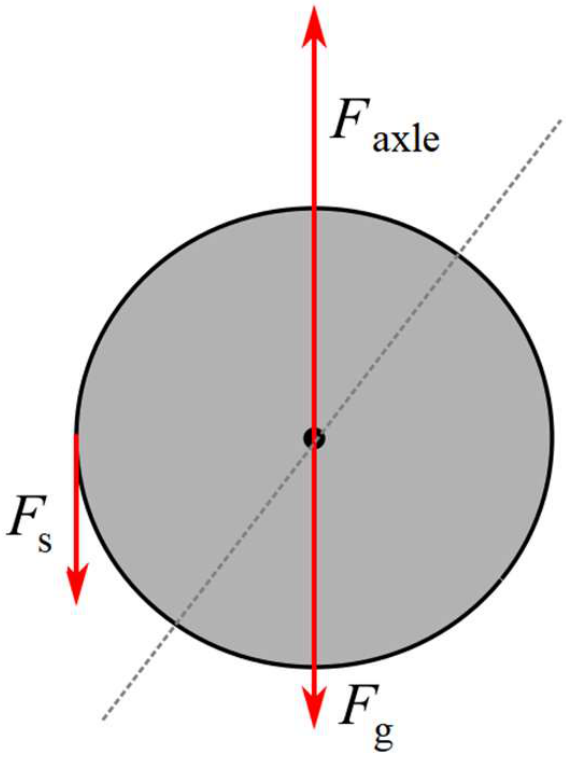
   Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(b) At time $t = 0$, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.



解：

剪去右绳后右张力消失。盘仍受左绳的切向拉力、中心向下的盘重、中心轴力；轴力保证盘心不平动。三支力画在各自作用点。

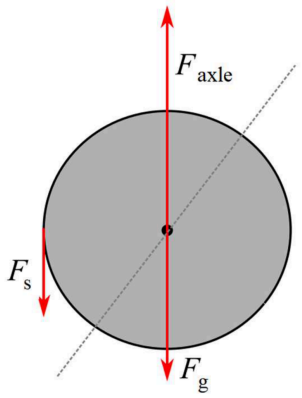


剪绳后的盘受力；不要保留已经消失的右侧张力。

官方评分

(b)	For drawing and labeling the force of the tension exerted on the disk anywhere between point P and the left edge of the disk, including point P and the left edge of the disk, tangent to the disk	**1 point**
	For correct location and label of the force due to gravity exerted on the disk, directed straight down	**1 point**
	For correct location and label of the force exerted on the disk by the axle, directed such that the disk remains in translational equilibrium (i.e., $\Sigma F = 0$)	**1 point**

Example Response



Scoring Notes:

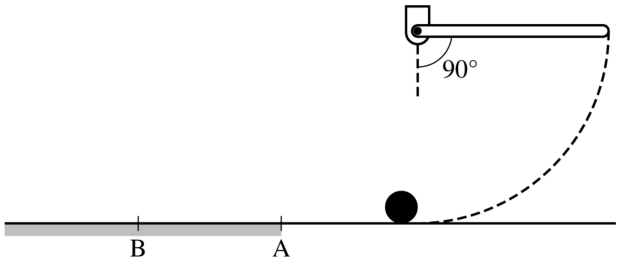
- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{Earth\ on\ block}$, $F_{E,Block}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force, which can be used instead of F_{axle}. F_{spring}, F_s, T_{spring}, T, "spring force," or similar labels may be used for the tension force exerted by the spring.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (b) 3 points

滑动球：摩擦画在接触点

球心向左运动但尚未开始转动时，接触点也相对地面向左滑，摩擦因此向右。重力作用在球心，支持力和摩擦作用在底部接触处；这正是摩擦能够同时改变平动和转动的原因。

例题 2023 Set 1 Q3 (c)



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

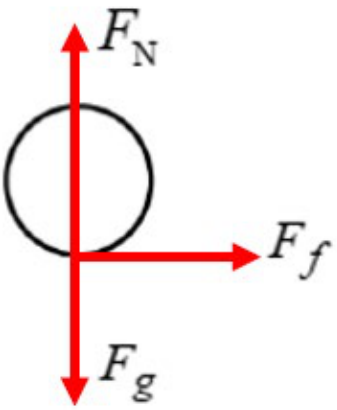
After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



解：

球向左滑，接触点也向左滑，动摩擦向右。重力从球心向下、支持力从底部向上、摩擦从底部向右。

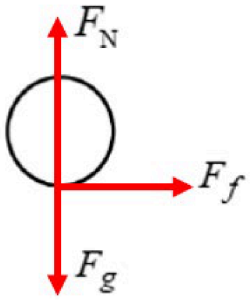


摩擦减小向左平动速率，同时使球逆时针转起来。

官方评分

(c)	For correctly drawing and labeling the force due to gravity and the normal force on the sphere	**1 point**
	For drawing the frictional force horizontally to the right at the bottom of the sphere	**1 point**

Example Solution



Scoring note: Examples of appropriate labels for the force due to gravity include: $F_G, F_g, F_{grav}, W, mg, Mg$, "grav force", "F Earth on sphere" , "F on sphere by Earth", $F_{\text{Earth on sphere}}$, $F_{\text{E,Sphere}}$, $F_{\text{Sphere,E}}$. The labels G or g are not appropriate labels for the force due to gravity. F_n , F_N , N , "normal force", "ground force", or similar labels may be used for the normal force.

Scoring Note: A response that includes extraneous vectors can earn a maximum of 1 point.

Scoring Note: Horizontally displacing the F_N and F_g vectors slightly is permitted in order to show the distinct points at which those forces are exerted on the sphere.

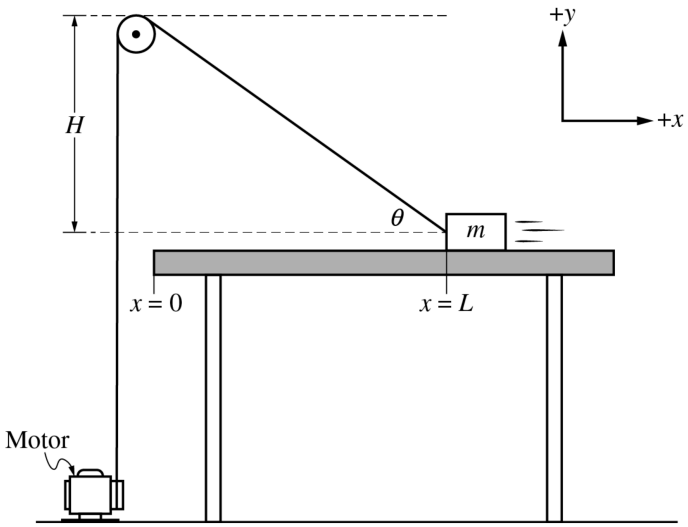
Total for part (c)	**2 points**

绳角：先找对边与邻边

绳、水平距离和竖直高度形成直角三角形。角从水平量起时，对边是高度、邻边是水平距离；角从竖直量起时，两边角色交换。先写正切等于对边除邻边，再用反正切求角。反正切的上标−1表示反函数，不是倒数。

$$\tan\theta = \frac{\text{对边}}{\text{邻边}}.$$

例题 2022 Set 1 Q1 (b)



1. A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

(b) Derive an expression for the angle θ that the string makes with the horizontal as a function of x.

解：

绳子、水平距离x和高度H构成直角三角形，题目的角从水平量起。因此对边是H、邻边是x。

$$\tan\theta = \frac{H}{x}, \qquad \theta = \tan^{-1}\frac{H}{x}.$$

官方评分

(b)	For a trigonometric expression relating the angle to the horizontal distance of the block from the left corner of the table, x.	**1 point**
	For any correct trigonometric expression for θ in terms of the given quantities	**1 point**

Example Responses

First Point	Second Point
$\sin\theta = \dfrac{H}{\sqrt{H^2 + x^2}}$	$\theta = \sin^{-1}\left(\dfrac{H}{\sqrt{H^2 + x^2}}\right)$
$\cos\theta = \dfrac{x}{\sqrt{H^2 + x^2}}$	$\theta = \cos^{-1}\left(\dfrac{x}{\sqrt{H^2 + x^2}}\right)$
$\tan\theta = \dfrac{H}{x}$	$\theta = \tan^{-1}\left(\dfrac{H}{x}\right)$

Total for part (b) 2 points

练习 2022 Set 2 Q1 (b)

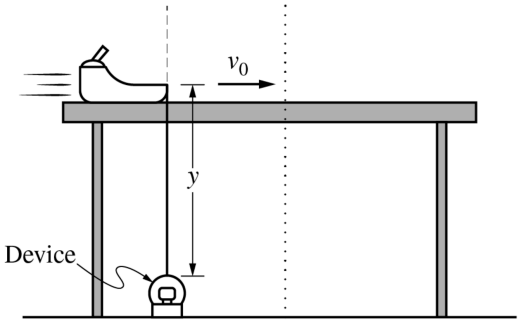


Figure 1

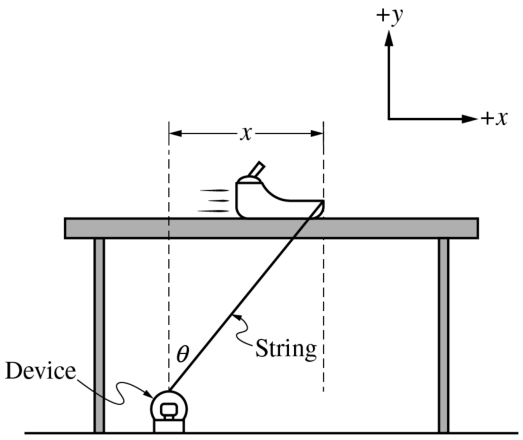


Figure 2

Note: Figures not drawn to scale.

1. A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.

(b) Determine an expression for the angle θ that the string makes with the vertical when the sled has traveled a horizontal distance x.

解：

这次角从竖直量起，不能照搬前题的正弦、余弦。直角三角形的水平边x是对边，竖直边y是邻边。

$$\tan\theta = \frac{x}{y}, \qquad \theta = \tan^{-1}\frac{x}{y}.$$

官方评分

(b)	For any correct trigonometric expression for θ in terms of the given quantities	**1 point**
	$\sin\theta = \dfrac{x}{\sqrt{y^2+x^2}}$ $\quad\therefore \theta = \sin^{-1}\left(\dfrac{x}{\sqrt{y^2+x^2}}\right)$	
	OR	
	$\cos\theta = \dfrac{y}{\sqrt{y^2+x^2}}$ $\quad\therefore \theta = \cos^{-1}\dfrac{y}{\sqrt{y^2+x^2}}$	
	OR	
	$\tan\theta = \dfrac{x}{y}$ $\quad\therefore \theta = \tan^{-1}\dfrac{x}{y}$	
	Total for part (b)	**1 point**

无竖直加速度：求实际支持力

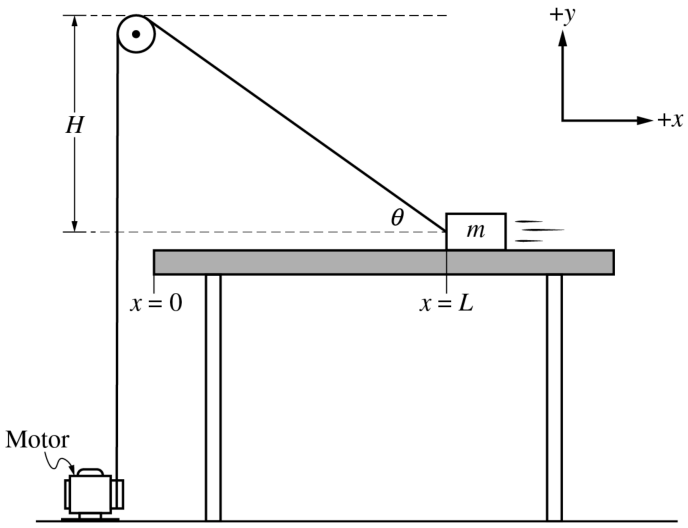
物体一直贴着水平桌面，只说明竖直加速度为零。斜向上的张力帮桌子托住物体，支持力会减小；斜向下的张力加重桌子的负担，支持力会增大。

物体质量为m，重力加速度为g，重力大小是mg。先取向上为正，写向上的力减去向下的力等于零。再从原题直角三角形求张力的竖直分量，不把支持力预先写成mg。

$$\sum F_y = 0.$$

例题 2022 Set 1 Q1 (c)(i)

本问承接 2022 Set 1 Q1 (b)。



1. A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

(c)

i. Derive an expression for the normal force F_N exerted on the block by the table as a function of the block's position x.

解：

物块没有竖直加速度。取向上为正，把向上的两力减去重力；绳长是直角三角形的斜边。

$$\sum F_y = 0, \qquad F_N + F_T \sin\theta - mg = 0.$$

$$F_N = mg - \frac{F_T H}{\sqrt{H^2 + x^2}}.$$

张力的竖直分量帮忙托住物块，所以支持力小于重力。题设始终接触桌面，对应支持力不能成为负值。

官方评分

(c)(i)	For using Newton's second law to sum the forces in the vertical direction and write an equation that is consistent with part (a) $\Sigma F_y = ma$ $F_{net,y} = F_N + F_{T,y} - F_g = 0$ $F_N = F_g - F_{T,y}$	**1 point**
	For correctly substituting the vertical component of the tension in terms of the given variables consistent with part (b) $F_N = mg - F_T \sin\theta$ $F_N = mg - F_T\left(\frac{H}{\sqrt{H^2 + x^2}}\right)$	**1 point**

练习 2022 Set 2 Q1 (c)(i)

本问承接 2022 Set 2 Q1 (b)。

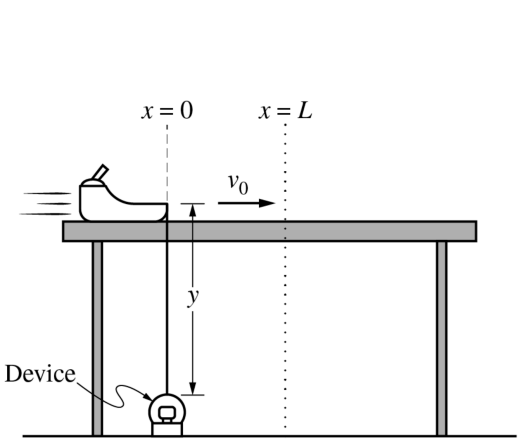


Figure 1

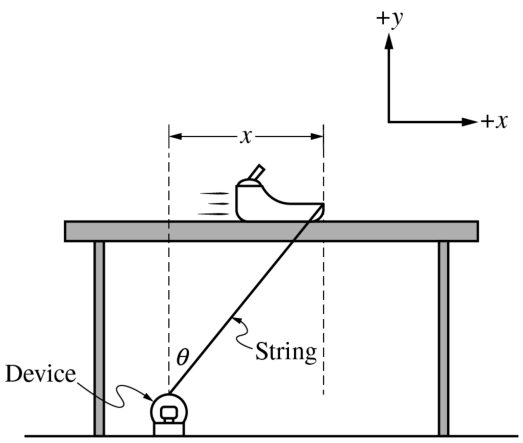


Figure 2

Note: Figures not drawn to scale.

1. A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.

(c)

i. Derive an expression for the normal force F_N exerted on the sled by the table as a function of the position x.

解：

向上为正，没有竖直加速度。向下的张力分量与重力都必须由支持力抵消。

$$\sum F_y = 0, \qquad F_N - mg - F_T \cos\theta = 0.$$

$$F_N = mg + \frac{F_T y}{\sqrt{x^2 + y^2}}.$$

官方评分

(c)(i)	For beginning the derivation with Newton's second law to write an equation that is consistent with part (a). $\Sigma F = ma$ $F_N - F_{T,y} - F_g = 0$ $F_N = F_{T,y} + F_g$ **Scoring Note:** Derivation must start with a statement of Newton's second law to earn this point.	**1 point**
	For correctly substituting the vertical component of the tension in terms of the given variables consistent with part (b) $F_N = mg + F_T \cos\theta$ $F_N = mg + F_T\left(\frac{y}{\sqrt{y^2 + x^2}}\right)$	**1 point**

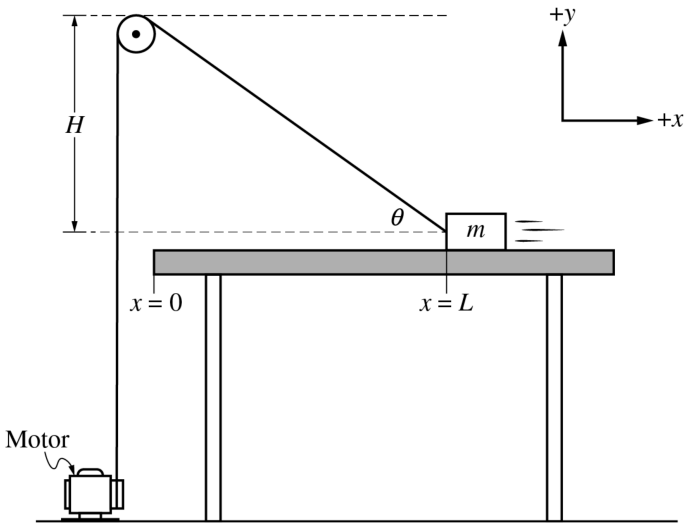
水平合力：先定符号，再代入实际摩擦

动摩擦系数μ_k描述这对接触表面的滑动摩擦强弱，动摩擦的大小是这个系数乘实际支持力。支持力已经由竖直受力求出，再沿水平方向把张力投影和摩擦相加。两力同向要相加，反向要相减；所求是大小时，最后取带方向结果的绝对值。

$$f_k = \mu_k F_N, \qquad F_{\text{net}} = \left|\sum F_x\right|.$$

例题 2022 Set 1 Q1 (c)(ii)

本问承接 2022 Set 1 Q1 (b)、2022 Set 1 Q1 (c)(i)。



1. A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

(c)

ii. Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the block as a function of the position x.

解：

沿运动方向向左为正，张力水平分量为正，摩擦为负。若合力实际反向，所求大小应再取绝对值。

$$F_{\text{left}} = \frac{F_T x}{\sqrt{H^2 + x^2}} - \mu_k\left(mg - \frac{F_T H}{\sqrt{H^2 + x^2}}\right), \qquad F_{\text{net}} = |F_{\text{left}}|.$$

官方评分

(c)(ii)	For using Newton's second law to sum the forces in the horizontal direction and write an equation that is consistent with part (a) $F_{net,x} = F_{T,x} - F_f$	**1 point**
	For correctly substituting the horizontal component of the force of tension in terms of the given variables consistent with part (b) $F_{net,x} = F_T \cos\theta - F_f$ $F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2 + x^2}}\right) - F_f$	**1 point**
	For correctly substituting the expression for the normal force from part (c)(i) into the expression for the force of friction $F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2 + x^2}}\right) - \mu_k F_N$ $F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2 + x^2}}\right) - \mu_k\left(mg - F_T\left(\frac{H}{\sqrt{H^2 + x^2}}\right)\right)$	**1 point**
		Total for part (c) 5 points

练习 2022 Set 2 Q1 (c)(ii)

本问承接 2022 Set 2 Q1 (b)、2022 Set 2 Q1 (c)(i)。

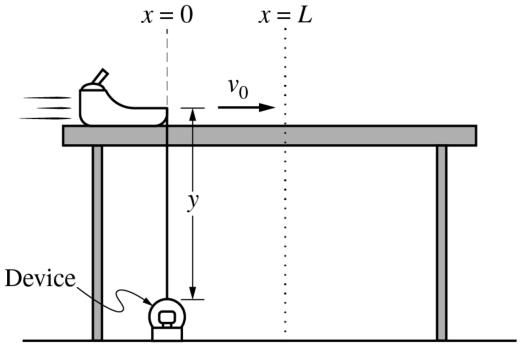


Figure 1

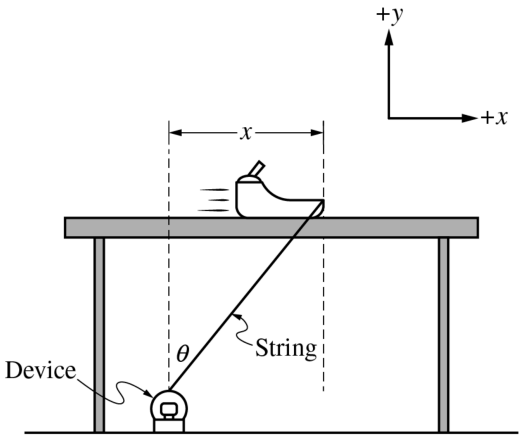


Figure 2

Note: Figures not drawn to scale.

1. A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.

(c)

ii. Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the sled as a function of the position x.

解：

两个水平力都朝左，因此合力大小是它们相加，不是相减。

$$F_{\text{net}} = \frac{F_T x}{\sqrt{x^2+y^2}} + \mu_k \left(mg + \frac{F_T y}{\sqrt{x^2+y^2}} \right).$$

若取向右为正，带方向的水平合力是以上表达式的负值。

官方评分

(c)(ii)	For summing the forces in the horizontal direction, consistent with parts (a) and (b) $F_{net} = -F_{T,x} - F_f$	**1 point**
	For correctly substituting the horizontal component of the force of tension in terms of the given variables $F_{net} = -F_T \sin\theta - F_f$ $F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - F_f$	**1 point**
	For correctly substituting the expression for the normal force from part (c)(i) into the expression for the force of friction $F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - \mu_k F_N$ $F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - \mu_k\left(mg + F_T\left(\frac{y}{\sqrt{y^2 + x^2}}\right)\right)$	**1 point**
		Total for part (c) 5 points

单个物体的加速度：只用它受到的水平力

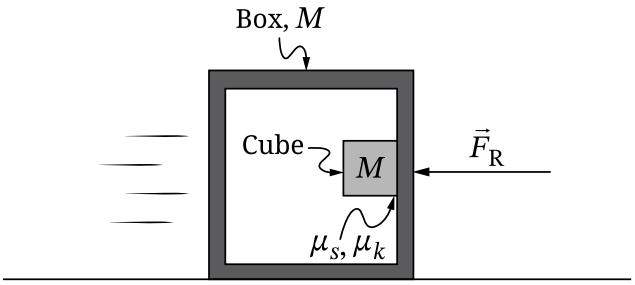
贴在箱壁上的小块没有竖直运动时，向上的摩擦抵消重力。水平只有箱壁的法向力。选小块本身而不是整个箱—块系统，用小块的质量把水平合力除开。

$$\sum F_x = ma_x.$$

例题 2026 Q1 (A)(i)

1. A box of mass M slides to the right on a horizontal surface. A small cube, also of mass M, is inside the box. The cube is against the right wall of the box and above the bottom of the box, as shown in Figure 1. The coefficients of static and kinetic friction between the cube and the wall of the box are μ_s and μ_k, respectively, where $\mu_s > \mu_k$.

Figure 1



The speed of the box is decreasing as a result of a resistive force that is exerted on the box. The resistive force $\vec{F}_{\mathrm{R}}$ is modeled by $\vec{F}_{\mathrm{R}} = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the box. Friction between the box and horizontal surface is negligible.

Consider the motion of the box and cube for the following times.

- At time $t = 0$, the box-cube system slides to the right with speed v_0.
- For $0 \le t < t_{\mathrm{crit}}$, the cube remains in contact with the wall of the box at a constant height above the bottom of the box.
- At $t = t_{\mathrm{crit}}$, the cube begins to slide downward along the wall of the box.

Part A

The magnitude of the normal force exerted on the cube by the wall of the box is F_N.

i. **Determine** an expression for the magnitude of the acceleration of the cube. Express your answer in terms of M, F_N, and physical constants, as appropriate.

解：

取地面为惯性参考系，向右为正。未下滑时小块没有竖直加速度，水平方向只有墙的法向力向左。题问加速度大小，所以最后取正值。

$$\sum F_x = Ma_x = -F_N, \qquad |a_x| = \frac{F_N}{M}.$$

官方评分

A	**(i)**	For determining a correct expression for the magnitude of the horizontal acceleration of the cube	**Point A1**

Example Response

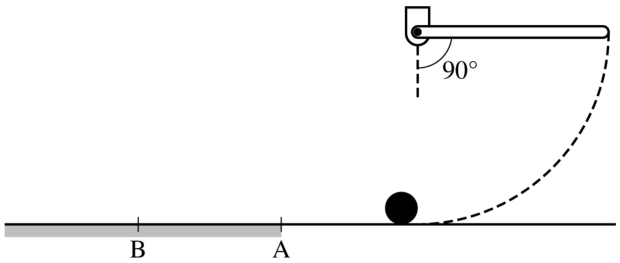
$a_{\text{cube}} = \dfrac{F_N}{M}$

滑动加速：先求恒定摩擦，再积分速度

球仍在滑动时，动摩擦大小不随时间改变。向左取正则向右摩擦带负号；由牛顿第二定律得到恒定加速度。速度的变化是加速度在时间上的累积；初速度由原题的v_0给出。

$$v(t) = v(0) + \int_0^t a\,dt.$$

例题 2023 Set 1 Q3 (d)(i)



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

(d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

i. The linear velocity v of the center of mass of the sphere as a function of time t

解：

取向左为平动正方向，摩擦为负；支持力等于重力，所以动摩擦恒定。

$$ma = -\mu mg, \qquad a = -\mu g, \qquad v(t) = v_0 - \mu g t.$$

官方评分

(d)(i) For using Newton's second law in the horizontal direction **1 point**

Example Response

$\Sigma F = ma$

$-\mu mg = ma$

For a correct derivation of acceleration **1 point**

Example Response

$a = -\mu g$

For a correct expression for the velocity **1 point**

Example Response

$\therefore v = v_0 - \mu g t$

Example Solution

$\Sigma F = ma$

$-\mu mg = ma$

$a = -\mu g$

$v = v_0 + at$

$\therefore v = v_0 - \mu g t$

Scoring Note: Only the final expression for velocity must have correct signs.

变力做功与储存的能量

力—位移图：用面积累积做功

把运动分成短小路段，每段的功约等于沿位移方向的力乘那段距离。很多小段相加，就成为力—位移图下的面积。力随形变改变时，不能只取终点的力再乘总距离。

$$W = \int F_x \, dx.$$

例题 2023 Set 1 Q1 (b)(i)

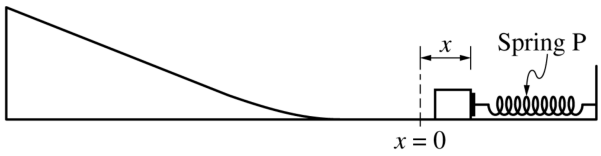


Figure 1

1. A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the maximum height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_{\text{P}}(x) = kx$, where $k = 100.0\ \text{N/m}$, and for Spring Q, $F_{\text{Q}}(x) = Cx^{1/2}$, where $C = 20.0\ \text{N/m}^{1/2}$.

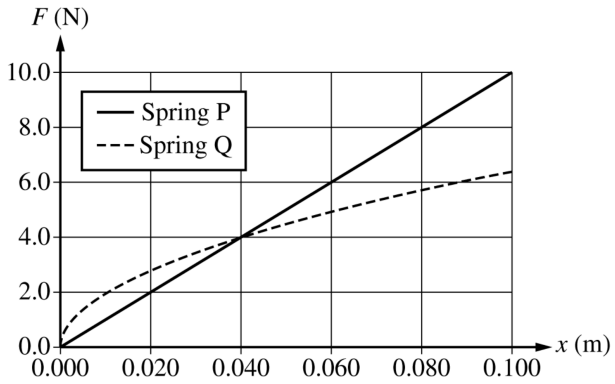


Figure 2

i. What feature(s) of the graph in Figure 2 could be used to estimate the work done on the block by each spring as each spring is compressed?

解：

力—压缩量图与横轴围出的面积是弹簧储能，也就是释放时能做的功。

官方评分

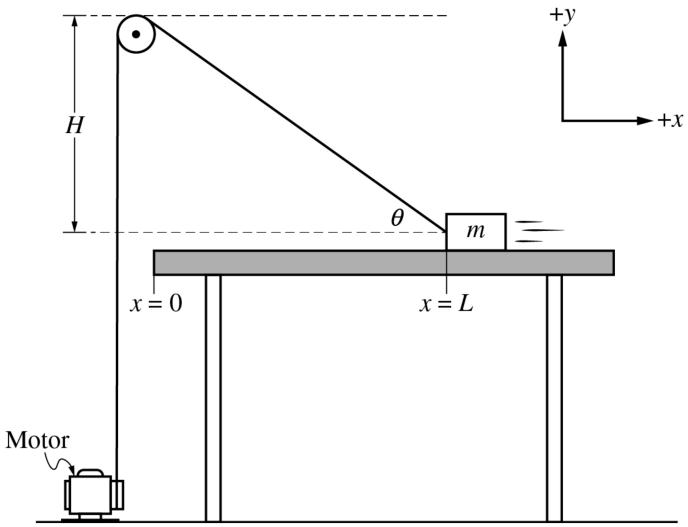
(b)(i)	For a statement that the work is equal to the area under the curve	**1 point**

变向张力：建立带方向的功积分

物块水平移动，所以绳的竖直分量不做功。坐标x向右为正时，向左的张力分量带负号；运动从L到0，积分的起点和终点也必须按这个方向写。只建立积分的题，到完整积分式为止，不额外求值。

例题 2022 Set 1 Q1 (d)

本问承接 2022 Set 1 Q1 (b)。



1. A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

(d) Write, but do not solve, an integral expression that could be used to solve for the work W done by the string on the block as the block moves from $x = L$ to $x = 0$.

解：

做功积分改用原题位置坐标x，坐标正向朝右。因此向左的张力水平分量带负号，运动的积分上限又小于下限，两者使功为正。题目要求只建立积分，不求值。

$$W = \int_L^0 -\frac{F_T x}{\sqrt{H^2 + x^2}}\, dx.$$

官方评分

(d)	For any indication that the work done on the block by the string is due only to the horizontal component of the tension in the string $W = \int F_{T,x} dx$	**1 point**
	For using the horizontal component of the force of tension consistent with part (c)	**1 point**
	For indicating that work is the integral of the force with respect to x, including limits or a constant of integration $W = \int_{x=L}^{x=0} -F_T \left(\frac{x}{\sqrt{H^2 + x^2}} \right) dx$	**1 point**
	Total for part (d)	**3 points**

根式分量：找到原函数再代上下限

若水平分量包含x除以根号(x^2+常数^2)，根式本身的导数正好产生这个比值。先检验这个导数，再把常量力提到积分外；求定积分时使用终点的原函数减去起点的原函数。

$$\frac{d}{dx}\sqrt{x^2 + y^2} = \frac{x}{\sqrt{x^2 + y^2}}.$$

例题 2022 Set 2 Q1 (d)

本问承接 2022 Set 2 Q1 (b)。

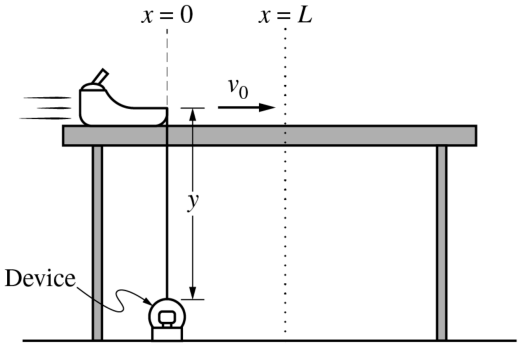


Figure 1

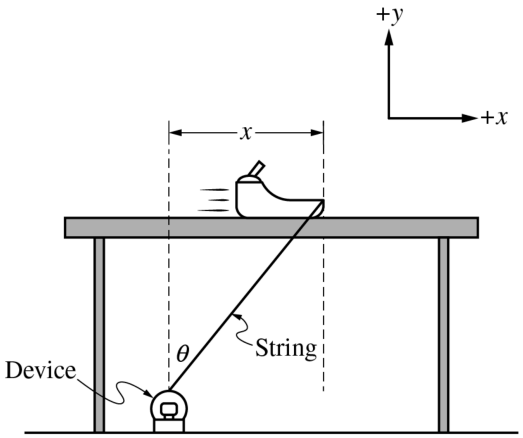


Figure 2

Note: Figures not drawn to scale.

1. A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.

(d) Derive an expression for the work W done by the string on the sled as the sled moves from $x = 0$ to $x = L$.

解：

雪橇的位移向右，张力水平分量向左，故张力做负功。根式的导数恰是积分中的比值。

$$W = \int_0^L -\frac{F_T x}{\sqrt{x^2+y^2}}\,dx = -F_T\left[\sqrt{x^2+y^2}\right]_0^L = -F_T\left(\sqrt{L^2+y^2}-y\right).$$

官方评分

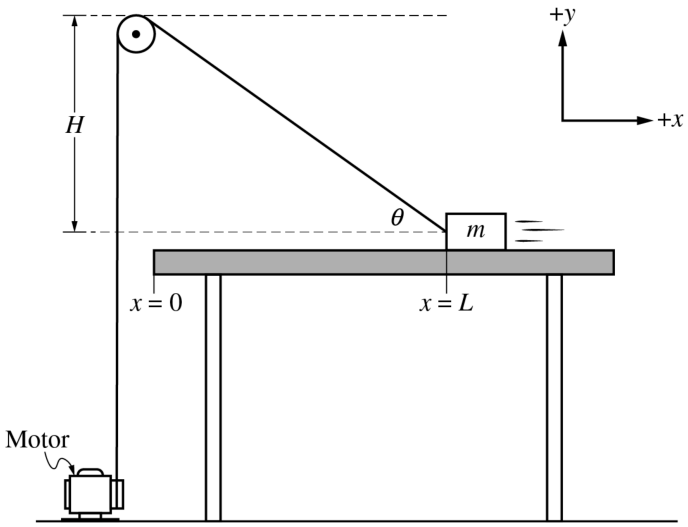
(d)	For using the integral definition of work to derive an expression for the work done by the force of tension $W = \int F \cdot dx$	**1 point**
	For any indication that the work done on the sled by the string is due only to the horizontal component of the tension in the string $W = \int F_{T,x} dx$	**1 point**
	For using the horizontal component of the force of tension consistent with part (b) or (c) $W = \int_{x=0}^{x=L} -F_T \left(\frac{x}{\sqrt{y^2 + x^2}} \right) dx$	**1 point**
	For correct limits of integration, x= 0 to x= L, and indicating that the work done is negative let $u = y^2 + x^2$, then $du = 2x\,dx$ $W = -F_T \int_{x=0}^{x=L} \frac{1}{2} \left(\frac{1}{\sqrt{u}} \right) du$ $W = -F_T \left(\sqrt{y^2 + x^2} \right) \Big\|_{x=0}^{x=L}$ $W = -F_T \left(\sqrt{y^2 + L^2} - \sqrt{y^2} \right)$	**1 point**
		Total for part (d) 4 points

两段等距离：比较每一段的做功分量

两段路程长度相同，不代表做功相同。看真正沿位移方向的力是否更大。角度增大时，恒定张力的水平分量减小；若比较的是摩擦耗能，还要看角度使实际支持力怎样改变。耗散量报告正值。

例题 2022 Set 1 Q1 (e)

本问承接 2022 Set 1 Q1 (b)。



1. A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

(e) Does the string do more, less, or the same amount of work on the block as the block moves from $x = L$ to $x = \frac{L}{2}$ compared to when the block moves from $x = \frac{L}{2}$ to $x = 0$?

_____ More work when the block moves from $x = L$ to $x = \frac{L}{2}$

_____ Less work when the block moves from $x = L$ to $x = \frac{L}{2}$

_____ The same amount of work when the block moves from $x = L$ to $x = \frac{L}{2}$

Justify your answer.

解：

选More work。两半的移动距离相同、张力大小相同，但靠近滑轮时绳子更陡，沿水平位移方向的张力分量更小。第一半的每一小段都由较大的水平分量做功，所以第一半的总功更多。

官方评分

(e)	For selecting "More work …" with an attempt at a relevant justification	**1 point**
	For a correct justification relating the smaller angle to a larger component of the force of tension, thus resulting in greater work.	**1 point**
	Example Response *F_T stays the same for both halves, the displacement is the same in both halves, but from $x = L$ to $x = L/2$ the angle is smaller, resulting in a larger component of the tension force that aligns with the displacement.*	
	Total for part (e)	**2 points**
	Total for question 1	**15 points**

例题 2022 Set 2 Q1 (e)

本问承接 2022 Set 2 Q1 (c)(i)。

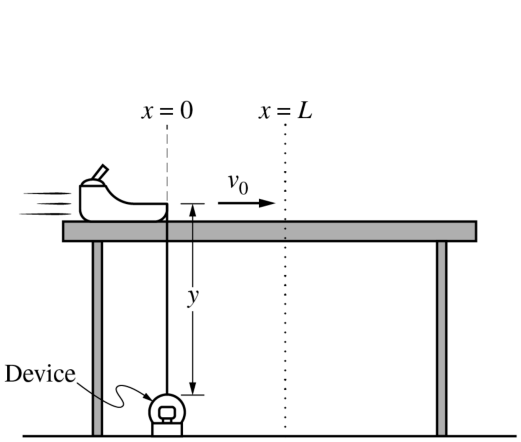


Figure 1

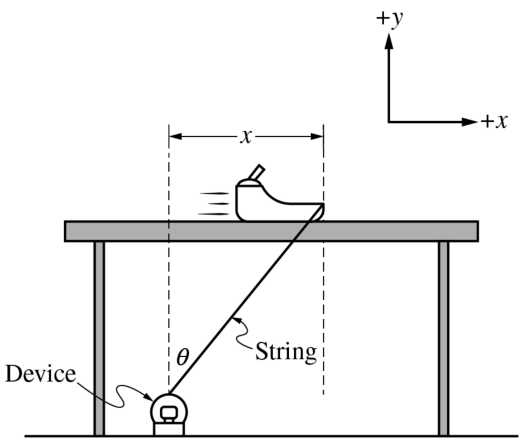


Figure 2

Note: Figures not drawn to scale.

1. A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.

(e) The sled comes to rest after traveling a horizontal distance $x = 2L$. As the system slides from $x = 0$ to $x = L$, the energy dissipated by friction is E_1. As the sled slides from $x = L$ to $x = 2L$, the energy dissipated by friction is E_2. Is E_1 greater than, less than, or equal to E_2 ?

___$E_1 > E_2$ ___$E_1 < E_2$ ___ $E_1 = E_2$

Justify your answer.

解：

选第一段耗散较多。x越小，绳子的向下分量越大，支持力和动摩擦也越大。两段的长度相同，第一段每一位置的摩擦都较大，故摩擦耗散的能量更多。

$$E_1 > E_2.$$

官方评分

(e)	For selecting "$E_1 > E_2$" with an attempt at a relevant justification	**1 point**
	For a justification that correctly relates the force of friction to the normal force, and the normal force to the position or the angle	**1 point**
	Example Response *The vertical component of the string force is the largest when the string is more vertical ($0 < x < L$). A larger vertical component of the string tension leads to a larger normal force and, hence, a larger friction force.*	
	Total for part (e)	**2 points**
	Total for question 1	**15 points**

相等储能：比较曲线面积，不只比较力

同一物块达到相同最高位置，需要相同重力势能，所以需要两种弹簧释放相同能量。两条力曲线交点只表示那一个压缩量处的力相等；储能是从零到该压缩量的整块面积。较小的一侧必须再压缩，才可能把面积追平。

例题 2023 Set 1 Q1 (b)(ii)

本问承接 2023 Set 1 Q1 (b)(i)。

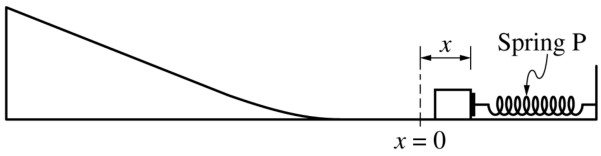


Figure 1

1. A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the maximum height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_{\text{P}}(x) = kx$, where $k = 100.0\,\text{N/m}$, and for Spring Q, $F_{\text{Q}}(x) = Cx^{1/2}$, where $C = 20.0\,\text{N/m}^{1/2}$.

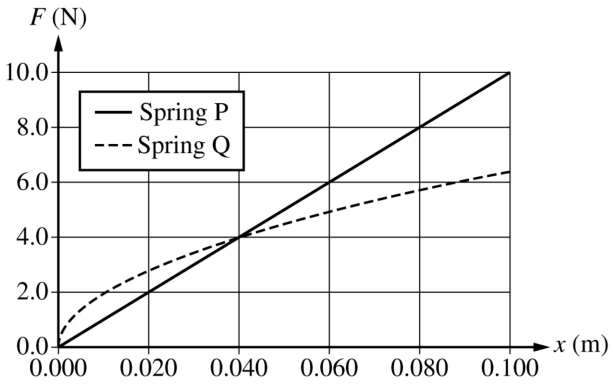


Figure 2

ii. There is one compression distance x_0 for which the maximum height h_{max} reached by the block is the same regardless of which spring, Spring P or Spring Q, is used. Predict whether the value of x_0 is greater than, less than, or equal to 0.040 m. Use the graph in Figure 2 to justify your answer.

解：

在0.040米之前Q曲线的面积较大；0.040米只是两弹力相等，并不表示两面积相等。必须再压缩一段，让P增加的面积追上Q，所以等高度的压缩量大于0.040米。

官方评分

(b)(ii)	For stating the spring compression will be greater than $0.040\ \text{m}$ with an attempt at a relevant justification	**1 point**
	For a justification that indicates that the heights are equal when the area between each function and the horizontal axis are equal, which happens after $x = 0.040\ \text{m}$ **Scoring Note:** While a mathematical solution is not required to earn credit for this point, students may reference a mathematical solution.	**1 point**
	Example Solution *The work done by each spring is equal to the area under their respective curves. This work is converted to the change in potential energy and therefore relates to the maximum height reached by the block. At $x = 0.040\ \text{m}$, the area under the curve for Spring Q is greater than that of Spring P. Therefore, it is not until a compression greater than $x = 0.040\ \text{m}$ that the areas under the two curves are equal, have the same work, and convert to the same maximum height.*	

非线性弹簧：积分力得到高度曲线

原题一种弹力正比于压缩量，另一种正比于压缩量的平方根。先把各自的力从零压缩到x累积成储能，再令它等于物块最高点的mgh。

积分幂函数时，把指数加一，再除以增加后的指数。例如平方根是x的1/2次方，积分成为(2/3)x的3/2次方。两条高度图从零开始，曲线形状应由所得幂次决定。

$$\int_0^x s^n\,ds = \frac{x^{n+1}}{n+1} \quad (n \neq -1).$$

例题 2023 Set 1 Q1 (b)(iii)

本问承接 2023 Set 1 Q1 (b)(i)。

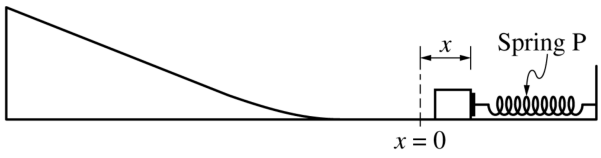


Figure 1

1. A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the maximum height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_{\text{P}}(x) = kx$, where $k = 100.0\,\text{N/m}$, and for Spring Q, $F_{\text{Q}}(x) = Cx^{1/2}$, where $C = 20.0\,\text{N/m}^{1/2}$.

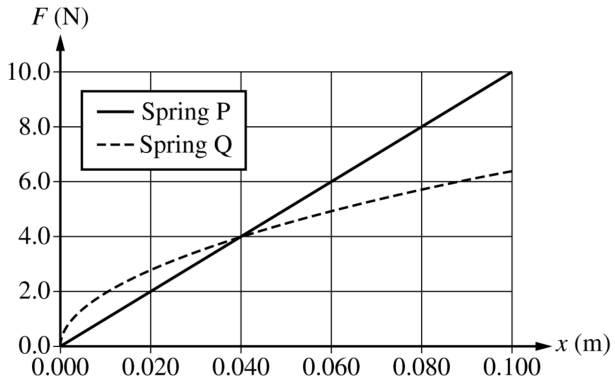
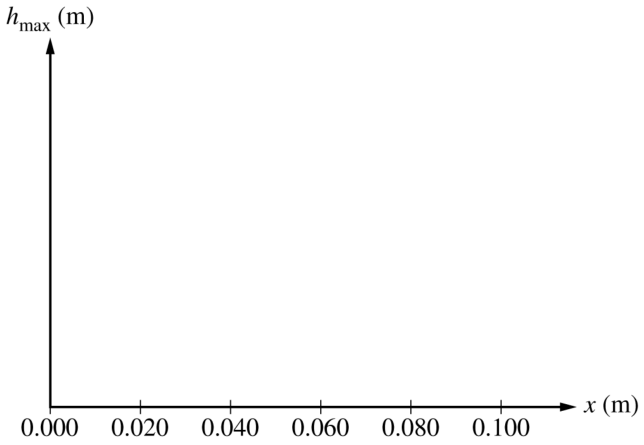


Figure 2

iii. On the axes provided, for both Spring P and Spring Q, sketch a graph of the maximum height h_{max} reached by the block as a function of the distance each spring is compressed for values of x ranging from 0 to 0.100 m. Clearly label the curve for Spring P and Spring Q.



解：

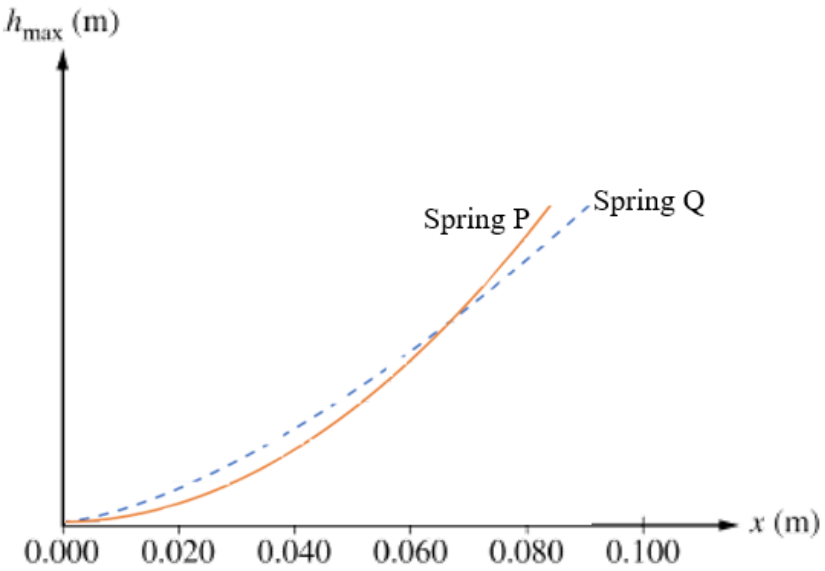
分别积分两种力，再令所得能量等于最高位置的重力势能。

$$U_P = \int_0^x ku\,du = \frac{1}{2}kx^2, \qquad U_Q = \int_0^x Cu^{1/2}\,du = \frac{2}{3}Cx^{3/2}.$$

$$h_P = \frac{kx^2}{2mg}, \qquad h_Q = \frac{2Cx^{3/2}}{3mg}.$$

两条高度曲线都向上凹；开始Q更高，非零交点以后P更高。求面积相等得到约0.0711米。

$$\frac{1}{2}kx_0^2 = \frac{2}{3}Cx_0^{3/2}, \qquad x_0 = \left(\frac{4C}{3k}\right)^2 \approx 0.0711\ \text{m}.$$

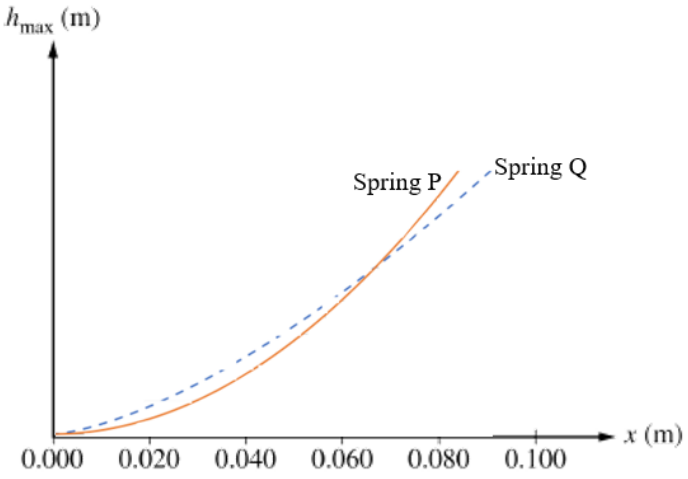


曲线的交点来自面积相等，不是弹力相等。

官方评分

(b)(iii)	For drawing smooth concave upward curves for both springs P and Q	**1 point**
	For showing that the curves intersect after $x = 0.05\ \text{m}$, but before $x = 0.10\ \text{m}$	**1 point**
	For showing the curve for Spring P below Spring Q before an intersection and the curve for Spring P above Spring Q after this intersection	**1 point**

Example Solution



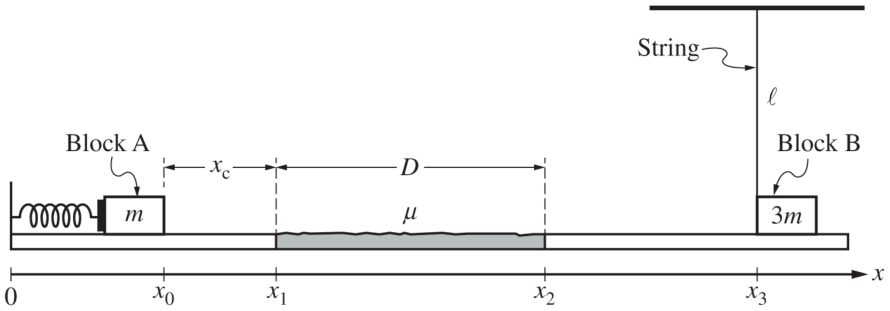
Total for part (b) **6 points**

弹簧释放：由储能求离开弹簧的速率

物块从静止释放、该段光滑时，弹簧原有的能量变成物块动能。先区分压缩量和以后行进的距离，再令初始弹性势能等于末动能。题目求速率，开根后取正值。

$$\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2.$$

例题 2024 Set 1 Q1 (a)(i)



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

解：

释放到弹簧恢复原长的路段光滑，初始弹性势能变为A的动能。

$$\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2, \qquad v = x_c\sqrt{\frac{k}{m}}.$$

官方评分

(a)(i)	For a multi-step derivation with an application of the conservation of mechanical energy that indicates that all of the energy of the system is initially U_s	**1 point**

Example Response

$$E_{\text{initial}} = E_{\text{final}}$$
$$\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$$

For a correct solution for v	**1 point**

Example Response

$$v = x_c\sqrt{\frac{k}{m}}$$

Example Solution

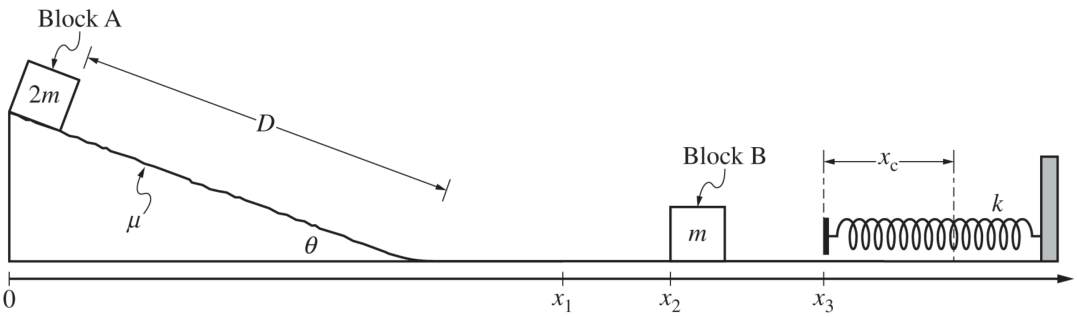
$$E_{\text{initial}} = E_{\text{final}}$$
$$U_s = K$$
$$\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{kx_c^2}{m}}$$
$$v = x_c\sqrt{\frac{k}{m}}$$

粗糙斜面：高度与支持力必须投影

沿斜面长度D不等于下降高度。角从水平量起，竖直高度差是Dsinθ；没有垂直斜面的加速度，支持力是重力的垂直分量mgcosθ。把重力势能降低量减去摩擦耗能，剩下的是动能。

$$h = D\sin\theta, \qquad N = mg\cos\theta, \qquad E_{\text{loss}} = \mu N D.$$

例题 2024 Set 2 Q1 (a)(i)



Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

解：

A质量为2m，下降高度为斜面长度乘正弦。支持力为重力的法向分量，摩擦耗能为摩擦乘斜面长度。

$$2mgD\sin\theta - 2\mu mgD\cos\theta = \frac{1}{2}(2m)v^2.$$

$$v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}.$$

官方评分

(a)(i) For a multi-step derivation that includes **one** of the following: **1 point**

- An application of conservation of energy that indicates that the initial mechanical energy of the system is U_g
- An application of Newton's second law that includes the frictional force, including sign

For **one** of the following that is consistent with the previous point: **1 point**

- An expression for the energy dissipated by friction, including the correct sign
- A substitution of acceleration in a kinematics equation

Example Responses

$\Delta E_{\text{friction}} = -F_f D$ **OR** $v^2 = v_0^2 + 2\left(\frac{F_{g,x} - F_f}{2m}\right)\Delta x$

For a correct expression for v **1 point**

Example Response

$v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$

Example Solutions

$E_{\text{initial}} = E_{\text{final}}$
$U_g - \Delta E_{\text{friction}} = K$
$m_A gD\sin\theta - \mu m_A gD\cos\theta = \frac{1}{2}m_A v^2$
$gD\sin\theta - \mu gD\cos\theta = \frac{1}{2}v^2$
$v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$

OR

$a = \frac{F_{\text{net}}}{m}$
$2ma = F_{g,x} - F_f$
$2ma = 2mg\sin\theta - \mu(2m)g\cos\theta$
$a = g\sin\theta - \mu g\cos\theta$

$v^2 = v_0^2 + 2a\Delta x$
$v^2 = 0^2 + 2(g\sin\theta - \mu g\cos\theta)D$
$v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$

能量条：分别计算两个弹簧与静止物块

两根弹簧形变大小相同，但常量不同，储能按各自的弹簧常量成比例。物块被保持静止，动能为零；零能量要在零线上画独立短横线，不能省略。

$$U = \frac{1}{2}kx^2, \qquad K = \frac{1}{2}mv^2.$$

例题 2025 Q2 (A)

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

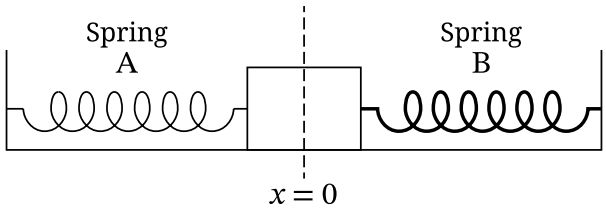


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

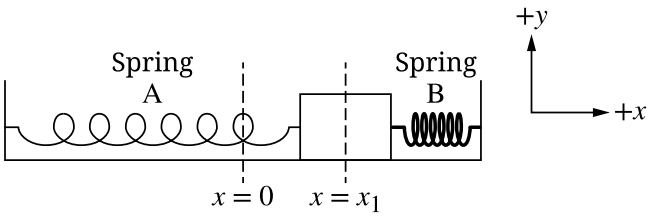


Figure 2

A. An energy bar chart can be used to represent the elastic potential energy U_{A} of Spring A, the elastic potential energy U_{B} of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.

- The height of the shaded bars should be proportional to the relative values of U_{A}, U_{B}, and K_{block}.
- Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

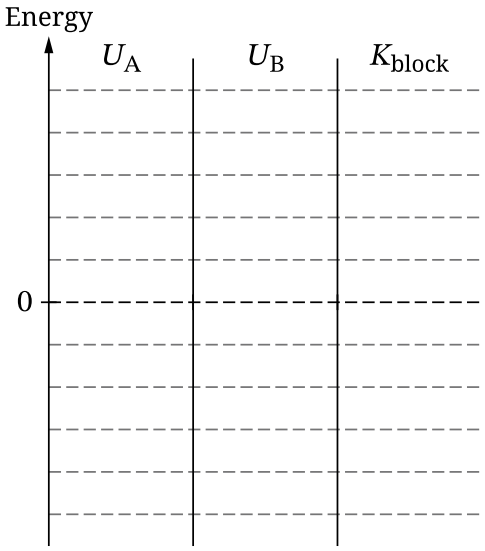


Figure 3

解：

两弹簧变形量大小相同，B常量为A的两倍；块被保持静止，动能为零。能量条的比例为一比二比零，在动能零线上画清楚横线。

$$U_A = \frac{1}{2}kx_1^2, \qquad U_B = kx_1^2, \qquad K = 0.$$

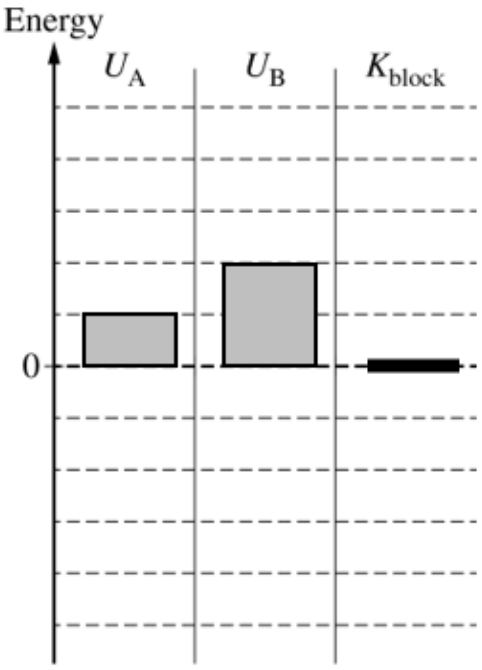


Figure 3

B储能为A的两倍，动能为零。

官方评分

A	For indicating that K_{block} is zero	**Point A1**
	For drawing bars with positive heights for U_A and U_B	**Point A2**
	For drawing a bar for U_B with a height that is twice the height of the bar drawn for U_A **Scoring Note:** This point may be earned regardless of the signs of either bar.	**Point A3**

Example Response

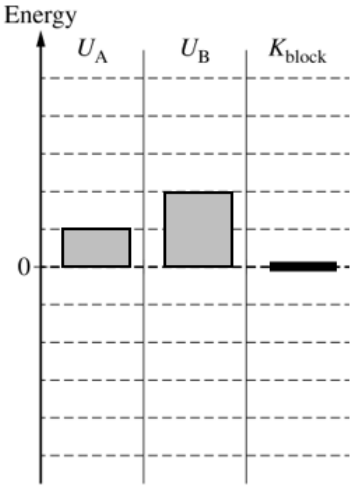


Figure 3

两弹簧共同回复：能量差等于动能

物块偏离平衡位置时，两根弹簧都提供指向平衡位置的回复力，储能要相加。把初始储能减去经过指定位置时仍留下的储能，剩下的就是动能；位移减为一半，平方关系使储能减为四分之一。

例题 2025 Q2 (B)

本问承接 2025 Q2 (A)。

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

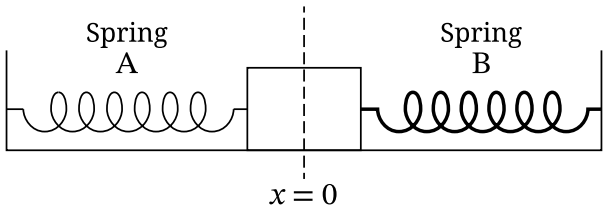


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

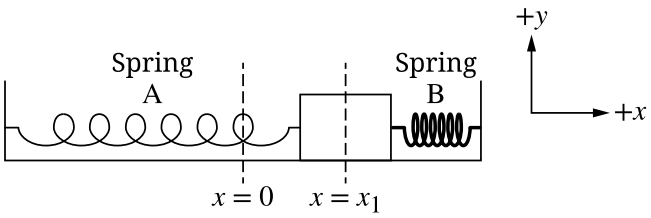


Figure 2

B. The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m, k, x_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

从能量守恒开始。两弹簧同时产生回复力，等效常量是3k；到半初位移时，弹簧势能是初值的四分之一。

$$\frac{1}{2}(3k)x_1^2 = \frac{1}{2}(3k)\left(\frac{x_1}{2}\right)^2 + \frac{1}{2}mv^2.$$

$$\frac{1}{2}mv^2 = \frac{9}{8}kx_1^2, \qquad v = \frac{3}{2}x_1\sqrt{\frac{k}{m}}.$$

第一次经过该位置时块向左，但原题求速率，所以给正值。

官方评分

B	For a multistep derivation that includes energy conservation or simple harmonic motion	**Point B1**
	For relating the presence of both springs to the behavior of the system	**Point B2**
	For relating positions $x = x_1$ and $x = \frac{1}{2}x_1$ to the oscillation of the block	**Point B3**
	For a correct expression for v in terms of given quantities	**Point B4**

Example Responses

$E_{\text{tot},0} = U_{\text{A}}(x_1) + U_{\text{B}}(x_1) + K_{\text{block}}(x_1)$

$E_{\text{tot},f} = U_{\text{A}}\left(\frac{1}{2}x_1\right) + U_{\text{B}}\left(\frac{1}{2}x_1\right) + K_{\text{block}}\left(\frac{1}{2}x_1\right)$

$E_{\text{tot},0} = \frac{1}{2}(k)(x_1)^2 + \frac{1}{2}(2k)(x_1)^2 + 0$

$E_{\text{tot},f} = \frac{1}{2}(k)\left(\frac{1}{2}x_1\right)^2 + \frac{1}{2}(2k)\left(\frac{1}{2}x_1\right)^2 + K_{\text{block},\,\frac{1}{2}x_1}$

$E_{\text{tot},0} = E_{\text{tot},f}$

$\frac{1}{2}kx_1^2 + kx_1^2 = \frac{1}{8}kx_1^2 + \frac{1}{4}kx_1^2 + K_{\text{block},\,\frac{1}{2}x_1}$

$\frac{3}{2}kx_1^2 = \frac{3}{8}kx_1^2 + K_{\text{block},\,\frac{1}{2}x_1}$

$K_{\text{block},\,\frac{1}{2}x_1} = \frac{9}{8}kx_1^2$

$\frac{1}{2}mv^2 = \frac{9}{8}kx_1^2$

$v = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$

OR

$x = x_{\max}\cos(\omega t + \phi)$

$x(t) = x_1\cos\omega t$

$v(t) = -x_1\omega\sin\omega t$

$k_{\text{eff}} = k + 2k = 3k$

$\omega = \sqrt{\frac{3k}{m}}$

$\frac{x_1}{2} = x_1\cos\omega t_{\frac{1}{2}x_1}$

$\cos\omega t_{\frac{1}{2}x_1} = \frac{1}{2}$

$t_{\frac{1}{2}x_1} = \frac{\pi}{3\omega}$

$v_{\frac{1}{2}x_1} = -x_1\omega\sin\omega t_{\frac{1}{2}x_1}$

$v_{\frac{1}{2}x_1} = -x_1\sqrt{\frac{3k}{m}}\sin\frac{\pi}{3}$

$v = \left|v_{\frac{1}{2}x_1}\right| = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$

阻力、终速与速度随时间改变

下落阻力：建立速度的微分方程

重力使物体向下加速，空气阻力反对下落。取向下为正，写重力减阻力等于质量乘加速度；加速度就是速度对时间的导数。先读取题目给的是bv还是bv^2，不把两种阻力模型混用。

$$m\frac{dv}{dt} = mg - F_{\text{drag}}.$$

例题 2024 Set 2 Q2 (a)

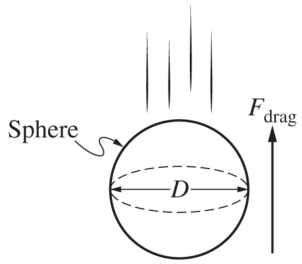


Figure 1

2. A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{\text{drag}} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

(a) **Derive**, but do NOT solve, a differential equation that could be used to determine the speed v of the sphere as a function of time t. Express your answer in terms of given quantities and physical constants, as appropriate.

解：

球从静止向下运动，重力向下、空气阻力向上。取向下为正，把加速度写成速率随时间的变化率，先写牛顿第二定律，再代入题给线性阻力。

$$\sum F = ma, \qquad mg - bv = m\frac{dv}{dt}, \qquad v(0) = 0.$$

原题只要求建立微分方程，不要求求出速度函数。

官方评分

(a)	For a multi-step derivation that includes Newton's second law of motion	**1 point**
	For indicating that the net force exerted on the sphere includes only the gravitational force and a drag force	**1 point**
	Example Response $F_{\text{net}} = F_g - F_{\text{drag}}$	
	For a correct differential equation that is in terms of the given variables **Scoring Note:** Variables do not have to be separated for this point to be earned.	**1 point**
	Example Response $m\frac{dv}{dt} = mg - bv$	
	Example Solution $\Sigma F = ma$ $F_g - F_{\text{drag}} = ma$ $mg - bv = ma$ $m\frac{dv}{dt} = mg - bv$	
		Total for part (a) 3 points

练习 2024 Set 1 Q2 (a)

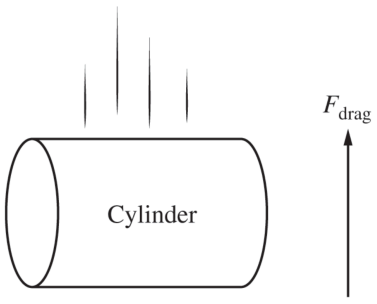


Figure 1

2. A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{\text{drag}} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

(a) **Derive**, but do NOT solve, a differential equation that could be used to determine the speed v of the cylinder as a function of time t. Express your answer in terms of given quantities and physical constants, as appropriate.

解：

取下落方向为正。只有重力和题给的平方阻力；加速度是速率对时间的导数。

$$\sum F = ma, \qquad mg - bv^2 = m\frac{dv}{dt}, \qquad v(0) = 0.$$

题目只要求建立方程。这里b的单位为kg/m，与线性阻力题的b不同。

官方评分

(a)	For a multi-step derivation that includes Newton's second law of motion	**1 point**
	For indicating that the net force exerted on the cylinder includes only the gravitational force and a drag force	**1 point**
	Example Response $F_{\text{net}} = F_g - F_{\text{drag}}$	
	For a correct differential equation that is in terms of the given variables **Scoring Note:** Variables do not have to be separated for this point to be earned.	**1 point**
	Example Response $m\frac{dv}{dt} = mg - bv^2$	
	Example Solution $\Sigma F = ma$ $F_g - F_{\text{drag}} = ma_y$ $mg - bv^2 = ma_y$ $m\frac{dv}{dt} = mg - bv^2$	
		Total for part (a) 3 points

阻力图的平台：找速率不再增加的时刻

线性阻力与速率成正比。阻力—时间曲线成为水平平台时，阻力不再改变，速率也不再改变。在原图开始近似变平的位置画竖线，并按原题在时间轴上标名称。真实模型可以逐渐接近终速，示意平台表示已可以近似当作终速。

例题 2024 Set 2 Q2 (b)(i)

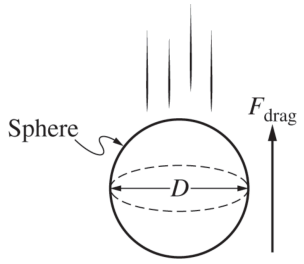


Figure 1

2. A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

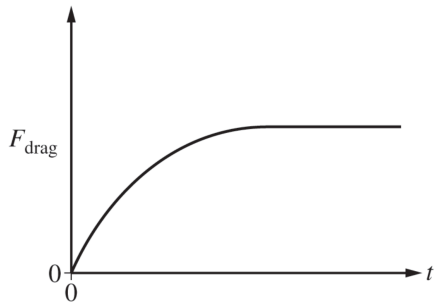


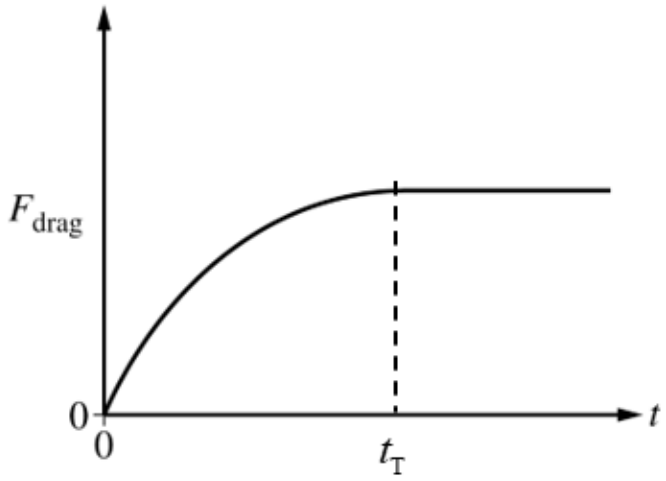
Figure 2

(b) The student sketches the drag force F_{drag} exerted on the sphere as a function of time t, as shown in Figure 2.

i. **Draw** a vertical line on the sketch in Figure 2 to indicate the earliest time at which F_{drag} is equal to the magnitude of the weight of the sphere, which occurs when the sphere reaches terminal speed. Label this time as t_T on the time axis.

解：

在阻力曲线开始近似成为水平线的位置画竖线，在时间轴上标终速时刻。

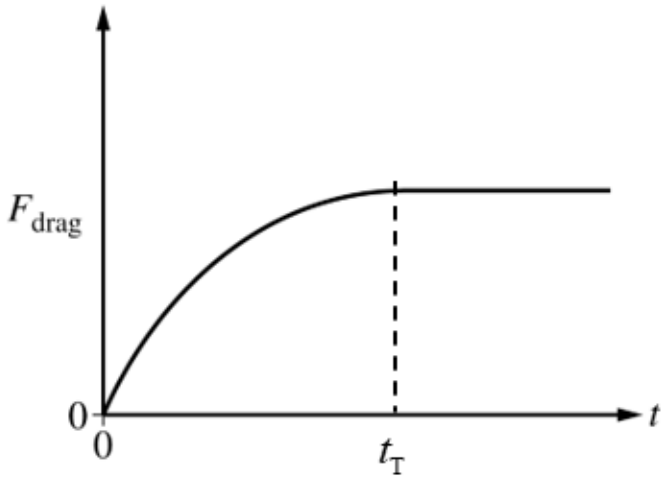


阻力图的平台与终速时刻；未给数值时间，所以横轴不添加秒数。

官方评分

(b)(i) For a vertical line labeled t_T at the approximate location at which the line becomes horizontal **1 point**

Example Response



速度图的平台：加速度由斜率判断

速度—时间图的斜率是加速度。曲线近似成为水平平台时，斜率为零，合力为零，向上的阻力恰好抵消重力。在速度仍上升的区间，两力尚未平衡。

例题 2024 Set 1 Q2 (b)(i)

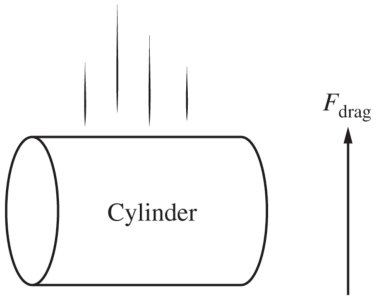


Figure 1

2. A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

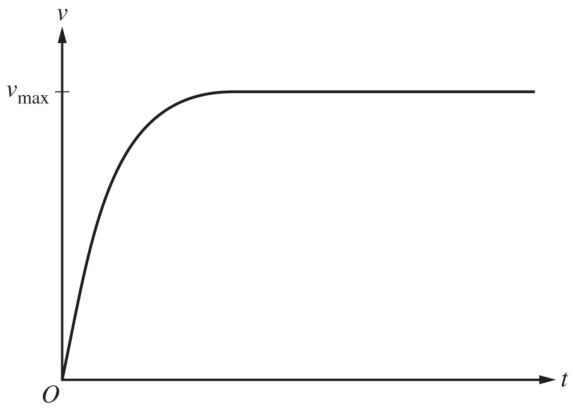


Figure 2

(b) The student correctly sketches the speed v of the cylinder as a function of time t, as shown in Figure 2.

i. **Draw** a vertical line on the sketch in Figure 2 to indicate the earliest time at which F_{drag} on the cylinder is equal to the magnitude of the weight of the cylinder. Label this time as t_1 on the time axis.

解：

在原速度曲线开始近似成为平台的位置画竖线，标出题给时刻。下图显示应取的图形特征。

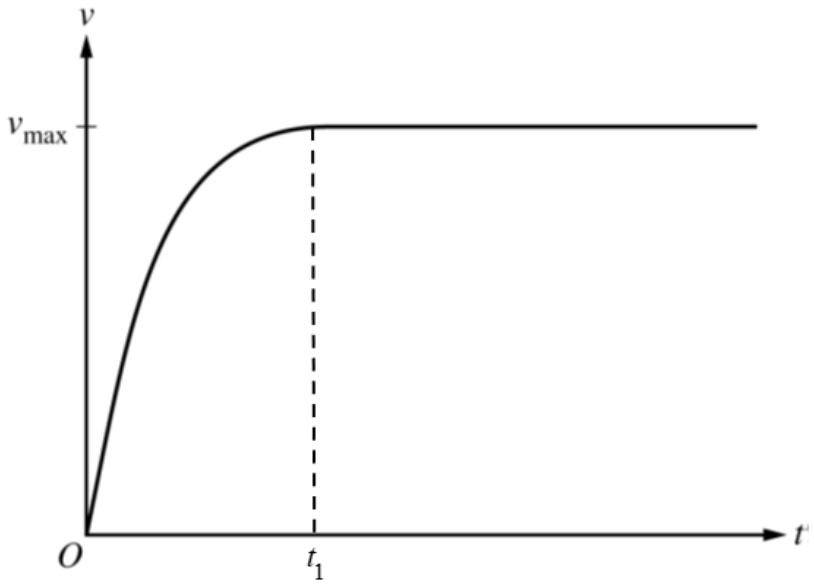


Figure 2

速度图的水平段表示加速度为零。

官方评分

(b)(i)	For a vertical line labeled t_1 at the approximate location at which the line becomes horizontal	**1 point**

Example Response

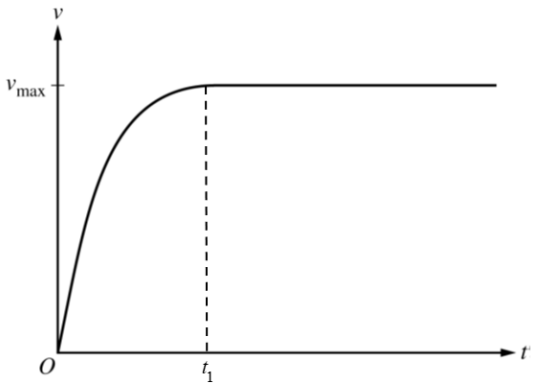


Figure 2

练习 2024 Set 1 Q2 (b)(ii)

本问承接 2024 Set 1 Q2 (b)(i)。

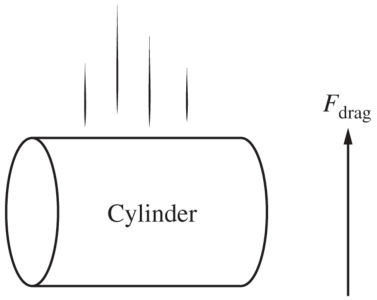


Figure 1

2. A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

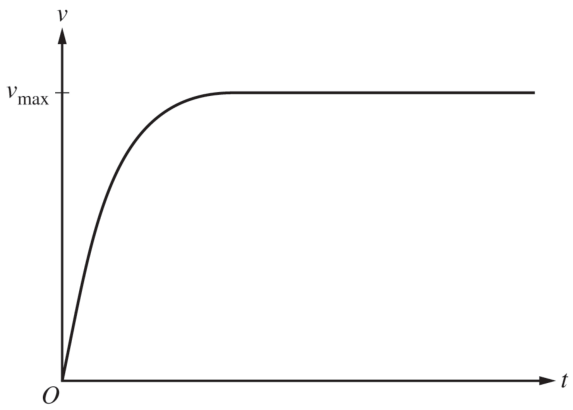


Figure 2

(b) The student correctly sketches the speed v of the cylinder as a function of time t, as shown in Figure 2.

ii. **Justify** the location of t_1. Explicitly reference appropriate features of the sketch in Figure 2.

解：

速度图的斜率就是加速度；平台斜率为零，所以合力为零，向上的阻力等于重力。平台之前速度上升，合力仍朝下，阻力小于重力。

官方评分

(b)(ii)	For relating t_1 to the time at which the velocity versus time graph is constant or the slope of the line is zero	**1 point**
	For indicating that a constant velocity indicates that the net force is zero	**1 point**
	Example Response *Because the sketched line is horizontal after t_1, the velocity is constant. If the velocity is constant, then the acceleration is zero. Therefore, the net force is zero, which means that the gravitational and drag forces are equal in magnitude.*	
	Total for part (b)	**3 points**

用阻力平台解释力平衡

读阻力图时，先用Fdrag=bv把阻力是否改变转成速率是否改变，再由速率恒定推得加速度为零。不能只说图是水平的就直接宣称所有作用力都为零；重力和阻力各自仍存在，只是互相平衡。

例题 2024 Set 2 Q2 (b)(ii)

本问承接 2024 Set 2 Q2 (b)(i)。

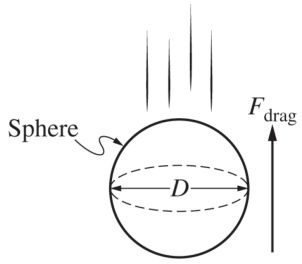


Figure 1

2. A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

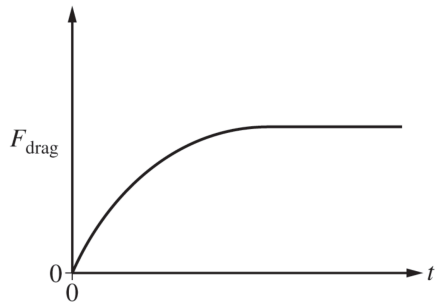


Figure 2

(b) The student sketches the drag force F_{drag} exerted on the sphere as a function of time t, as shown in Figure 2.

ii. **Justify** the location of t_{T}. Explicitly reference appropriate features of the sketch in Figure 2.

解：

平台之前曲线斜率为正，阻力仍增大；因为阻力正比于速率，速率也仍增大。平台上阻力不再变，速率恒定，因而加速度和合力都为零。

$$F_{\text{drag}} = bv, \qquad a = 0 \implies F_{\text{drag}} = mg.$$

理想微分模型渐近接近终速；原题的平台表示已可近似按终速处理。

官方评分

(b)(ii)	For a response that references the slope of the graph or the rate at which the slope changes	**1 point**
	For correctly relating a feature of the graph to the forces exerted on the sphere as the sphere reaches terminal speed	**1 point**

Example Response

For the times leading up to t_{T}, the slope of the graph is positive which means that the magnitude of the drag force is still increasing. After t_{T}, the slope of the graph is zero which means that the magnitude of the drag force is constant and equal to the downward gravitational force, which indicates that the net force is zero and that the sphere has reached a constant terminal velocity.

Total for part (b) 3 points

改变初速度：终速由最终平衡决定

同一个物体、同一种阻力规律达到终速时，仍满足重力等于阻力。初始向下投掷或先向上抛，只改变到达终速的过程。比较最终终速或终速阻力时，先检查质量、大小形状和姿态是否保持不变。

$$mg = bv_T \quad \text{或} \quad mg = bv_T^2.$$

例题 2024 Set 2 Q2 (c)

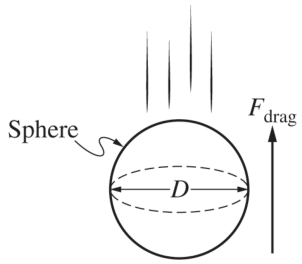


Figure 1

2. A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

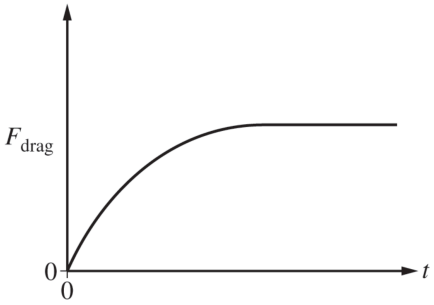


Figure 2

(b) The student sketches the drag force F_{drag} exerted on the sphere as a function of time t, as shown in Figure 2.

(c) Suppose the student throws the same sphere downward with a nonzero initial speed. The magnitude of the new drag force at terminal speed after being thrown downward is F_{new}.

Indicate whether F_{new} would be greater than, less than, or equal to the magnitude of F_{drag} at terminal speed represented in Figure 2.

_____ Greater than _____ Less than _____ Equal to

Briefly **justify** your answer.

解：

选Equal to。终速时两力平衡；同一个球的质量没有变，所以终速阻力始终等于同一个重力。最初是否向下投掷，只改变到达该状态的过程。

官方评分

(c)	For selecting "Equal to" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**
	Example Response *The magnitude of the drag force at terminal speed does not change since the mass of the sphere is not changed and the drag force at terminal speed does not depend on the initial speed of the sphere.*	
	Total for part (c)	**2 points**

练习 2024 Set 1 Q2 (c)

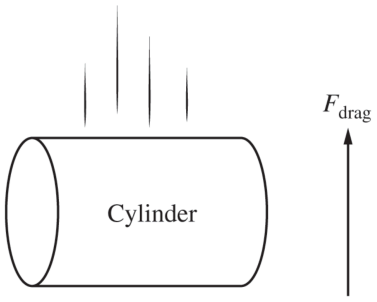


Figure 1

2. A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

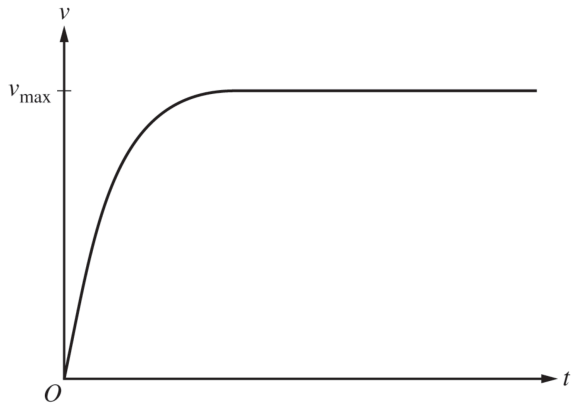


Figure 2

(b) The student correctly sketches the speed v of the cylinder as a function of time t, as shown in Figure 2.

(c) Rather than dropping the cylinder from rest, the student throws the cylinder upward with a nonzero initial speed. The cylinder is in the same orientation as when the cylinder was previously dropped. The student allows the cylinder to fall toward the ground.

Indicate whether the magnitude of the cylinder's maximum downward speed after being thrown upward would be greater than, less than, or equal to the maximum speed v_{max} in Figure 2.

_____ Greater than _____ Less than _____ Equal to

Briefly **justify** your answer.

解：

选Equal to。向上抛后先到最高点，再下落；同一圆柱的质量和阻力规律不变。题设允许达到终速，故最大向下速率仍由同一力平衡决定。

$$mg = bv_T^2, \qquad v_T = \sqrt{\frac{mg}{b}}.$$

官方评分

(c)	For selecting "Equal to" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**
	Example Response *At its peak, the cylinder will have a speed of* $0\ \text{m/s}$. *Therefore, the cylinder would reach the same* v_{max} *as if the student had dropped the cylinder from rest at that height. Because the cylinder reached* v_{max} *from the initial drop, the two max speeds are equal.*	
	Total for part (c)	**2 points**

速度阻力：先求整体运动，再回到单个物块

箱和块在水平方向共同运动，把二者选成整体可以抵消内部的法向力。总质量是两质量相加；外力为 −bv，所以整体速度不是匀减速。先分离速度和时间，再由初始速度积分得到指数衰减。最后选小块本身，用它的减速求法向力。

$$\frac{dv}{v} = -\frac{b}{2M}\,dt, \qquad \int_{v_0}^{v(t)} \frac{dv}{v} = -\frac{b}{2M}\int_0^t dt.$$

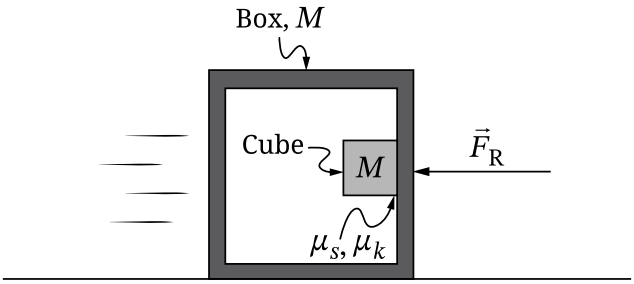
左边是ln(v/v_0)，自然对数是指数函数的反函数；把两边取指数即可还原速度。

例题 2026 Q1 (A)(ii)

本问承接 2026 Q1 (A)(i)。

1. A box of mass M slides to the right on a horizontal surface. A small cube, also of mass M, is inside the box. The cube is against the right wall of the box and above the bottom of the box, as shown in Figure 1. The coefficients of static and kinetic friction between the cube and the wall of the box are μ_s and μ_k, respectively, where $\mu_s > \mu_k$.

Figure 1



The speed of the box is decreasing as a result of a resistive force that is exerted on the box. The resistive force $\vec{F}_R$ is modeled by $\vec{F}_R = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the box. Friction between the box and horizontal surface is negligible.

Consider the motion of the box and cube for the following times.

- At time $t = 0$, the box-cube system slides to the right with speed v_0.
- For $0 \le t < t_{crit}$, the cube remains in contact with the wall of the box at a constant height above the bottom of the box.
- At $t = t_{crit}$, the cube begins to slide downward along the wall of the box.

Part A

The magnitude of the normal force exerted on the cube by the wall of the box is F_N.

ii. **Derive** an expression for F_N as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box. Express your answer in terms of M, b, v_0, t, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

解：

小块贴着右壁，箱和块的水平加速度相同。把二者作为整体，内部作用力抵消，外部水平力只剩速度阻力，总质量是2M。

$$2M\frac{dv}{dt} = -bv.$$

不能用常加速度公式，因为阻力随速度变化。把速度放在一边、时间放在另一边，并由初值积分。

$$\frac{dv}{v} = -\frac{b}{2M}\,dt, \qquad \int_{v_0}^{v(t)} \frac{du}{u} = -\frac{b}{2M}\int_0^t ds.$$

$$\ln\frac{v(t)}{v_0} = -\frac{bt}{2M}, \qquad v(t) = v_0 e^{-bt/(2M)}.$$

再回到小块单独的水平受力，法向力提供它的减速；不能把整个系统受到的阻力直接当作法向力。

$$F_N = M\left|\frac{dv}{dt}\right| = \frac{b}{2}v, \qquad F_N(t) = \frac{bv_0}{2}e^{-bt/(2M)}.$$

官方评分

(ii)	For a multistep derivation that includes Newton's second law	**Point A2**
	For indicating that the magnitude of the acceleration is $\frac{bv}{2M}$	**Point A3**
	For separating dv and dt variables	**Point A4**
	For an exponential expression for F_N that evaluates to zero at infinite time	**Point A5**

Example Response

$a_{\text{cube}} = a_{\text{box}} = a$

$a = \frac{F_{\text{net}}}{m_{\text{total}}} = -\frac{bv}{2M}$

$a = -\frac{bv}{2M}$

$a = \frac{dv}{dt} = -\frac{bv}{2M}$

$\frac{dv}{v} = -\frac{b}{2M}dt$

$\int_{v_0}^{v} \frac{1}{v}dv = -\frac{b}{2M}\int_0^t dt$

$\ln(v) - \ln(v_0) = -\frac{bt}{2M}$

$\ln\left(\frac{v}{v_0}\right) = -\frac{bt}{2M}$

$\frac{v}{v_0} = e^{-\frac{bt}{2M}}$

$v = v_0 e^{-\frac{bt}{2M}}$

$\vec{F}_N = Ma = M\left(\frac{-bv}{2M}\right)$

$\vec{F}_N = -\frac{b}{2}v_0 e^{-\frac{bt}{2M}}$

$F_N = \left|\vec{F}_N\right| = \frac{b}{2}v_0 e^{-\frac{bt}{2M}}$

静摩擦转为动摩擦：图像要分两段

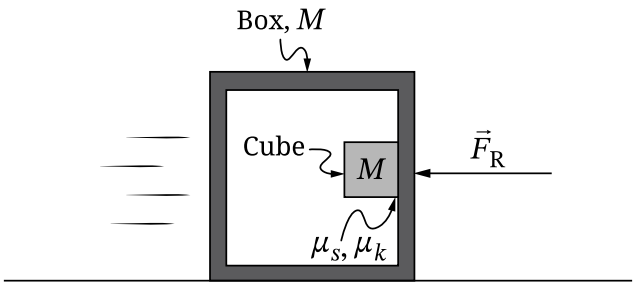
块未下滑时，静摩擦恰好向上平衡重力，实际摩擦为常数mg；最大静摩擦才是μsN。法向力逐渐减小，达到临界后块下滑，摩擦转为μkN。不同系数使刚开始滑动时可能出现跳变；以后跟随法向力的指数衰减。

例题 2026 Q1 (A)(iii)

本问承接 2026 Q1 (A)(ii)。

1. A box of mass M slides to the right on a horizontal surface. A small cube, also of mass M, is inside the box. The cube is against the right wall of the box and above the bottom of the box, as shown in Figure 1. The coefficients of static and kinetic friction between the cube and the wall of the box are μ_s and μ_k, respectively, where $\mu_s > \mu_k$.

Figure 1



The speed of the box is decreasing as a result of a resistive force that is exerted on the box. The resistive force $\vec{F}_{\mathrm{R}}$ is modeled by $\vec{F}_{\mathrm{R}} = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the box. Friction between the box and horizontal surface is negligible.

Consider the motion of the box and cube for the following times.

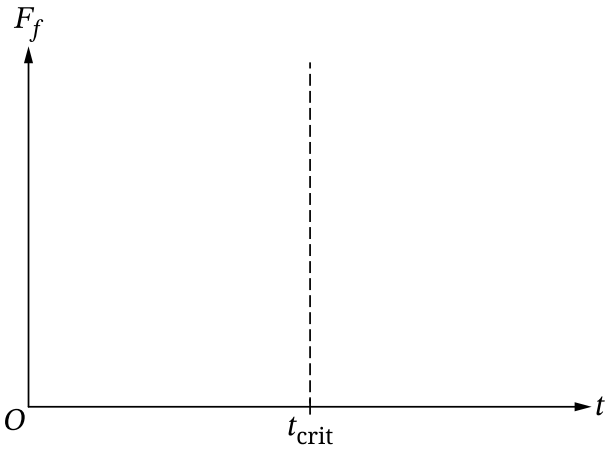
- At time $t = 0$, the box-cube system slides to the right with speed v_0.
- For $0 \leq t < t_{\mathrm{crit}}$, the cube remains in contact with the wall of the box at a constant height above the bottom of the box.
- At $t = t_{\mathrm{crit}}$, the cube begins to slide downward along the wall of the box.

Part A

The magnitude of the normal force exerted on the cube by the wall of the box is F_N.

iii. On the axes shown in Figure 2, **sketch** a graph of the magnitude F_f of the frictional force exerted on the cube by the wall of the box as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box.

Figure 2

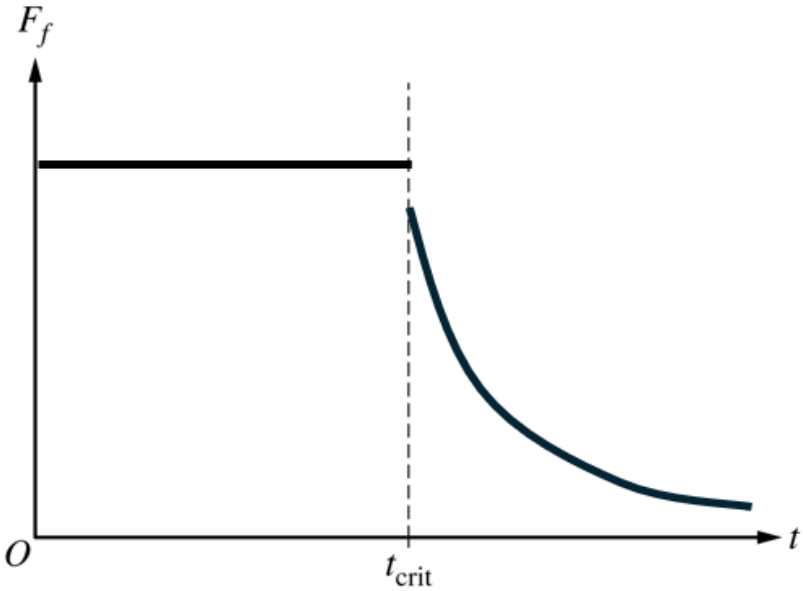


解：

未下滑时实际静摩擦始终向上抵消重力，所以图是水平线，不是一直等于静摩擦上限。下滑后动摩擦改为法向力乘动摩擦系数。

$$F_f(t) = \begin{cases} Mg, & 0 \le t < t_{\text{crit}}, \\ \mu_k \dfrac{bv_0}{2} e^{-bt/(2M)}, & t > t_{\text{crit}}. \end{cases}$$

临界之后摩擦减小且向上凹；因为动摩擦系数较小，模型允许刚滑动时出现向下跳变。图只画到小块到达底部之前。

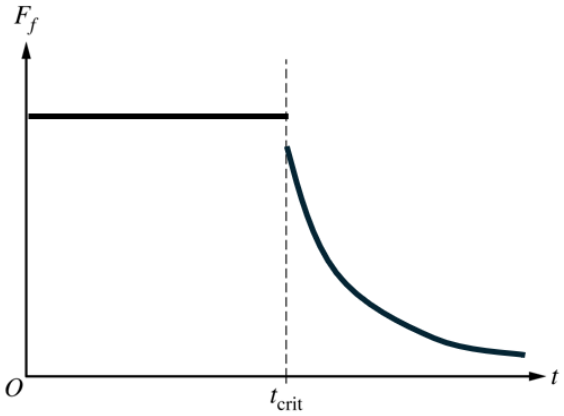


静摩擦先保持重力大小，随后动摩擦随法向力减小。

官方评分

(iii)	For sketching a horizontal line from $t = 0$ to $t = t_{crit}$	**Point A6**
	For sketching a concave up curve that never increases for $t > t_{crit}$ **Scoring Note:** This point can be earned regardless of the discontinuity at $t = t_{crit}$.	**Point A7**

Example Response



开始滑动的时刻：静摩擦上限等于重力

小块留在固定高度，需要向上的静摩擦抵消重力。当最大静摩擦降到mg，恰好不能继续维持静止。先写这个临界等式，代入已经求得的法向力函数，再用自然对数把时间从指数里取出。

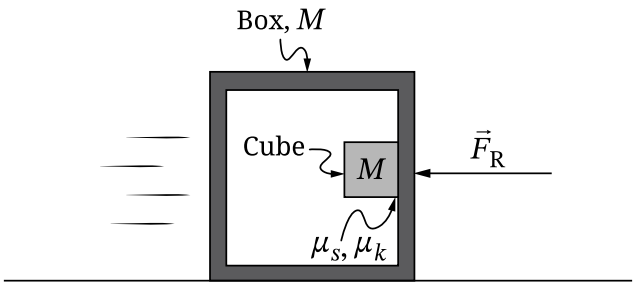
$$\mu_s N(t_{\text{crit}}) = Mg.$$

例题 2026 Q1 (B)

本问承接 2026 Q1 (A)(ii)。

1. A box of mass M slides to the right on a horizontal surface. A small cube, also of mass M, is inside the box. The cube is against the right wall of the box and above the bottom of the box, as shown in Figure 1. The coefficients of static and kinetic friction between the cube and the wall of the box are μ_s and μ_k, respectively, where $\mu_s > \mu_k$.

Figure 1



The speed of the box is decreasing as a result of a resistive force that is exerted on the box. The resistive force $\vec{F}_{\mathrm{R}}$ is modeled by $\vec{F}_{\mathrm{R}} = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the box. Friction between the box and horizontal surface is negligible.

Consider the motion of the box and cube for the following times.

- At time $t = 0$, the box-cube system slides to the right with speed v_0.
- For $0 \le t < t_{\mathrm{crit}}$, the cube remains in contact with the wall of the box at a constant height above the bottom of the box.
- At $t = t_{\mathrm{crit}}$, the cube begins to slide downward along the wall of the box.

Part B

Derive an expression for t_{crit} in terms of M, b, μ_s, v_0, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

解：

临界时，能够提供的最大静摩擦恰好等于重力。代入已经求得的法向力，再取自然对数。

$$\mu_s F_N(t_{\mathrm{crit}}) = Mg, \qquad \frac{\mu_s b v_0}{2} e^{-bt_{\mathrm{crit}}/(2M)} = Mg.$$

$$t_{\mathrm{crit}} = \frac{2M}{b} \ln\left(\frac{\mu_s b v_0}{2Mg}\right).$$

题述正时间的静止阶段要求初始静摩擦上限大于重力，否则小块从开始就会滑。指数中的量无单位，前面的2M/b有时间单位。

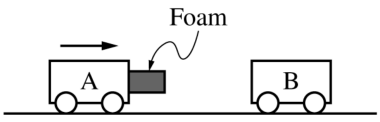
官方评分

B	For a multistep derivation that includes Newton's second law	**Point B1**
	For indicating that the sum of the frictional force and gravitational force exerted on the cube is zero at t_{crit}	**Point B2**
	For substituting the expression for F_N derived in part A into the expression for static friction **Scoring Note:** Responses that omit the subscript on μ_s may earn this point.	**Point B3**
	Example Response $\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = 0$ $F_f = F_g$ $\mu_s F_{N,\text{cube}} = F_g$ $\mu_s\left(\frac{b}{2}v_0 e^{-\frac{bt_{\text{crit}}}{2M}}\right) = Mg$ $e^{-\frac{bt_{\text{crit}}}{2M}} = \frac{2Mg}{\mu_s b v_0}$ $-\frac{bt_{\text{crit}}}{2M} = \ln\left(\frac{2Mg}{\mu_s b v_0}\right)$ $t_{\text{crit}} = -\frac{2M}{b}\ln\left(\frac{2Mg}{\mu_s b v_0}\right)$ $t_{\text{crit}} = \frac{2M}{b}\ln\left(\frac{\mu_s b v_0}{2Mg}\right)$	

速度函数：求导得到力，再找极值

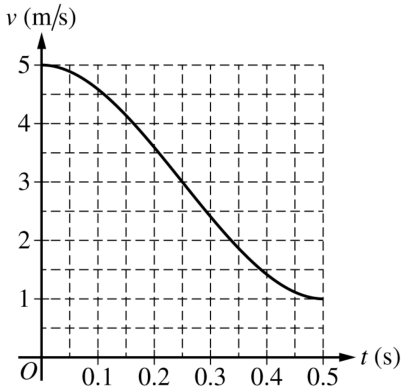
速度函数给出了每时刻的变化。先求导得到加速度，再乘质量得到合力；题目求大小，需处理加速度的负号。得到力大小函数后，再求它的导数为零，并与区间两端比较，确认内点确实是最大值。

例题 2023 Set 2 Q1 (b)(i)



1. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.

(b) For $0 \le t \le 0.50$ s, the velocity v of Cart A can be described by the function $v(t) = 64t^3 - 48t^2 + 5$.



(a) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

i. Calculate the magnitude of the maximum net force acting on Cart A during this interval.

解：

先对速度求导得到加速度，再乘质量。此区间加速度非正；求力大小时取负号。

$$a_A = \frac{dv_A}{dt} = 192t^2 - 96t, \qquad |F_A| = 96000t - 192000t^2.$$

这是开口向下的二次函数。峰值处斜率为零，区间两端都为零，所以内点确实给最大值。

$$\frac{d|F_A|}{dt} = 96000 - 384000t = 0, \qquad t = 0.25\ \text{s}.$$

$$|F_A|_{\max} = 12000\ \text{N}.$$

官方评分

(b)(i) For indicating the derivative of the velocity function is the acceleration of the cart **1 point**

Example Response

$$a(t) = \frac{dv}{dt}$$
$$\rightarrow a(t) = \frac{d(64t^3 - 48t^2 + 5)}{dt}$$
$$\therefore a(t) = 192t^2 - 96t$$

For setting the derivative of the previously derived expression for acceleration equal to zero **1 point**

Example Response

$$\frac{d}{dt}a(t) = 0$$
$$0 = 384t - 96$$

For indicating the maximum acceleration is $12\ \text{m/s}^2$ or maximum acceleration occurs at time $t = 0.25\ \text{s}$ **1 point**

Example Response

$$0 = 384t - 96$$
$$\therefore t_{\text{max}} = 0.25\ \text{s}$$

For substituting the time at which the acceleration is a maximum or the maximum acceleration into an expression of Newton's second law to calculate the value of the maximum force **1 point**

Example Response

$$F(t) = ma(t)$$
$$F_{\text{max}} = ma(0.25\ \text{s})$$

Example Solution

$$a(t) = \frac{dv}{dt}$$
$$\rightarrow a(t) = \frac{d(64t^3 - 48t^2 + 5)}{dt}$$
$$\therefore a(t) = 192t^2 - 96t$$

$$\frac{d}{dt}a(t) = 0$$
$$\frac{d\left(192t^2 - 96t\right)}{dt} = 0$$
$$\rightarrow 0 = 384t - 96$$
$$\therefore t_{\text{max}} = 0.25\ \text{s}$$

$$F(t) = ma(t)$$
$$F_{\text{max}} = ma(0.25\ \text{s})$$
$$F_{\text{max}} = (1000\ \text{kg})\left(192(0.25\ \text{s})^2 - 96(0.25\ \text{s})\right)$$
$$\therefore |F_{\text{max}}| = 12{,}000\ \text{N}$$

Alternate Solution

For indicating the derivative of the momentum function is the force exerted on the cart ***1 point***

Alternate Example Response

$$F(t) = \frac{dp}{dt}$$

$$F(t) = \frac{d}{dt}(64000t^3 - 48000t^2 + 5000)$$

For setting the derivative of the previously derived expression for force equal to zero ***1 point***

Alternate Example Response

$$\frac{d}{dt}F(t) = 0$$

$$\frac{d}{dt}\left(192000t^2 - 96000t\right) = 0$$

For indicating the maximum force occurs at time $t = 0.25\,\text{s}$ ***1 point***

Alternate Example Response

$$0 = 384000t - 96000$$

$$\therefore t_{\max} = 0.25\,\text{s}$$

For substituting the time at which the force is at a maximum into an expression for momentum to calculate the value of the maximum force ***1 point***

Alternate Example Response

$$F(t) = \frac{d}{dt}p(t)$$

$$F_{\max} = \frac{d}{dt}p(0.25\,\text{s})$$

Alternate Example Solution

$$p(t) = mv(t)$$

$$\rightarrow p(t) = (1000\,\text{kg})(64t^3 - 48t^2 + 5)$$

$$\therefore p(t) = 64000t^3 - 48000t^2 + 5000$$

$$F(t) = \frac{dp}{dt}$$

$$\rightarrow F(t) = \frac{d}{dt}(64000t^3 - 48000t^2 + 5000)$$

$$\therefore F(t) = 192000t^2 - 96000t$$

$$\frac{d}{dt}F(t) = 0$$

$$\frac{d}{dt}\left(192000t^2 - 96000t\right) = 0$$

$$\rightarrow 0 = 384000t - 96000$$

$$\therefore t_{\max} = 0.25\,\text{s}$$

$$F(t) = \frac{d}{dt}p(t)$$

$$F_{\max} = \frac{d}{dt}p(0.25\,\text{s})$$

$$F_{\max} = 192000(0.25\,\text{s})^2 - 96000(0.25\,\text{s})$$

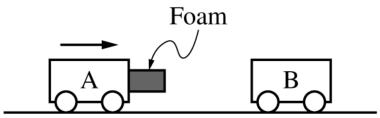
$$\therefore |F_{\max}| = 12{,}000\,\text{N}$$

力—时间图：端点、峰值与凹向同时核对

把上一问求得的力大小函数放回原题时间区间。二次函数的开口决定凹向；先标两端是否为零和内点最大值，再连接成光滑曲线。不能把车的速度曲线形状直接照搬成力的曲线。

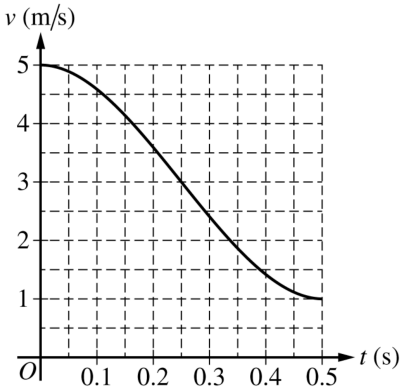
例题 2023 Set 2 Q1 (b)(ii)

本问承接 2023 Set 2 Q1 (b)(i)。



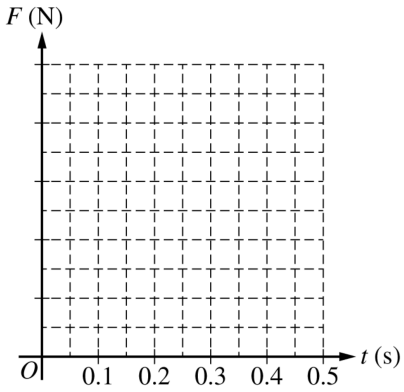
1. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.

(b) For $0 \le t \le 0.50$ s, the velocity v of Cart A can be described by the function $v(t) = 64t^3 - 48t^2 + 5$.



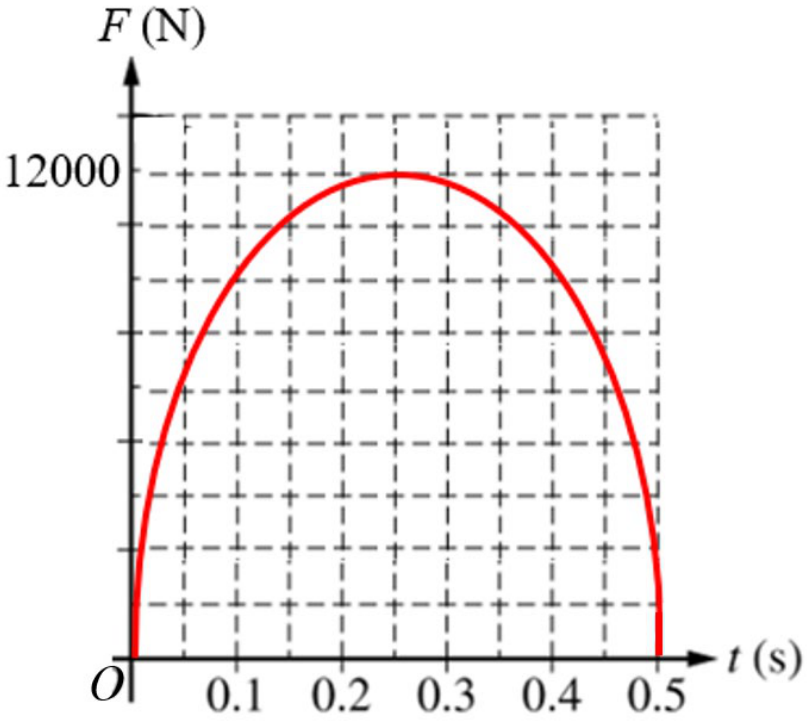
(a) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

ii. On the following grid, draw a smooth curve of the magnitude of the force acting on Cart A as a function of time. Clearly indicate the value of the maximum force on the vertical axis.



解：

画两端为零、始终向下凹的抛物线，并在纵轴标峰值。

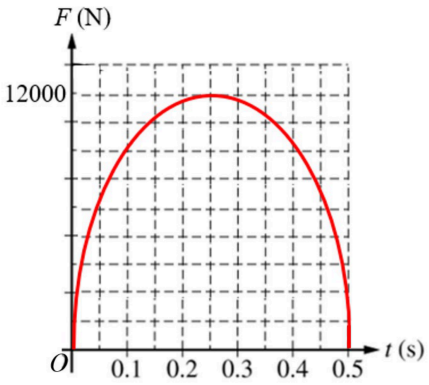


峰值在0.25秒；纵轴画的是力大小。

官方评分

(b)(ii)	For a curve that increases and then decreases in value that is only concave down	**1 point**
	For a labeled maximum value consistent with the calculated value in part (b)(i)	**1 point**
	For the graph having values of 0 N at $t = 0$ s and $t = 0.5$ s	**1 point**

Example Solution



Total for part (b)　7 points

碰撞、冲量与质心

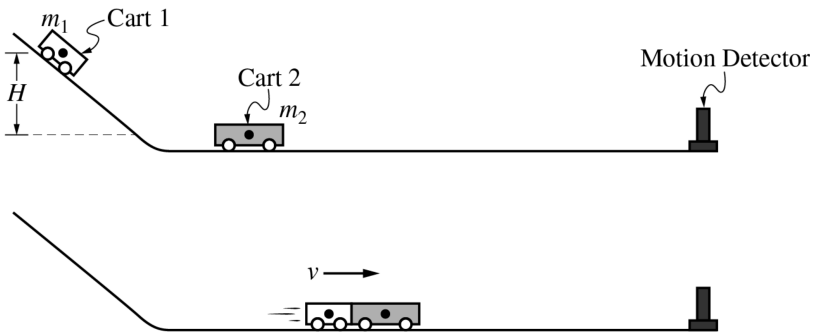
冲量比较：逐段追踪动量变化

冲量是力在时间上的累积，等于该物体的动量变化。两物体在同一次碰撞中相互作用，牛顿第三定律让每一时刻的力等大反向，持续时间相同，故冲量大小相同。

若比较的是先由弹簧推动、后与另一个块碰撞，必须比较两个不同过程的动量变化，不能套用同一次碰撞的等冲量结论。

$$J = \int F\,dt = \Delta p.$$

例题 2022 Set 1 Q2 (a)



2. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

___ Greater than ___ Less than ___ Equal to

Justify your answer.

解：

选Equal to。碰撞中两车的相互作用力每一时刻大小相等、方向相反，且持续时间相同。对同一时间段积累这两个力，冲量大小必相等。

官方评分

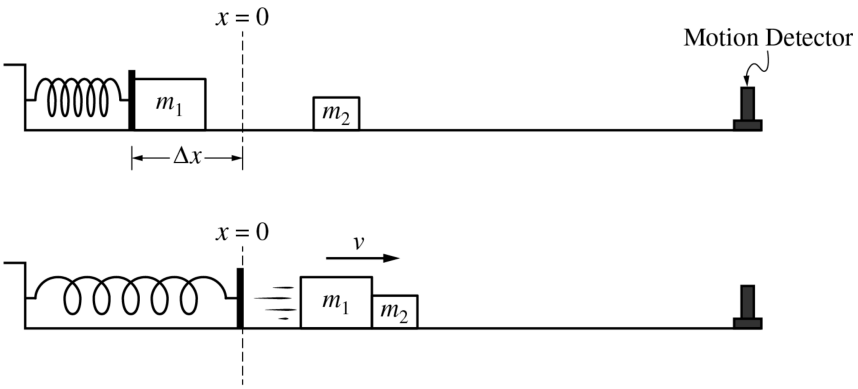
(a)	For selecting "Equal to" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**

Example Response

Newton's third law says equal/opposite forces, time intervals are same during same collision, so magnitudes of impulse must be equal.

Total for part (a) 2 points

例题 2022 Set 2 Q2 (a)



2. Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

(a) The impulse on Block 1 from the spring during the time interval $0 < t < t_C$ is J_S. The impulse on Block 1 from Block 2 during the collision is J_2. Which of the following expressions correctly compares the magnitudes of J_S and J_2 ?

___$J_S > J_2$ ___$J_S < J_2$ ___ $J_S = J_2$

Justify your answer.

解：

弹簧先让块1从静止获得正向动量；碰撞只拿走其中一部分，因为碰后块1仍向前运动。因此弹簧冲量大小大于碰撞使块1损失的动量。

$$J_S > J_2.$$

官方评分

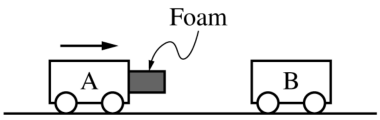
(a)	For selecting "$J_S > J_2$" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**
	Example Response *The impulse between the two blocks is the same, so because Block 1 is moving at the end, the impulse given by the spring must be greater than the impulse given to Block 2 by Block 1.*	
	Total for part (a)	**2 points**

速度—时间图：面积给出位移

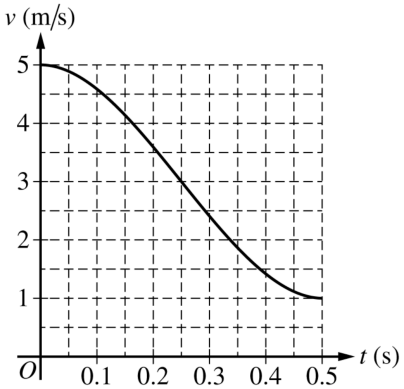
速度表示每秒移动多少距离。把每小段时间的速度乘那段时间，再相加，就是曲线与时间轴之间的有符号面积。原图速度全为正，因此这里的位移直接等于图下的正面积。

$$\Delta x = \int v\,dt.$$

例题 2023 Set 2 Q1 (a)(i)



1. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.



(a) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

i. What feature(s) of the graph could be used to estimate the displacement of Cart A during the collision?

解：

速度—时间图下的有符号面积是位移；这段速度为正，因此直接取曲线与时间轴之间的面积。

官方评分

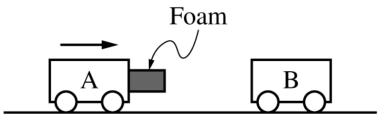
(a)(i)	For stating that the area bounded by the curve is the displacement	**1 point**

动量守恒：由一车末速求另一车末速

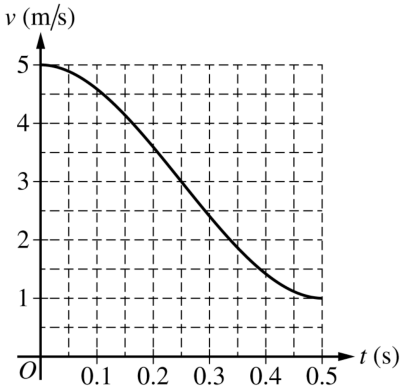
碰撞或接触期间，系统水平外冲量可忽略，两个车的总水平动量不变。先用原图读取车A的初末速度，再把未知的车B末速度放进同一个总动量等式。车B起初静止，它的初动量为零。

$$m_A v_{A0} + m_B v_{B0} = m_A v_A + m_B v_B.$$

例题 2023 Set 2 Q1 (a)(ii)



1. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.



(a) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

ii. Using the information shown in the graph, determine the speed of Cart B at $t = 0.5$ s.

解：

水平外力冲量可忽略，系统动量守恒。由原图，车A速率从5降到1米每秒。

$$1000(5) + 2000(0) = 1000(1) + 2000v_B, \qquad v_B = 2\ \mathrm{m/s}.$$

官方评分

(a)(ii) For indicating momentum is conserved **1 point**

Example Response

$\Sigma p_0 = \Sigma p_f$

$m_{\mathrm{A}}v_{\mathrm{A0}} + m_{\mathrm{B}}v_{\mathrm{B0}} = m_{\mathrm{A}}v_{\mathrm{Af}} + m_{\mathrm{B}}v_{\mathrm{Bf}}$

$m_{\mathrm{A}}v_{\mathrm{A0}} - m_{\mathrm{A}}v_{\mathrm{Af}} = m_{\mathrm{B}}v_{\mathrm{Bf}}$

For the correct answer for speed with units (2 m/s) **1 point**

Example Solution

$m_{\mathrm{A}}v_{\mathrm{A0}} + m_{\mathrm{B}}v_{\mathrm{B0}} = m_{\mathrm{A}}v_{\mathrm{Af}} + m_{\mathrm{B}}v_{\mathrm{Bf}}$

$m_{\mathrm{A}}v_{\mathrm{A0}} - m_{\mathrm{A}}v_{\mathrm{Af}} = m_{\mathrm{B}}v_{\mathrm{Bf}}$

$v_{\mathrm{Bf}} = \dfrac{m_{\mathrm{A}}}{m_{\mathrm{B}}}(v_{\mathrm{A0}} - v_{\mathrm{Af}})$

$\rightarrow v_{\mathrm{Bf}} = \dfrac{1000\ \mathrm{kg}}{2000\ \mathrm{kg}}(5\ \mathrm{m/s} - 1\ \mathrm{m/s})$

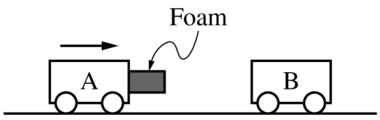
$\therefore v_{\mathrm{Bf}} = 2\ \mathrm{m/s}$

每一时刻守恒：由一车曲线画另一车曲线

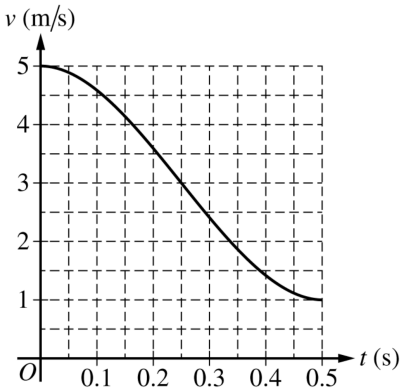
总动量守恒不只比较碰撞前后，也适用于接触中的每一时刻。车A减少多少动量，车B就增加多少。先把vB写成vA的函数，再对题目给定的时间函数逐点判断起点、终点和凹向；曲线保持光滑。

例题 2023 Set 2 Q1 (a)(iii)

本问承接 2023 Set 2 Q1 (a)(ii)。

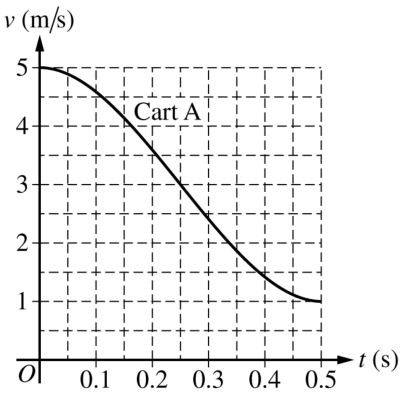


1. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.



(a) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

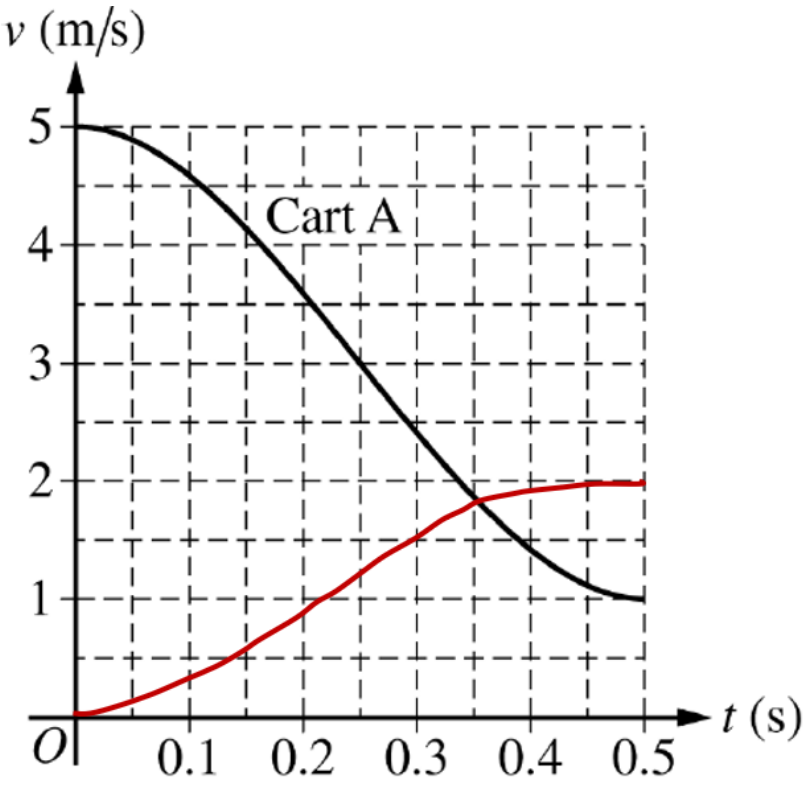
iii. On the following grid, draw a smooth curve of the velocity of Cart B as a function of time.



解：

对每一时刻都用动量守恒，车B增加的动量等于车A减少的动量。代入给定车A速度，得到应画的光滑曲线。

$$v_B(t) = \frac{1000}{2000}[5 - v_A(t)] = 24t^2 - 32t^3.$$

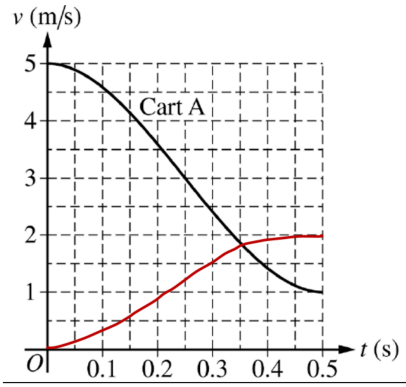


车B先向上凹后向下凹，在0.5秒到达2米每秒。

官方评分

(a)(iii)	For a graph that starts at $(0,0)$ and ends at $(0.5,2)$ or value consistent with (a)(ii)	**1 point**
	For a smooth, continuous curve that transitions from concave up to concave down	**1 point**

Example Solution



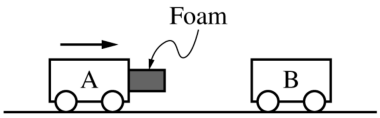
Total for part (a)　5 points

同一冲量：接触时间越长，平均力越小

同一车具有相同初末速度，质量也相同，动量变化就相同。平均力等于冲量除以接触时间。缓冲材料延长接触时间，使同一个动量变化分散在更长时间内；这改变平均力，不改变总冲量。

$$F_{\text{avg}} = \frac{|\Delta p|}{\Delta t}.$$

例题 2023 Set 2 Q1 (c)



1. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.

The foam is removed from the front of Cart A and the experiment is repeated. The carts collide, with both Cart A and Cart B having the same initial and final velocities as in the original collision. The time intervals during which the carts are in contact are different in the collision with the foam and the collision without the foam. In the collision without the foam, Cart A is in contact with Cart B for a shorter duration than in the original collision, when the foam was present.

For the original collision when the foam is present, the magnitude of the average net force exerted on Cart B is F_1. For the collision without the foam, the magnitude of the average net force exerted on Cart B is F_2.

(c) What is the relationship between the magnitude of the average net force F_1 exerted on Cart B for the collision with the foam and the magnitude of the average net force F_2 exerted on Cart B for the collision without the foam?

_____ $F_1 > F_2$ _____ $F_1 < F_2$ _____ $F_1 = F_2$

Justify your answer.

解：

选有泡沫时平均力较小。两种情形的初末速度相同，块B的动量变化和冲量相同；同一冲量分散到更长时间，平均力就更小。

$$\bar{F} = \frac{\Delta p}{\Delta t}, \qquad F_1 < F_2.$$

官方评分

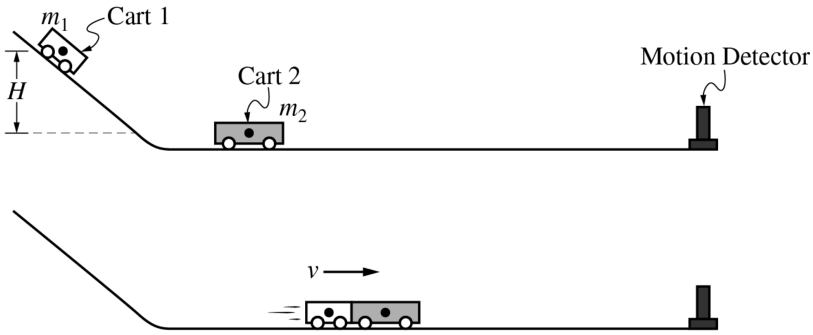
(c)	For selecting $F_1 < F_2$ with an attempt at a relevant justification	**1 point**
	For indicating that the impulse or change in momentum of each cart in both collisions is the same	**1 point**
	For indicating that decreasing the time of collision means the average force must be greater	**1 point**
	Example Solution *Since the initial and final velocities are the same for both collisions, Δp is the same for both collisions; as a result, the impulse is the same for both collisions. So, if Δt is smaller, F_{avg} is larger.*	
	Total for part (c)	**3 points**
	Total for question 1	**15 points**

先获得速度再粘连：能量与动量分阶段

物体在光滑斜面或弹簧上获得速度，先用机械能守恒。随后粘连碰撞，选两个物体为系统，只守恒总动量。把碰撞前速率暂记为u，先求u，再求共同末速度；不能把机械能守恒直接跨过粘连过程。

$$m_1 u = (m_1 + m_2)v.$$

例题 2022 Set 1 Q2 (c)



2. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}.$$

解：

先处理下坡，再处理粘连，不能跨过碰撞使用机械能守恒。把车1碰前速率记为u。

$$m_1 gH = \frac{1}{2} m_1 u^2, \qquad u = \sqrt{2gH}.$$

$$m_1 u + m_2(0) = (m_1 + m_2)v, \qquad v = \frac{m_1\sqrt{2g}}{m_1 + m_2}\sqrt{H}.$$

官方评分

(c)	For using conservation of energy to find the speed of Cart 1 at the bottom of the incline **OR** For a correct substitution of acceleration and displacement in a kinematics equation to find the speed of Cart 1 at the bottom of the incline	**1 point**
	For using conservation of momentum to find the speed of the two-cart system after the collision	**1 point**
	For combining correct equations from above	**1 point**

Example Response

Conservation of energy:

$m_1 gH = \frac{1}{2} m_1 v_1^2$

$v_1 = \sqrt{2gH}$

OR

$v_f^2 = v_i^2 + 2g\sin(\theta)L$

$H = L\sin(\theta)$

$v_f^2 = v_i^2 + 2g\sin(\theta)\left(\frac{H}{\sin\theta}\right)$

$v_f^2 = 2gH$

Conservation of momentum:

$m_1 v_1 = (m_1 + m_2)v_f$

$v_f = \frac{m_1}{m_1 + m_2} v_1$

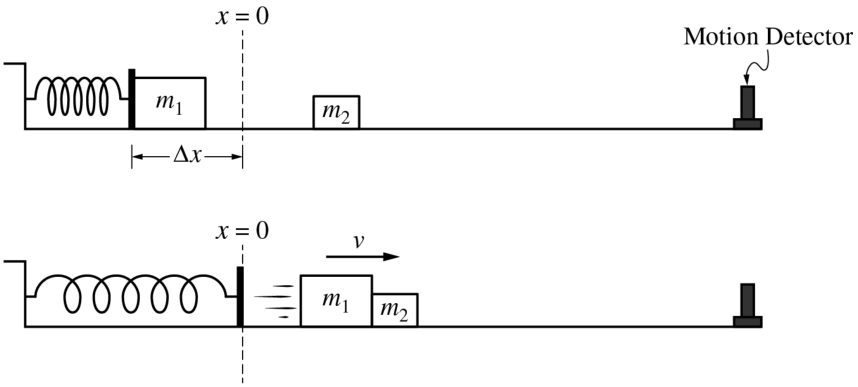
Combining:

$v_f = \frac{m_1}{m_1 + m_2}\sqrt{2gH}$

$v_f = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}$

Total for part (c) 3 points

练习 2022 Set 2 Q2 (c)



2. Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

(c) Show that the velocity v of the two-block system after the collision is given by the equation $v = \dfrac{\sqrt{km_1}}{m_1 + m_2}\Delta x$.

解：

弹簧释放阶段守恒机械能，粘连阶段守恒水平动量。

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m_1u^2, \qquad u = \Delta x\sqrt{\frac{k}{m_1}}.$$

$$m_1u = (m_1 + m_2)v, \qquad v = \frac{\sqrt{km_1}}{m_1 + m_2}\Delta x.$$

官方评分

(c)	For using conservation of energy to find the speed of Block 1 at $x = 0$	**1 point**
	For using conservation of momentum to find the speed of the two-block system after the collision	**1 point**
	For combining correct equations from above	**1 point**

Example Response

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m_1 v_1^2$$

$$v_1 = \sqrt{\frac{k}{m_1}}\Delta x$$

$$m_1 v_1 = (m_1 + m_2)v$$

$$v = \frac{m_1 v_1}{m_1 + m_2}$$

$$\therefore v = \frac{\Delta x\sqrt{km_1}}{m_1 + m_2}$$

Total for part (c) 3 points

练习 2023 Set 1 Q1 (c)(i)

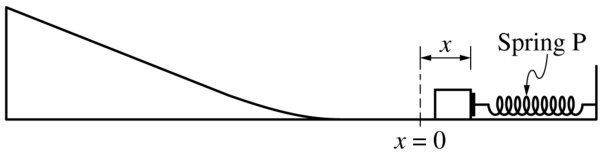


Figure 1

1. A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{max,P}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the maximum height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_{\text{P}}(x) = kx$, where $k = 100.0\,\text{N/m}$, and for Spring Q, $F_{\text{Q}}(x) = Cx^{1/2}$, where $C = 20.0\,\text{N/m}^{1/2}$.

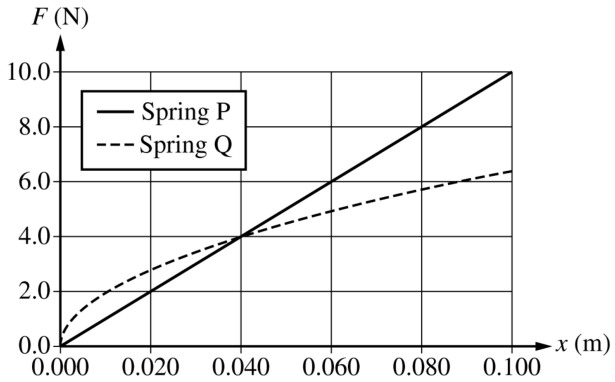


Figure 2

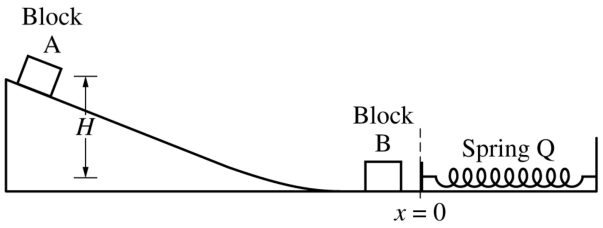


Figure 3

(c) Spring Q is attached to the wall and is at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp and Block B is at rest on the horizontal surface. The student releases Block A, and it moves down the ramp and collides with Block B. The change in vertical height of Block A is $H = 0.75$ m. After the collision, the blocks stick together and move to the right, compressing the spring. Frictional forces between the blocks and all surfaces are negligible.

i. Calculate the velocity of the two-block system immediately after the collision between Blocks A and B.

解：

下坡先用能量求A碰前速度，碰撞再用动量求两块共同速度。取向右为正。

$$m_A g H = \frac{1}{2} m_A u^2, \qquad u = \sqrt{2gH} = 3.83\,\text{m/s}.$$

$$m_A u = (m_A + m_B) v, \qquad v = \frac{0.120}{0.190}(3.83) = 2.42\,\text{m/s}.$$

官方评分

(c)(i) For correctly applying conservation of energy to solve for the velocity of Block A just before it collides with Block B **1 point**

Example Response

$m_A gH = \frac{1}{2} m_A v_A^2$

$v_A = \sqrt{2gH}$

$v_A = \sqrt{2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(0.75\ \text{m})}$

$\therefore v_A = 3.8\ \text{m/s}$

For substituting the calculated value for v_A into a correct conservation of linear momentum expression and solving for v_f **1 point**

Example Response

$m_A v_A = (m_A + m_B) v_f$

$v_f = \frac{m_A v_A}{(m_A + m_B)}$

$v_f = \frac{(0.120\ \text{kg})(3.8\ \text{m/s})}{(0.120\ \text{kg} + 0.070\ \text{kg})}$

$\therefore v_f = 2.4\ \text{m/s}$

Example Solution

$m_A gH = \frac{1}{2} m_A v_A^2$

$v_A = \sqrt{2gH}$

$v_A = \sqrt{2\left(9.8 \frac{m}{s^2}\right)(0.75\ m)}$

$\therefore v_A = 3.8\ \text{m/s}$

$m_A v_A = (m_A + m_B) v_f$

$v_f = \frac{m_A v_A}{(m_A + m_B)}$

$v_f = \frac{(0.120\ \text{kg})(3.8\ \text{m/s})}{(0.120\ \text{kg} + 0.070\ \text{kg})}$

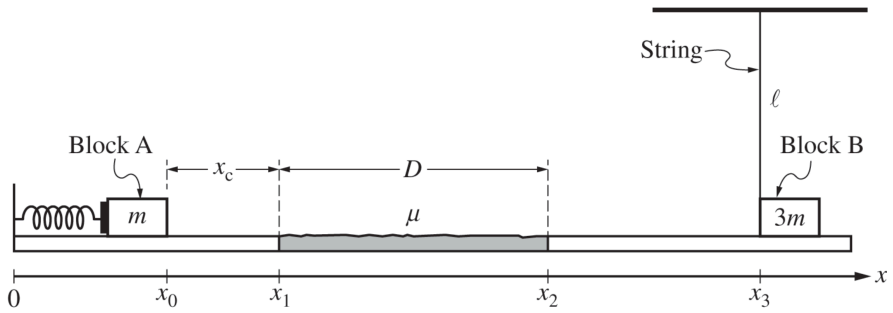
$\therefore v_f = 2.4\ \text{m/s}$

能量先被摩擦扣除：再处理粘连碰撞

经过粗糙水平段时，先从原来弹簧能量中扣掉μmgD；碰撞前剩下的动能确定A的速率。再用总动量守恒求两块共同速率。根号里的量需要为正，正是原题确实到达碰撞位置的条件。

例题 2024 Set 1 Q1 (a)(ii)

本问承接 2024 Set 1 Q1 (a)(i)。



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

解：

通过粗糙区先扣除摩擦耗能，再处理粘连碰撞。把A碰前速率记为u。

$$\frac{1}{2}mu^2 = \frac{1}{2}kx_c^2 - \mu mgD, \qquad u = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}.$$

$$mu + 3m(0) = 4mv_{A,B}, \qquad v_{A,B} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}.$$

根式需为正，对应原题确实穿过粗糙区并发生碰撞；碰撞期间动能并不守恒。

官方评分

(a)(ii) For a derivation to solve for the speed at x_2 that includes **one** of the following: **1 point**

- An appropriate application of the conservation of energy
- An appropriate kinematics equation

Example Responses

$K_{\text{initial}} - \Delta E_{\text{friction}} = K_{\text{final}}$ **OR** $v^2 = v_0^2 + 2a\Delta x$

For **one** of the following that is consistent with the previous point in the response for part (a)(ii): **1 point**

- A correct expression for the energy dissipated by friction
- A correct expression for the acceleration of the block in the region with nonnegligible friction

Example Responses

$\Delta E_{\text{friction}} = \mu mgD$ **OR** $a = -\mu g$

For attempting to derive an expression for $v_{\text{A,B}}$ by using the conservation of momentum **1 point**

Example Response

$m_{\text{initial}} v_{\text{initial}} = m_{\text{A,B}} v_{\text{A,B}}$

For substituting the expression for the speed at x_2 that is consistent with the first point of the response in part (a)(ii) and substituting the correct masses into an expression for conservation of momentum **1 point**

Example Response

$$v_{\text{A,B}} = \frac{m\sqrt{\frac{kx_c^{\,2}}{m} - 2\mu gD}}{4m}$$

Example Solutions

$E_{x_1} = E_{\text{before collision}}$

$K_{x_1} - \Delta E_{\text{friction}} = K_{\text{before collision}}$

$\frac{1}{2}m\left(\sqrt{\frac{kx_c^2}{m}}\right)^2 - \mu mgD = \frac{1}{2}mv_2^2$

$v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$

$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$

$m_A v_2 = m_{A,B} v_{A,B}$

$m_A v_2 = (m + 3m) v_{A,B}$

$v_{A,B} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$

$v_{A,B} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}$

OR

$v_2^2 = v^2 + 2a\Delta x$

$v_2^2 = \left(x_c\sqrt{\frac{k}{m}}\right)^2 + 2aD$

$v_2 = \sqrt{\frac{kx_c^2}{m} + 2aD}$

$\Sigma F_x = -F_f = ma$

$-\mu mg = ma$

$a = -\mu g$

$v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$

$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$

$m_A v_2 = m_{A,B} v_{A,B}$

$m_A v_2 = (m + 3m) v_{A,B}$

$v_{A,B} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$

$v_{A,B} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}$

Total for part (a) 6 points

粘连以后压弹簧：动量先给速率，能量再给形变

粘连瞬间用动量守恒；之后两个块作为共同物体，在光滑路段把共同动能转为弹簧储能。线性弹簧用$kx^2/2$；若题目给的是平方根力，仍要用前面已经求出的力积分储能。依题目所求，最后反解最大压缩量或弹簧常量。

例题 2023 Set 1 Q1 (c)(ii)

本问承接 2023 Set 1 Q1 (c)(i)、2023 Set 1 Q1 (b)(iii)。

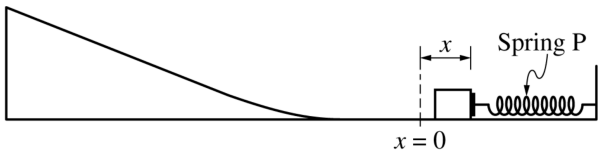


Figure 1

1. A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the maximum height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_{\text{P}}(x) = kx$, where $k = 100.0\,\text{N/m}$, and for Spring Q, $F_{\text{Q}}(x) = Cx^{1/2}$, where $C = 20.0\,\text{N/m}^{1/2}$.

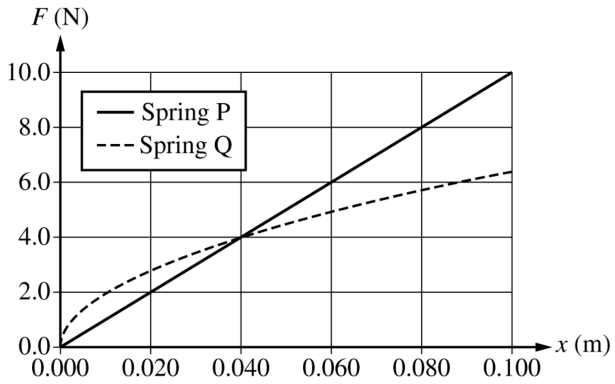


Figure 2

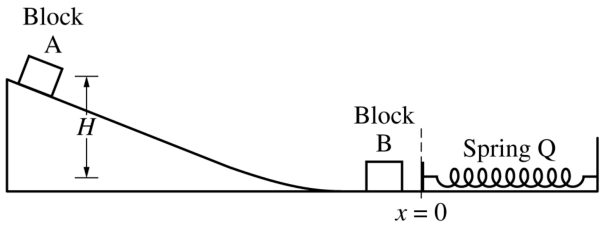


Figure 3

(c) Spring Q is attached to the wall and is at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp and Block B is at rest on the horizontal surface. The student releases Block A, and it moves down the ramp and collides with Block B. The change in vertical height of Block A is $H = 0.75$ m. After the collision, the blocks stick together and move to the right, compressing the spring. Frictional forces between the blocks and all surfaces are negligible.

ii. Calculate the maximum compression of the spring.

解：

碰后到最大压缩阶段没有耗散，用碰后动能等于Q弹簧的储能。

$$\frac{1}{2}(0.190)v^2 = \frac{2}{3}Cx_{\text{max}}^{3/2}, \qquad x_{\text{max}} = \left(\frac{3(0.190)v^2}{4C}\right)^{2/3} \approx 0.120\ \text{m}.$$

官方评分

(c)(ii) For equating kinetic energy of the two-block system and elastic potential energy of the spring **1 point**

Example Response

$$\frac{1}{2}mv^2 = \int F(x)\,dx$$

For correctly integrating to determine the elastic potential energy at the spring's maximum compression **1 point**

Example Response

$$\frac{1}{2}mv^2 = \int_0^{x_{\max}} Cx^{1/2}dx$$
$$\frac{1}{2}mv^2 = \frac{2}{3}Cx_{\max}{}^{3/2}$$

For substituting the value for v_f from part (c)(i) into the integrated equation and solving for $x_{\max}$ **1 point**

Example Response

$$x_{\max} = \left(\frac{3}{4}\frac{mv^2}{C}\right)^{2/3}$$
$$x_{\max} = \left(\frac{3}{4}\frac{(0.190\text{ kg})(2.4\text{ m/s})^2}{20.0\text{ N/m}^{1/2}}\right)^{2/3}$$

Example Solution

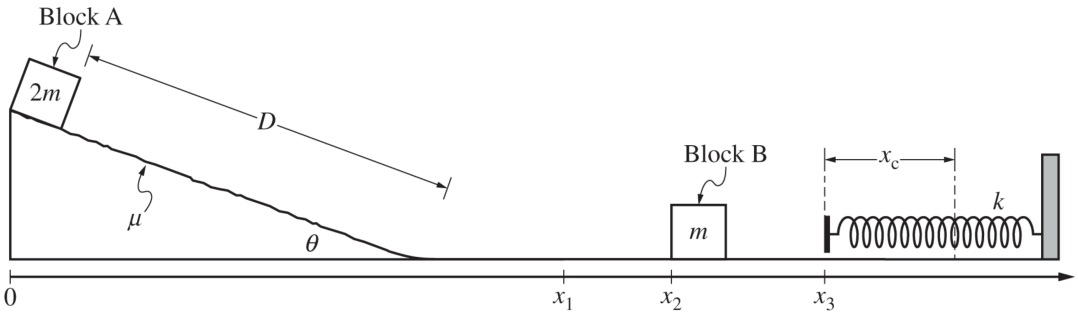
$$K = U_s$$
$$\frac{1}{2}mv^2 = \int F(x)\,dx$$
$$\frac{1}{2}mv^2 = \int_0^{x_{\max}} Cx^{1/2}dx$$
$$\frac{1}{2}mv^2 = \frac{2}{3}Cx_{\max}{}^{3/2}$$

$$\rightarrow x_{\max} = \left(\frac{1}{2}mv^2\frac{3}{2C}\right)^{2/3}$$
$$x_{\max} = \left(\frac{3}{4}\frac{mv^2}{C}\right)^{2/3}$$
$$x_{\max} = \left(\frac{3}{4}\frac{(0.190\text{ kg})(2.4\text{ m/s})^2}{20.0\text{ N/m}^{1/2}}\right)^{2/3}$$
$$\therefore x_{\max} = 0.12\text{ m}$$

Total for part (c) 5 points

例题 2024 Set 2 Q1 (a)(ii)

本问承接 2024 Set 2 Q1 (a)(i)。



Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

ii. **Derive** an expression for the spring constant k of the spring.

解：

水平光滑段速度不变。粘连先用动量求共同速度，压缩再用能量求弹簧常量。

$$2mv = 3mu, \qquad u = \frac{2}{3}v.$$

$$\frac{1}{2}(3m)u^2 = \frac{1}{2}kx_c^2, \qquad k = \frac{8mgD(\sin\theta - \mu\cos\theta)}{3x_c^2}.$$

官方评分

(a)(ii) For using the conservation of momentum to find $v_{A,B}$ **1 point**

For equating the kinetic energy after the collision between the blocks to the maximum elastic potential energy of the compressed spring **1 point**

Example Response

$K_{\text{after collision}} = U_{s,\,\max}$

For indicating v before the collision between the blocks and the spring is equal to $v_{A,B}$ **1 point**

Example Solution

$$p_{\text{before collision}} = p_{\text{after collision}}$$
$$2mv = (2m + m)v_{A,B}$$
$$v_{A,B} = \frac{2m\sqrt{2gD(\sin\theta - \mu\cos\theta)}}{(3m)}$$
$$v_{A,B} = \frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}$$

$$E_{\text{after collision}} = E_{\text{max compression of spring}}$$
$$K_{\text{after collision}} = U_{s,\,\max}$$
$$\frac{1}{2}(2m + m)\left(v_{A,B}\right)^2 = \frac{1}{2}kx^2$$
$$(3m)\left(\frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}\right)^2 = k{x_c}^2$$

$$k = \frac{(3m)}{{x_c}^2}\left(\frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}\right)^2$$

$$k = \frac{8}{3}\frac{mgD(\sin\theta - \mu\cos\theta)}{{x_c}^2}$$

Total for part (a) 6 points

同样输入能量：压缩更多意味着弹力系数更小

两个不同的非线性弹簧接收相同动能，停止时必须储存相同能量。同一幂次的力模型中，储能正比于力系数，也随压缩量增大。需要压缩得更多才存下相同能量的一根弹簧，其系数较小。

例题 2023 Set 1 Q1 (d)

本问承接 2023 Set 1 Q1 (c)(ii)。

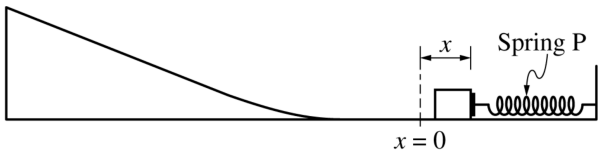


Figure 1

1. A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the maximum height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_{\text{P}}(x) = kx$, where $k = 100.0\,\text{N/m}$, and for Spring Q, $F_{\text{Q}}(x) = Cx^{1/2}$, where $C = 20.0\,\text{N/m}^{1/2}$.

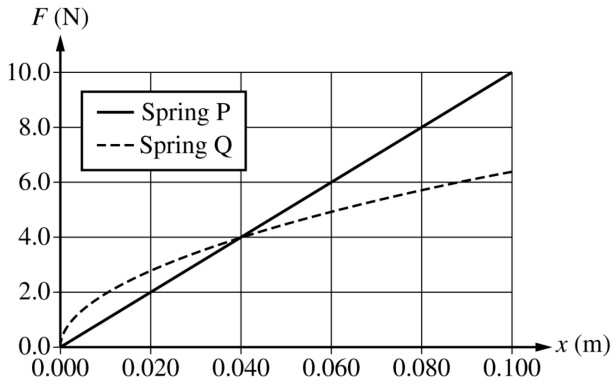


Figure 2

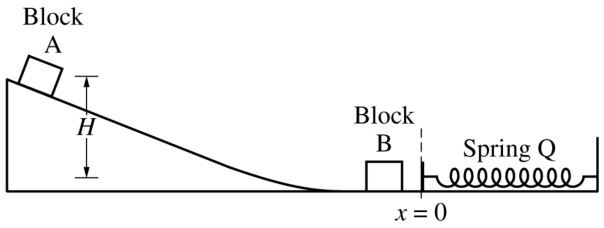


Figure 3

(c) Spring Q is attached to the wall and is at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp and Block B is at rest on the horizontal surface. The student releases Block A, and it moves down the ramp and collides with Block B. The change in vertical height of Block A is $H = 0.75$ m. After the collision, the blocks stick together and move to the right, compressing the spring. Frictional forces between the blocks and all surfaces are negligible.

(d) Spring Q where $F_{\mathrm{Q}}(x) = Cx^{1/2}$ is replaced by a different nonlinear spring, Spring R, and the procedure described in part (c) is repeated. For Spring R, $F_{\mathrm{R}}(x) = Dx^{1/2}$. The maximum compression of Spring R is greater than the maximum compression of Spring Q. Which of the following correctly compares the constants C and D ?

_____$C < D$ _____$C > D$ _____ $C = D$

Briefly justify your answer.

解:

选C大于D。两次进入弹簧前的动能相同，最后储能相同；R在更大压缩量时才储存同样能量，说明R的力系数较小。

$$\frac{2}{3}Cx_Q^{3/2} = \frac{2}{3}Dx_R^{3/2}, \qquad x_R > x_Q \implies C > D.$$

官方评分

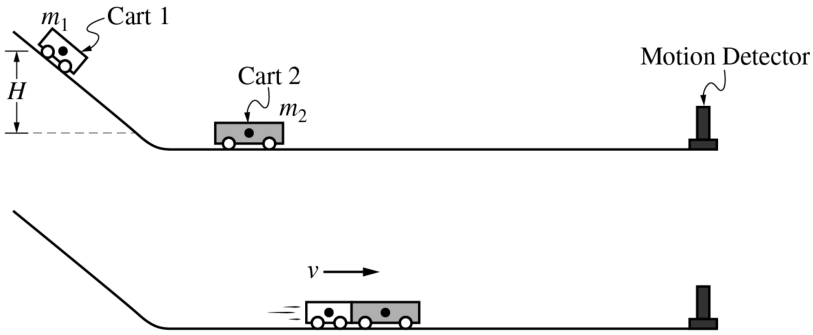
(d)	For selecting $C > D$ with an attempt at a relevant justification	**1 point**
	For one of the following: • a statement that correctly relates the total energy of the system, the maximum compression distance of the springs, and the spring constant • a statement that correctly relates the force exerted on the blocks by the spring, the maximum compression distance of the springs, and the spring constant	**1 point**
	Example Solutions *The energy of the two-block-spring system before the blocks compress the spring is the same in both procedures, so the total potential energy of both springs must be the same when each spring is at its maximum compression. Since Spring Q is compressed less than Spring R, C must be greater than D.* **OR** *The blocks are traveling at the same speed before colliding with the spring in each procedure. The maximum compression of Spring Q is less than Spring R, so the average force exerted on the blocks by Spring Q to stop the blocks must be greater than that of Spring R. Therefore, C must be greater than D.*	
	Total for part (d)	**2 points**
	Total for question 1	**15 points**

动量图分阶段：力决定曲线斜率

动量用p表示，即质量乘速度；动量对时间的斜率是物体受到的合力。恒定沿斜面合力给直线；弹簧力逐渐减小给向下凹的上升曲线；光滑路段给水平线。粘连持续时间可忽略时，在碰撞时刻画跳变，并使两物体动量变化等大反向。

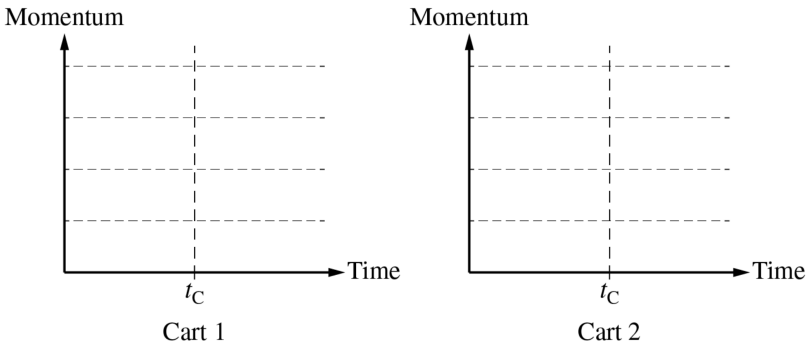
$$\frac{dp}{dt} = F_{\text{net}}.$$

例题 2022 Set 1 Q2 (b)



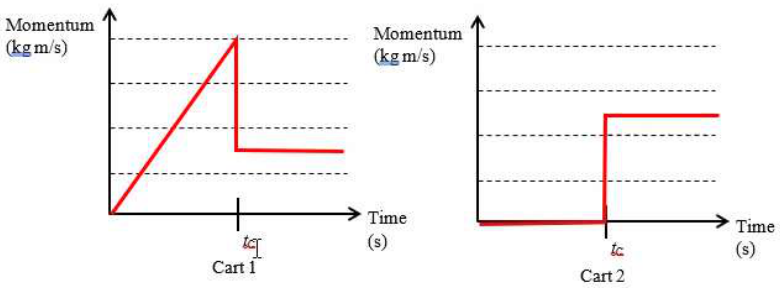
2. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



解：

碰撞前车1沿斜面恒加速，动量大小线性增加；车2静止，动量为零。碰后两车共速，较重的车2动量较大；各自匀速，图为水平线。车1损失的动量等于车2增加的动量。

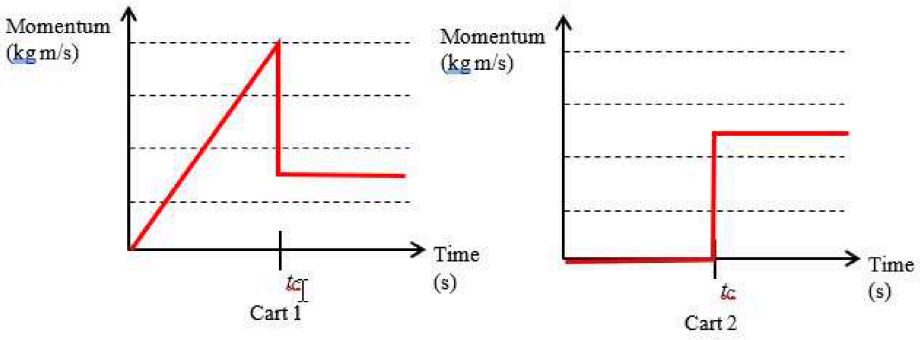


两车动量用同一纵向尺度；碰撞使等量动量从车1转移给车2。

官方评分

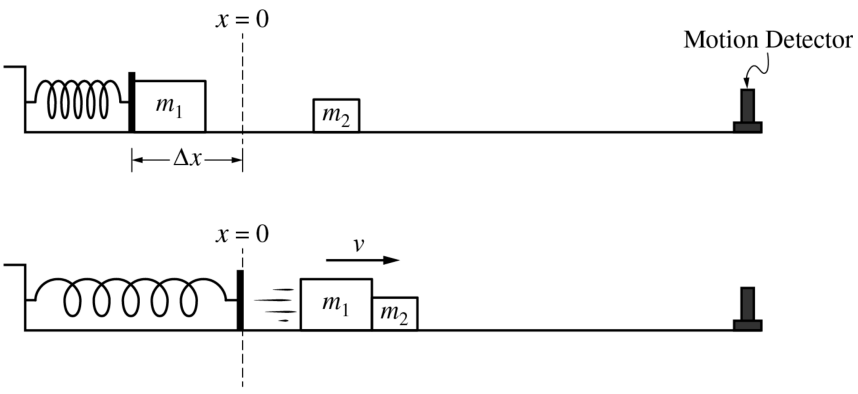
(b)	For correctly drawing the momentum for carts 1 and 2 for the time interval $0 < t < t_C$: Linear increasing momentum for Cart 1 and zero for Cart 2	**1 point**
	For drawing a horizontal line for Cart 1 when $t > t_C$ that is smaller in magnitude than the momentum of Cart 1 at time $t = t_C$	**1 point**
	For drawing a horizontal line for Cart 2 when $t > t_C$ that is greater in magnitude than the momentum of Cart 1 after time $t = t_C$	**1 point**
	For carts 1 and 2 having a change in momentum that is equal in magnitude, such that Cart 1 loses momentum and Cart 2 gains momentum or a response with changes in momentum consistent with the response in part (a)	**1 point**

Example Response



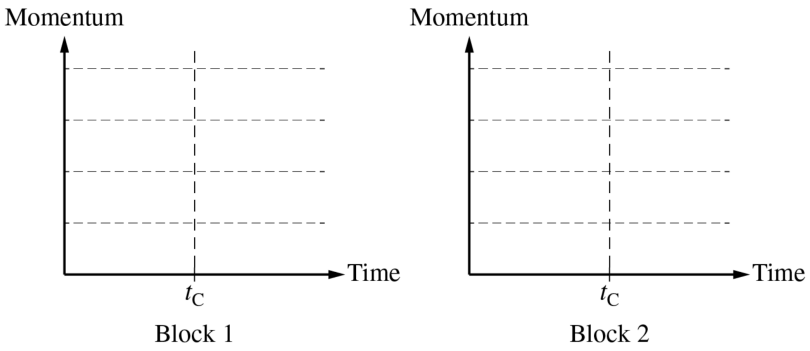
Total for part (b)　4 points

例题 2022 Set 2 Q2 (b)



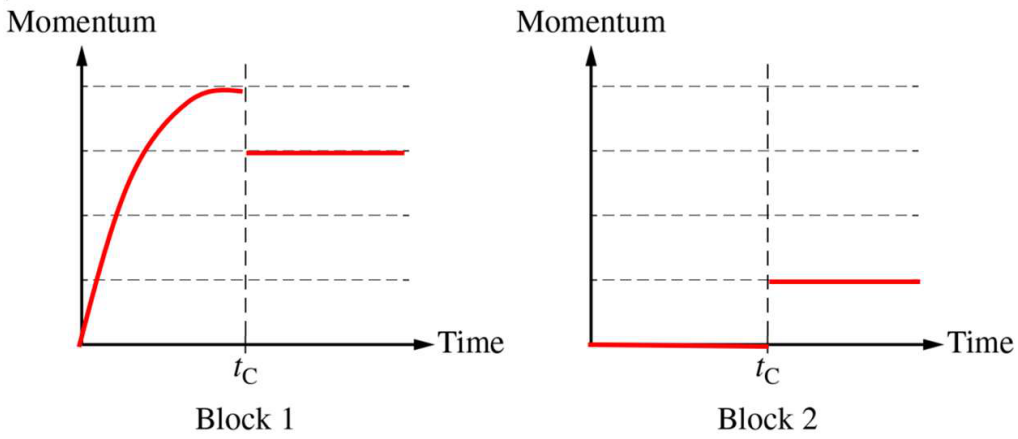
2. Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

(b) On the following axes, draw graphs of the magnitude of the momentum of each block as a function of time, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



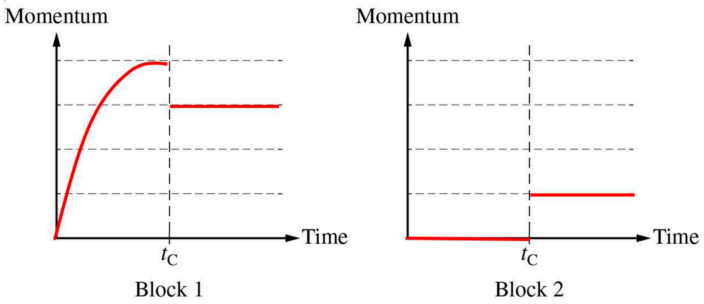
解：

弹簧力随释放而减小，所以块1动量先增加，但斜率逐渐降至零，曲线向下凹；块2仍为零。碰后两块共速而块1更重，故块1水平平台较高。两图碰撞跳变的大小相同、方向相反。



弹簧释放阶段的斜率递减；碰后重块1动量较大。

官方评分

(b)	For correctly drawing the momentum for blocks 1 and 2 for the time interval $0 < t < t_C$: Block 1 shows an increasing curve with a decreasing slope (concave down) to highest value (technically a cosine curve) and Block 2 shows zero momentum up to t_C (a horizontal line along the time axis).	**1 point**
	For drawing a horizontal line for Block 1 when $t > t_C$ that is smaller in magnitude than the momentum of Block 1 at time $t = t_C$	**1 point**
	For drawing a horizontal line for Block 2 when $t > t_C$ that is smaller in magnitude than the momentum of Block 1 after time $t = t_C$	**1 point**
	For blocks 1 and 2 having a change in momentum that is equal in magnitude such that Block 1 loses momentum and Block 2 gains momentum	**1 point**
	Example Response  	
		Total for part (b) 4 points

系统动量箭头：先按质量和方向数格

动量是质量乘带方向的速度。取一个mv_0为一格，向右箭头为正，向左为负；系统动量是两物体的有符号和。系统水平外冲量可忽略，碰前和碰后的系统箭头相同，即使两物体已经粘连。

例题 2025 Q1 (A)(i)

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

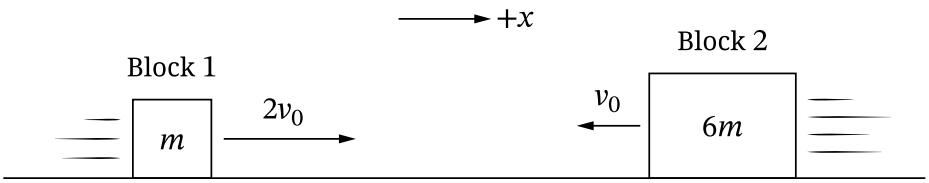


Figure 1

A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

i. **Draw** arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.

- Arrows should start at the zero-momentum line.
- The length of the arrows should be proportional to the relative magnitudes of the vectors.
- Represent an arrow of zero length by drawing a dot at zero.

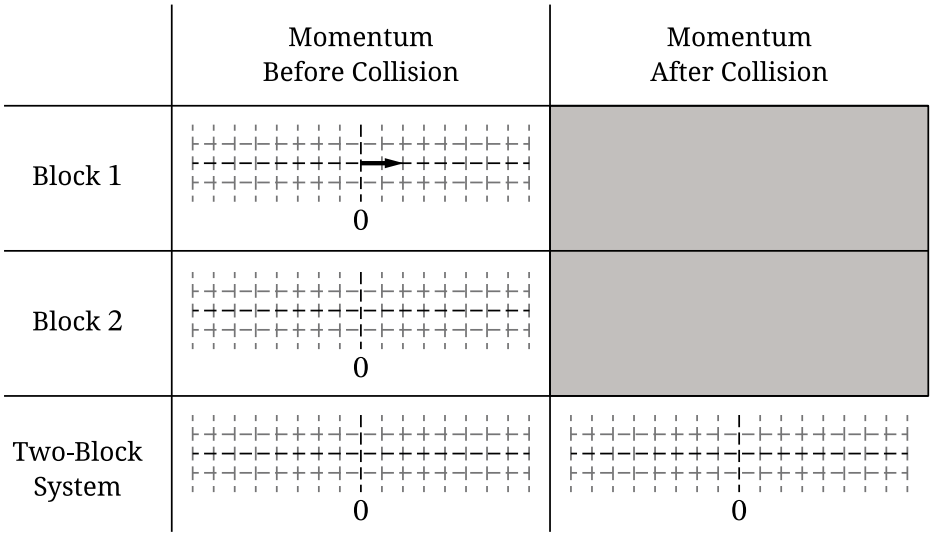


Figure 2

解：

以一个mv_0为一格，块1原有向右两格，块2原有向左六格。系统总动量是向左四格；粘连不改变系统总动量，因此碰前和碰后系统箭头相同。

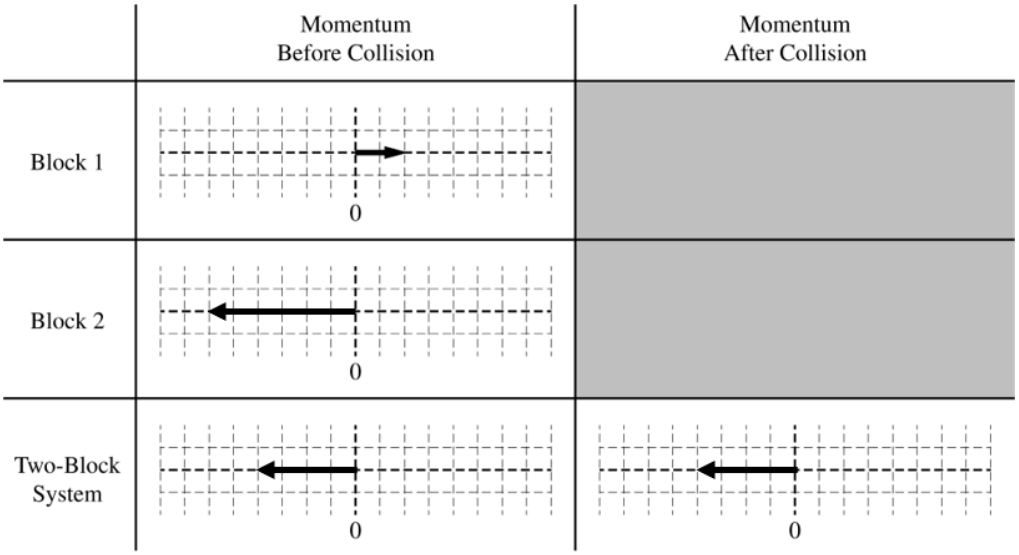


Figure 2

箭头从零线出发；正负动量不能先都取大小再相加。

官方评分

A	(i)	For drawing only one arrow for the momentum of Block 2 before the collision that points leftward and is 6 units long	**Point A1**
		For drawing only identical arrows for the momentum of the two-block system before and after the collision that are equal to the sum of the momentums drawn for blocks 1 and 2 before the collision	**Point A2**

Example Response

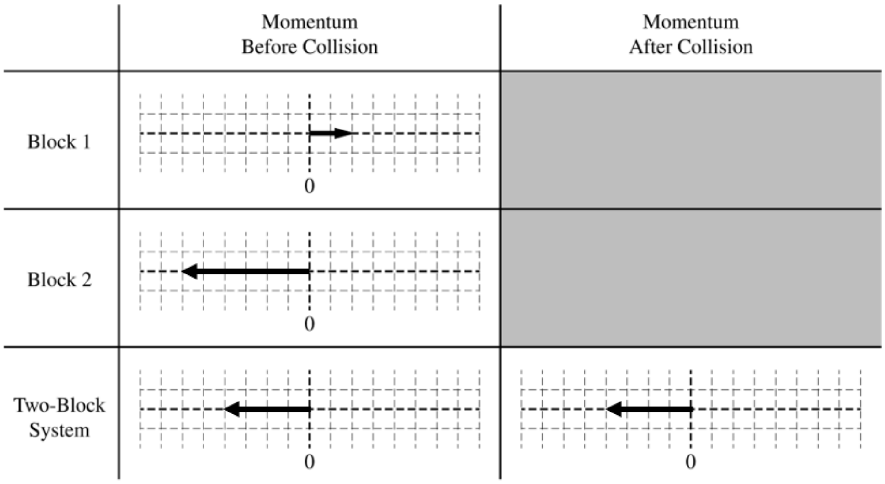


Figure 2

正弦碰撞力：冲量积分反求峰值

先用总动量守恒得到共同末速度，再只看块2的动量变化。题给力为Fmax sin(At)，冲量要从0积分到碰撞结束tc。正弦的原函数为负余弦；因为内部是At，还要除以常量A。最后解出未知峰值。

$$\int_0^{t_c} \sin(At)\,dt = \frac{1-\cos(At_c)}{A}.$$

例题 2025 Q1 (A)(ii)

本问承接 2025 Q1 (A)(i)。

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

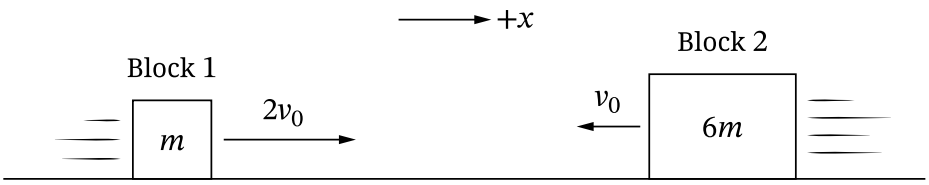


Figure 1

A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

ii. During the time interval $0 \le t \le t_c$, a force F is exerted on Block 2 by Block 1 along the x-direction as a function of t that is modeled by $F(t) = F_{\max}\sin(At)$, where A is a positive constant and $F_{\max}$ is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.

Derive an expression for $F_{\max}$. Express your answer in terms of m, v_0, A, t_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

先用系统动量求共同末速度，再单独求块2的动量变化。取向右为正。

$$m(2v_0) + 6m(-v_0) = 7mv_f, \qquad v_f = -\frac{4}{7}v_0.$$

$$\Delta p_2 = 6m\left(-\frac{4}{7}v_0 - (-v_0)\right) = \frac{18}{7}mv_0.$$

力随时间变化，冲量须积累整个碰撞期间的力。正弦的原函数带负余弦，并且要除以常量A。

$$\Delta p_2 = \int_0^{t_c} F_{\max} \sin(At)\, dt = \frac{F_{\max}}{A}[1 - \cos(At_c)].$$

$$F_{\max} = \frac{18Amv_0}{7[1 - \cos(At_c)]}.$$

官方评分

(ii)	For a multistep derivation that includes the integral form of impulse or the differential form of Newton's second law	**Point A3**
	For indicating **one** of the following: • The final speed of Block 1 or Block 2 is $\frac{4}{7}v_0$. • The change in the momentum of Block 2 is $+\frac{18}{7}mv_0$. • The change in the momentum of Block 1 is $-\frac{18}{7}mv_0$. • The change in the velocity of Block 2 is $+\frac{3}{7}v_0$. • The change in the velocity of Block 1 is $-\frac{18}{7}v_0$.	**Point A4**
	For substituting the given expression for $F(t)$ into an expression for the impulse exerted on either block or the differential form of Newton's second law	**Point A5**
	For an integral with appropriate limits or a constant of integration	**Point A6**
	For a correct expression for $F_{\max}$ in terms of given quantities	**Point A7**

Example Responses

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}(t)\,dt = \Delta\vec{p} \qquad \text{OR} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}, \quad \int \vec{F}_{\text{net}}\,dt = \Delta\vec{p}$$

$$\sum \vec{p}_0 = \sum \vec{p}_f$$
$$(m)(2v_0) + (6m)(-v_0) = (m + 6m)(v_f)$$
$$v_f = -\frac{4}{7}v_0$$

$$\Delta p_{\text{Block 2}} = -\Delta p_{\text{Block 1}} = 6m\left(-\frac{4}{7}v_0 - (-v_0)\right)$$

$$\Delta p_{\text{Block 2}} = \int_0^{t_c} F_{\max} \sin(At)\,dt$$

$$\frac{18}{7}mv_0 = \frac{F_{\max}}{A}[-\cos(At)]\Big|_0^{t_c}$$
$$\frac{18}{7}mv_0 = \frac{F_{\max}}{A}[1 - \cos(At_c)]$$

$$F_{\max} = \frac{18}{7}\left(\frac{Amv_0}{1 - \cos(At_c)}\right)$$

给末速率：先判末方向，再反求初速

速率只有大小，速度还含方向。粘连后题目只给速率时，分别检查正负两个末速度会给出什么初速度；排除与原题初始运动方向矛盾的分支，才写最终初速。

例题 2025 Q1 (B)

本问承接 2025 Q1 (A)(i)。

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

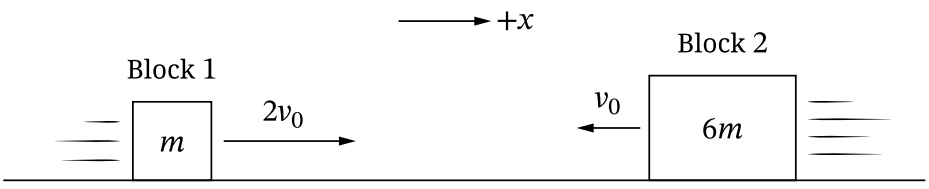


Figure 1

B. Consider a new scenario where Block 1 initially slides in the $+x$-direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$-direction with constant speed v_0. The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Derive an expression for v_1 in terms of v_0. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

题目给末速率而未先指定末方向。若末速度仍为向左的负v_0，动量守恒会使块1的初速度也向左，与题设冲突。所以新系统末速度向右。

$$mv_1 - 6mv_0 = 7mv_0, \qquad v_1 = 13v_0.$$

官方评分

B	For a multistep derivation that includes conservation of momentum	**Point B1**
	For indicating that the magnitude of the final momentum of the two-block system is $7mv_0$	**Point B2**
	For a correct expression for v_1 in terms of v_0	**Point B3**
	Example Response $\sum \vec{p}_0 = \sum \vec{p}_f$ $mv_1 + 6m(-v_0) = 7m(v_0)$ $mv_1 = 13mv_0$ $v_1 = 13v_0$	

顶点爆炸：水平动量与等时下落

抛体在顶点的竖直速度为零。两片爆后只水平运动，从顶点落地的时间等于之前上升的时间。Q最终回到发射点，爆后水平速度就与爆前水平速度等大反向；再用水平动量守恒求另一片的动量。竖直动量为零，图上仍需明确画零线。

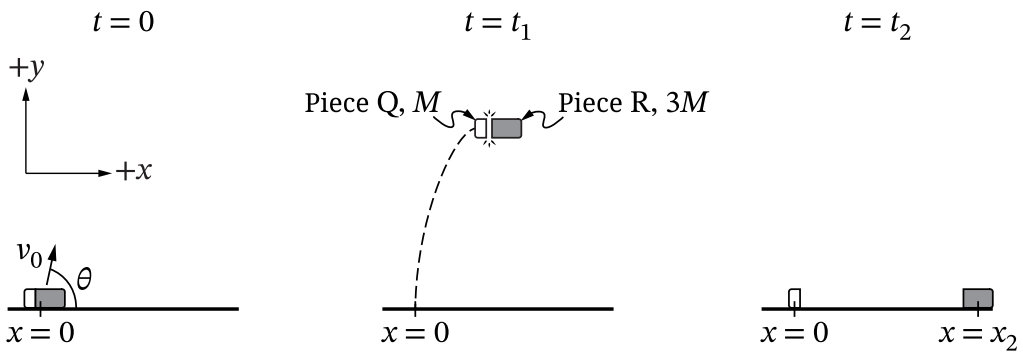
例题 2026 Q2 (A)

2. A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally.
- At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

Figure 1



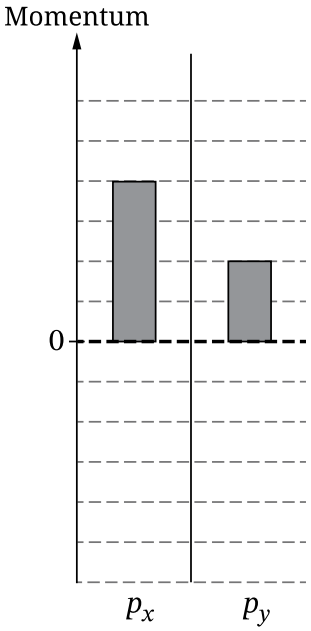
Note: Figure not drawn to scale.

Part A

The horizontal and vertical components of a momentum vector are represented by p_x and p_y, respectively. The shaded bars in Figure 2 represent p_x and p_y of the projectile immediately after $t = 0$.

Figure 2

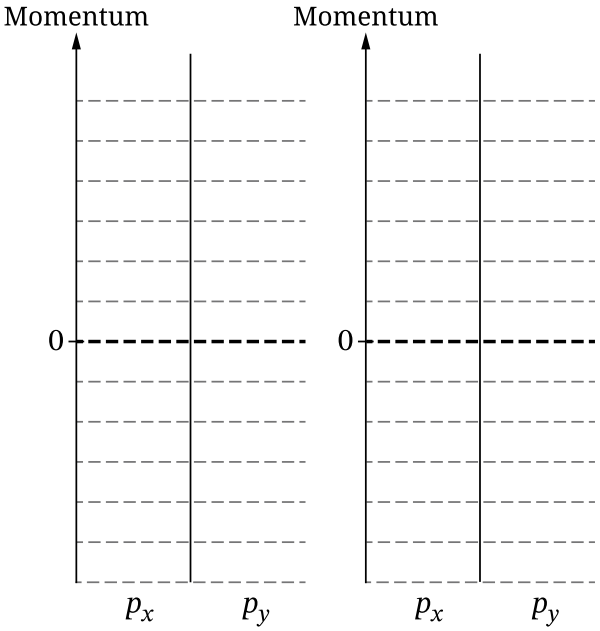
$t = 0$



Complete Projectile

Figure 3

$t = t_1$



Piece Q Piece R

On Figure 3, **draw** shaded bars to represent p_x and p_y of Pieces Q and R at $t = t_1$.

- Start the shaded bars at the dashed line that represents zero momentum.
- Represent the magnitudes of the components of the momentum by using the heights of the shaded bars, consistent with the scale used in Figure 2.
- Represent any momentum component that is equal to zero by drawing a distinct line on the zero-momentum line.

解：

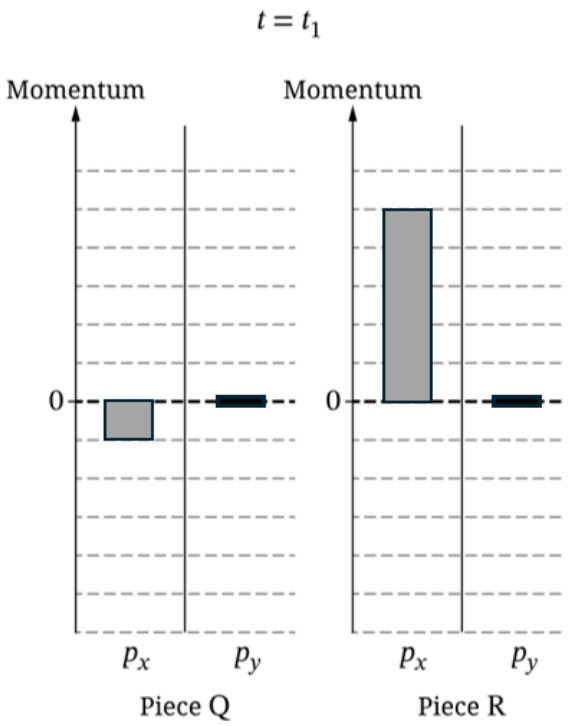
先取向右、向上为正。把发射速度的水平分量暂记为u，竖直上升到顶点的时间记为上升时间。爆炸发生在顶点，两片爆后都只有水平速度，因此此后下落时间也等于上升时间。

$$u = v_0 \cos\theta, \qquad t_{\mathrm{up}} = \frac{v_0 \sin\theta}{g}.$$

Q最终回到原点，它从顶点到原点的水平位移与爆炸前的水平位移等大反向，时间也相同，所以Q爆后的水平速度为负u。爆炸的水平外冲量可忽略，系统水平动量守恒。

$$4Mu = M(-u) + 3Mv_R, \qquad v_R = \frac{5}{3}u.$$

以Mu为一格，Q水平动量为负一格，R为正五格；两片的竖直动量都为零，分别在零线上画清楚短横线。

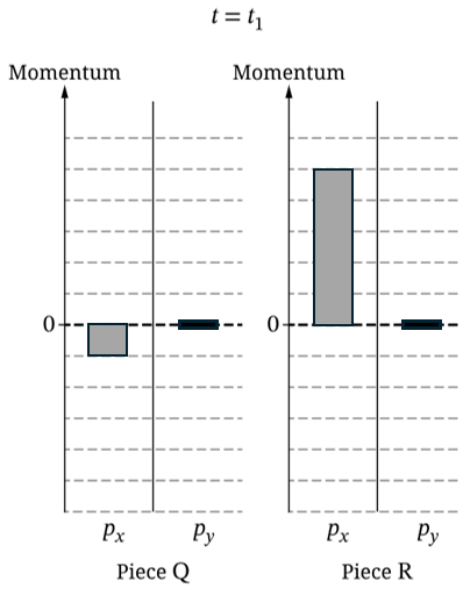


水平动量相加仍是正四格，竖直动量均为零。

官方评分

A	For drawing bars for p_x of Pieces Q and R that have heights which add to 4 units	**Point A1**
	For drawing lines on the zero-momentum line for p_y of Pieces Q and R	**Point A2**
	For drawing **one** of the following: • A bar for p_x of Piece Q with a height of -1 unit • A bar for p_x of Piece R with a height of 5 units	**Point A3**

Example Response



爆炸后落点：把两段水平位移相加

爆炸前整体到顶点的水平位移，与爆炸后R的水平位移分开计算。两段时间相同，但水平速度不同。先由竖直运动求上升时间，再从上一问的水平动量得到R速度，最后加上两段距离。

例题 2026 Q2 (B)

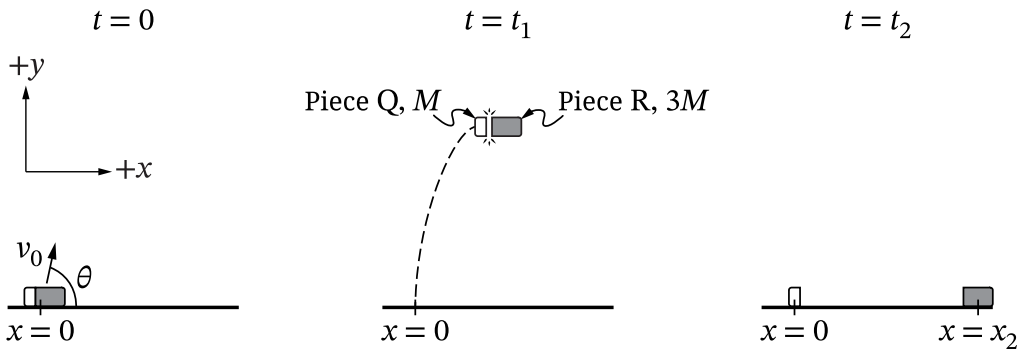
本问承接 2026 Q2 (A)。

2. A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally.
- At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

Figure 1



Note: Figure not drawn to scale.

Part B

Derive an expression for x_2 in terms of v_0, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

R先随原物体到达顶点，再以新的速度继续向右。两段时间相同，把两段水平位移相加。

$$x_2 = ut_{\text{up}} + \frac{5}{3}ut_{\text{up}} = \frac{8v_0^2 \sin\theta\cos\theta}{3g}.$$

官方评分

B	For a multistep derivation that includes **one** of the following: • Any kinematics equation relevant to constant acceleration • Conservation of momentum	**Point B1**
	For indicating **one** of the following: • That the vertical component of the velocity of the projectile is zero at $t = t_1$ • That the vertical displacement of the projectile is zero at $t = t_2$	**Point B2**
	For **one** of the following: • Correctly using conservation of momentum to determine the horizontal component of the velocity of Piece R immediately after the projectile breaks apart • Correctly indicating the position of Piece R relative to the position of the center of mass of the projectile at $t = t_2$	**Point B3**
	For a correct expression for x_2 in terms of given quantities	**Point B4**

Example Responses

Solution 1

$\Delta y = v_{y0}t + \frac{1}{2}at^2$

$v_{y0} = v_0 \sin\theta$

$0 = v_0 \sin\theta t_2 + \frac{1}{2}(-g){t_2}^2$

$t_2 = \frac{2v_0 \sin\theta}{g}$

$\sum \vec{p}_{0x} = \sum \vec{p}_{fx}$

$\vec{p}_{0x} = \vec{p}_{Qx} + \vec{p}_{Rx}$

$4Mv_0 \cos\theta = -Mv_0 \cos\theta + 3Mv_{Rf}$

$5Mv_0 \cos\theta = 3Mv_{Rf}$

$v_{Rf} = \frac{5}{3}v_0 \cos\theta$

$x_2 = x_{proj} + x_R$

$x_2 = v_{x0}t_{up} + v_{Rf}t_{down}$

$x_2 = (v_0 \cos\theta)\left(\frac{v_0 \sin\theta}{g}\right) + \left(\frac{5}{3}v_0 \cos\theta\right)\left(\frac{v_0 \sin\theta}{g}\right)$

$x_2 = \frac{{v_0}^2}{g}\cos\theta\sin\theta + \frac{5}{3}\frac{{v_0}^2}{g}\cos\theta\sin\theta$

$x_2 = \frac{8{v_0}^2}{3g}\cos\theta\sin\theta$

Solution 2

$v_y = v_{y0} + a_y t$

$0 = v_0 \sin\theta + (-g)t_{up}$

$t_1 = \frac{v_0 \sin\theta}{g}$

$t_2 = 2t_1 = \frac{2v_0 \sin\theta}{g}$

$v_x = v_0 \cos\theta$

$x_{CM} = v_x t_2$

$x_{CM} = \frac{2{v_0}^2 \cos\theta\sin\theta}{g}$

$x_{CM} = \frac{M(0) + 3M(x_2)}{M + 3M}$

$x_{CM} = \frac{3}{4}x_2$

$\frac{3}{4}x_2 = \frac{2{v_0}^2 \cos\theta\sin\theta}{g}$

$x_2 = \frac{8v_0^2}{3g}\cos\theta\sin\theta$

质心速度：内部分离不改变系统的水平运动

爆炸后没有水平外力，Q和R各自的水平速度恒定；系统质心的水平速度也保持爆炸前的水平速度。用上一问得到的两片速度画有名称的水平线，不能把两片速度都画成质心速度。

例题 2026 Q2 (C)

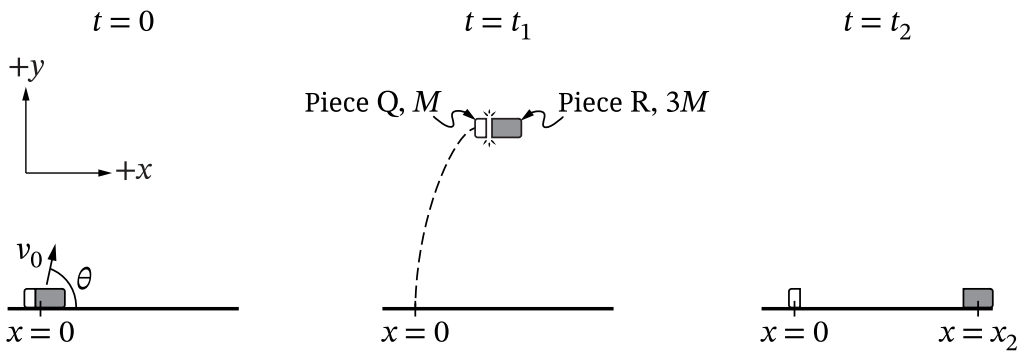
本问承接 2026 Q2 (A)。

2. A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally.
- At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

Figure 1

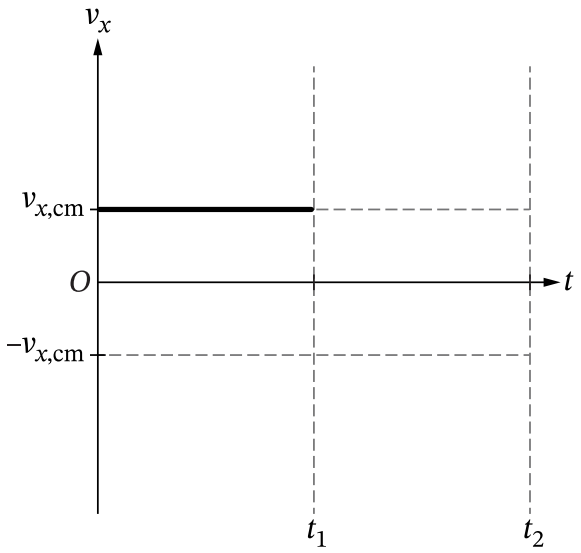


Note: Figure not drawn to scale.

Part C

The horizontal component of a velocity vector is represented by v_x. Figure 4 shows the horizontal component $v_{x,\mathrm{cm}}$ of the velocity of the center of mass of the projectile as a function of t during the time interval $0 < t < t_1$.

Figure 4



On Figure 4, **sketch** a line or curve to represent v_x as a function of t for the time interval $t_1 < t < t_2$ for each of the following.

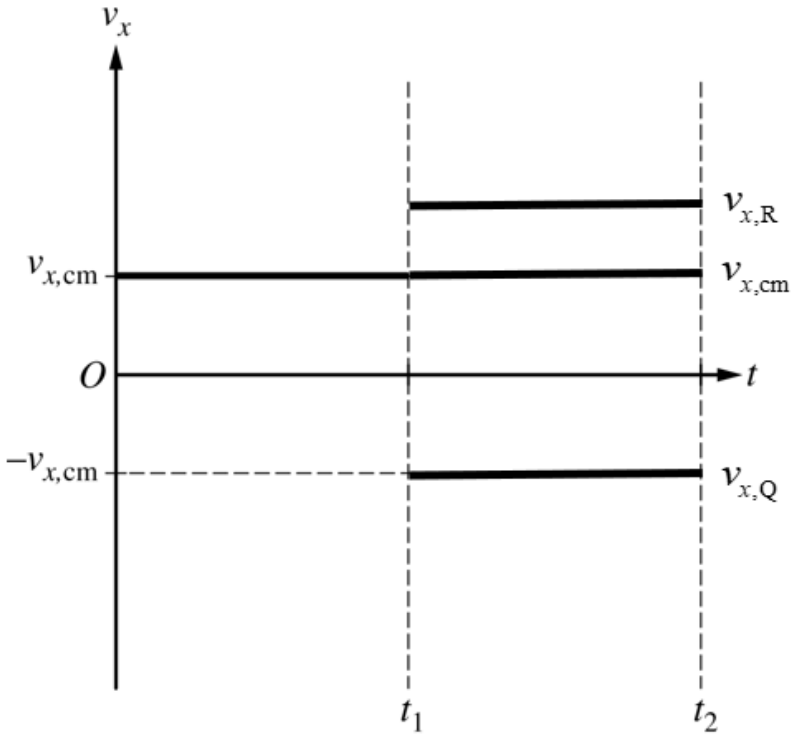
- Piece Q
- Piece R
- The center of mass of the two-piece system

Clearly **label** all lines or curves.

解：

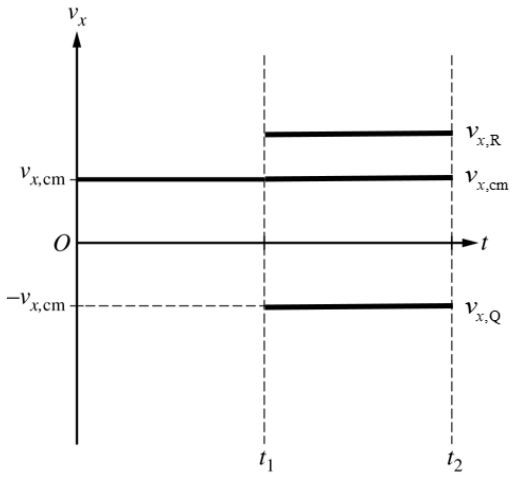
爆炸以后没有水平外力，Q、R和系统质心的水平速度都恒定，分别画三条带名称的水平线。

$$v_{Qx} = -u, \qquad v_{Rx} = \frac{5}{3}u, \qquad v_{\mathrm{CM},x} = u.$$



爆炸改变各片速度，系统质心的水平速度连续且不变。

官方评分

C	For sketching a labeled horizontal line for the center of mass that has a value equal to $v_{x,\mathrm{cm}}$ **Scoring Note:** Any clearly defined labels may earn this point.	**Point C1**
	For sketching a labeled horizontal line for Piece R that has a value greater than $v_{x,\mathrm{cm}}$	**Point C2**
	For sketching a labeled horizontal line for Piece Q that has a value equal to $-v_{x,\mathrm{cm}}$	**Point C3**
	Example Response  	

改变一片方向：用总动量判断另一片落点

把Q的水平速度由向左改为零，它的水平动量增加。系统总水平动量仍相同，因此R得到的向右动量减少。R质量和下落时间不变，所以向右速度和落地水平位置都减小。判断落点比较时，要同时检查速度和飞行时间。

例题 2026 Q2 (D)

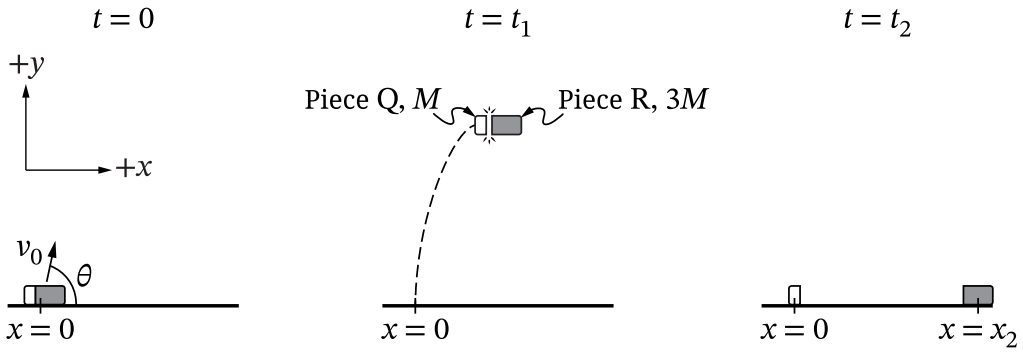
本问承接 2026 Q2 (A)、2026 Q2 (B)、2026 Q2 (C)。

2. A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally.
- At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

Figure 1



Note: Figure not drawn to scale.

Part D

Consider a case in which the projectile is launched at the same angle and initial speed as initially described. When the projectile breaks into Pieces Q and R, Piece Q falls straight down. In this case, Piece R reaches the ground at $x = x_{\text{new}}$.

Indicate whether x_{new} is greater than, less than, or equal to x_2 by writing one of the following.

- $x_{\text{new}} > x_2$
- $x_{\text{new}} < x_2$
- $x_{\text{new}} = x_2$

Briefly **justify** your answer either by referencing a feature of the representations you drew in part A or C or by using conceptual reasoning beyond algebraic solutions.

解：

选新落点较近。若Q直落，其水平动量从原来的负值改为零；系统总水平动量仍相同，R得到的正动量就比原来少。R质量不变，因此其向右速度更小，而下落时间相同，所以新落点更靠左。

$$x_{\text{new}} < x_2.$$

官方评分

D	For indicating $x_{\text{new}} < x_2$	**Point D1**
	For a correct justification, such as: • Comparing the change in momentum of either piece in the two cases, the speed of the corresponding piece after the projectile breaks apart, and the distance traveled before reaching the ground • Relating the motion of the center of mass of the projectile to the mass of the pieces and the distance each piece lands from the center of mass	**Point D2**
	Example Responses *In the case where Piece Q falls straight to the ground, it has zero momentum, as opposed to a negative momentum when it moved backwards. Because momentum is conserved when the pieces break apart, Piece R must have less positive momentum in the new case. Because Piece R has less momentum it has a lower speed. It takes the same time to fall to the ground in both cases, so Piece R travels a shorter distance before landing in the new case.* OR *In the case where Piece Q falls straight to the ground, Piece Q lands closer to the center of mass of the system than when it moved backward. Because the smaller mass piece lands closer to the CM, the larger mass piece also must land closer to the CM than it did before. This means Piece R travels a shorter distance before landing in the second case.*	

绕轴转动：力矩、惯量与角动量

绕支点平衡：垂直距离才是力臂

力矩记作τ，垂直力臂写成r的垂直分量。选支点为转轴，支点力的力臂为零，可以先消去这个未知力。每个力矩是力乘从转轴到作用线的垂直距离；角从竖直量起和从水平量起时，正弦、余弦的对应会交换。平衡要求两个转动方向的力矩相等。

$$\tau = F r_{\perp}, \qquad \sum \tau = 0.$$

例题 2024 Set 1 Q3 (b)

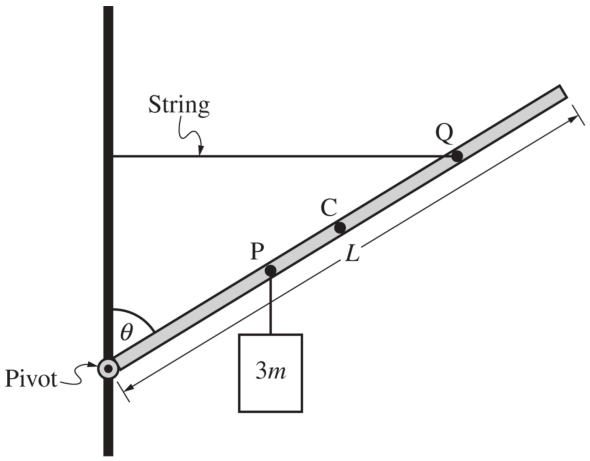


Figure 1

Note: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot. **Derive** an equation for the tension F_{T} in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

解：

选支点为力矩中心，支点力力臂为零。角从竖直量起，两个重力的力臂用正弦，水平绳力的力臂用余弦。平衡时顺时针、逆时针力矩相等。

$$\sum \tau = 0, \qquad 3mg\left(\frac{3L}{8}\right)\sin\theta + mg\left(\frac{L}{2}\right)\sin\theta = F_T\left(\frac{6L}{8}\right)\cos\theta.$$

$$F_T = \frac{13}{6}mg\tan\theta.$$

官方评分

(b) For indicating that the net torque exerted on the rod is equal to zero **1 point**

Example Response

$\Sigma\tau = 0$

For a correct expression for the torque exerted on the rod by the hanging mass **1 point**

Example Response

$3mg\sin\theta\left(\frac{3L}{8}\right)$

For a correct expression for the torque exerted on the rod by the gravitational force **1 point**

Example Response

$mg\sin\theta\left(\frac{4L}{8}\right)$

For a correct expression for the torque exerted on the rod by the string **1 point**

Example Responses

$F_{\mathrm{T}}\sin(90° - \theta)\left(\frac{6L}{8}\right)$ **OR** $F_{\mathrm{T}}\cos\theta\left(\frac{6L}{8}\right)$

Example Solution

$\Sigma\tau = 0$

$3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_{\mathrm{T}}\sin(90° - \theta)\left(\frac{6L}{8}\right) = 0$

$3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_{\mathrm{T}}\cos\theta\left(\frac{6L}{8}\right) = 0$

$\left(\frac{9}{8}\right)mg\sin\theta + \left(\frac{4}{8}\right)mg\sin\theta = \left(\frac{6}{8}\right)F_{\mathrm{T}}\cos\theta$

$13mg\sin\theta = 6F_{\mathrm{T}}\cos\theta$

$F_{\mathrm{T}} = \frac{13}{6}mg\tan\theta$

Scoring Note: A maximum of three points may be earned if the trigonometric functions (sin and cos) are reversed for all three torque terms.

Total for part (b) 4 points

练习 2024 Set 2 Q3 (b)

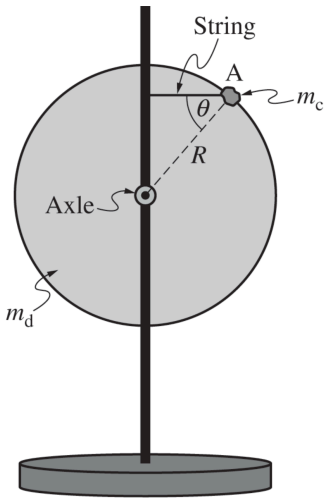


Figure 1

Note: Figure not drawn to scale.

3. A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(b) **Derive** an expression for the tension F_T in the string when the clay is at Point A, as shown in Figure 1, in terms of R, m_d, m_c, θ, and physical constants, as appropriate.

解：

关于轴求力矩。盘重与轴力都过轴，无力矩；只剩黏土重力矩与张力矩。原题角是径向线与水平绳之间的角。

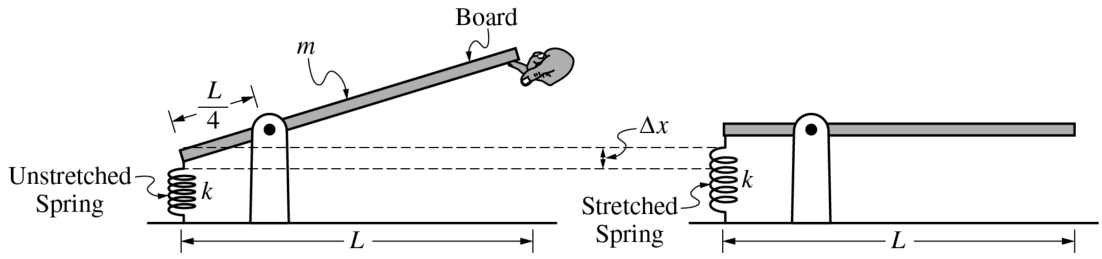
$$\sum \tau = 0, \qquad Rm_c g\cos\theta = RF_T \sin\theta, \qquad F_T = m_c g \cot\theta.$$

官方评分

(b)	For a multi-step derivation that indicates the net torque is zero	**1 point**
	For indicating only the weight of the clay and the tension in the string exert torques on the system about the axle	**1 point**
	Example Response $\tau_{\text{net}} = \tau_{\text{clay}} - \tau_{\text{string}}$	
	For a correct expression for the torque exerted on the system by the weight of the clay	**1 point**
	Example Response $\tau_{\text{clay}} = Rm_{\text{c}}g\cos\theta$	
	For a correct expression for the torque exerted on the system by the tension in the string	**1 point**
	Example Response $\tau_{\text{string}} = RF_{\text{T}}\sin\theta$	
	Example Solution $\Sigma\tau = 0$ $\tau_{\text{clay}} - \tau_{\text{string}} = 0$ $F_{g,\text{clay}}R\cos\theta - RF_{\text{T}}\sin\theta = 0$ $Rm_{\text{c}}g\cos\theta = RF_{\text{T}}\sin\theta$ $F_{\text{T}} = \frac{Rm_{\text{c}}g\cos\theta}{R\sin\theta}$ $F_{\text{T}} = m_{\text{c}}g\cot\theta$ **Scoring Note:** A maximum of three points can be earned if the trigonometric functions (sin and cos) are reversed for both torque expressions.	
		Total for part (b) 4 points

练习 2022 Set 2 Q3 (b)

本问承接 2022 Set 2 Q3 (a)。



Note: Figures not drawn to scale.

3. A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I.

(b) Derive an expression for the distance the spring stretches, Δx, when the board is in equilibrium. Express your answer in terms of k, L, m, and physical constants, as appropriate.

解：

关于支点求力矩。弹簧与重力的力臂同为四分之一板长，方向相反。

$$\sum \tau = 0, \qquad k\Delta x\frac{L}{4} = mg\frac{L}{4}, \qquad \Delta x = \frac{mg}{k}.$$

官方评分

(b)	For indicating that the sum of the torques equals zero $\Sigma\tau = 0$	**1 point**
	For substituting mg for the force of gravity and $k\Delta x$ for the spring force into a torque equation $\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$	**1 point**
	For a correct expression for Δx $\Delta x = \frac{mg}{k}$	**1 point**

Example Response

$\Sigma\tau = 0$

$F_{\text{spring}}d_{\text{spring}} - F_{\text{weight}}d_{\text{weight}} = 0$

$\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$

$\Delta x = \frac{mg}{k}$

Scoring Note: This point can be earned regardless of whether a negative sign is present.

Total for part (b) 3 points

盘转过的小角θ以弧度计量。盘缘的弧长是Rθ，连接在盘缘的弹簧也因此伸长Rθ；把这个伸长量代入胡克定律，再比较两侧张力矩。

$$\Delta x = R\theta, \qquad F_{\text{spring}} = kR\theta.$$

例题 2022 Set 1 Q3 (a)

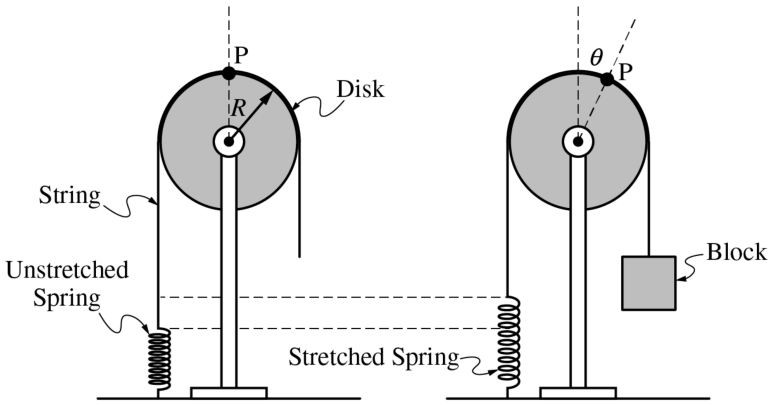


Figure 1 Figure 2

Note: Figures not drawn to scale.

3. A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_{d} and R, respectively. The rotational inertia of the disk is $I_{\mathrm{d}} = \frac{1}{2} M_{\mathrm{d}} R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

   A block of mass m_{B} is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

   Give all algebraic answers in terms of M_{d}, R, k, θ, and physical constants, as appropriate.

   (a) Derive an expression for the mass m_{B} of the block.

解：

盘顺时针转过小角后，弹簧端移动弧长Rθ。静止时关于轴的两侧张力矩平衡；两侧力臂都是R，故弹簧拉力等于挂块重力。

$$\Delta x = R\theta, \qquad m_B g R = k(R\theta)R, \qquad m_B = \frac{kR\theta}{g}.$$

官方评分

(a)	For indicating that the sum of the torques on the disk equals zero $\Sigma\tau_{\text{on disk}} = 0$ $\tau_{\text{g}} = \tau_{\text{s}}$ **OR** For indicating that the sum of the forces equals zero $\Sigma F = 0$ $F_{\text{g}} = F_{\text{s}}$	**1 point**
	For correctly substituting the expressions for the forces $F_{\text{g}}R = F_{\text{s}}R$ $m_{\text{B}}gR = k\Delta xR$ $m_{\text{B}}g = k\Delta x$ **OR** $F_{\text{g}} = F_{\text{s}}$ $m_B g = k\Delta x$	**1 point**
	For correctly substituting for Δx $m_{\text{B}}g = kR\theta$ $m_{\text{B}} = \dfrac{kR\theta}{g}$	**1 point**
		Total for part (a) 3 points

改变作用线或作用点：比较单位力的力矩

原来需要抵消的重力矩不变时，绳的单位张力能产生更大的力矩，所需张力就减小。反过来，若绳力臂不变而重力力臂增大，所需张力就增大。先说明哪一处改变，不能只凭绳长或图中角度大小猜结论。

例题 2024 Set 1 Q3 (c)

本问承接 2024 Set 1 Q3 (b)。

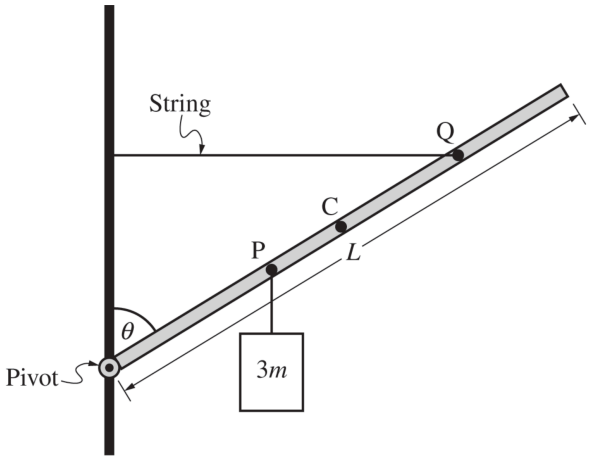


Figure 1

Note: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

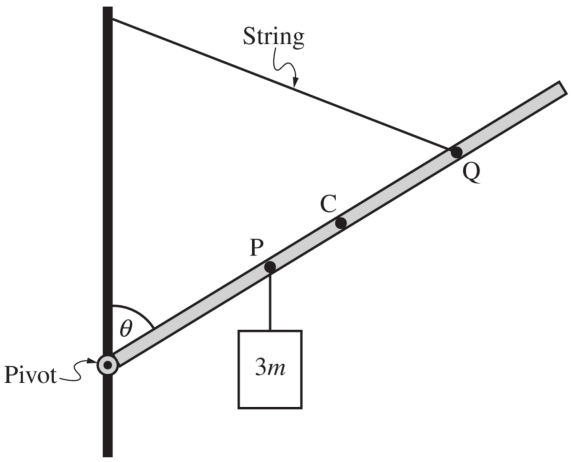


Figure 2

Note: Figure not drawn to scale.

(c) The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\,new}$ compare with the original tension F_T from part (b) ? **Justify** your reasoning.

解：

杆的角度、两处重力及作用点不变，需由绳抵消的力矩相同。换绳后绳与杆更接近垂直，单位张力产生的力矩更大，所以新张力较小。

官方评分

(c)	For indicating that the torque exerted on the rod by the string is always the same	**1 point**
	For stating that as the angle between the string and the rod increases, the force exerted on the rod by the string decreases	**1 point**
	Example Response *Because the torque exerted on the rod by the string is always the same, as the angle between the string and the rod increases, the tension $F_{T,\text{new}}$ must decrease.*	
	Total for part (c)	**2 points**

例题 2024 Set 2 Q3 (c)

本问承接 2024 Set 2 Q3 (b)。

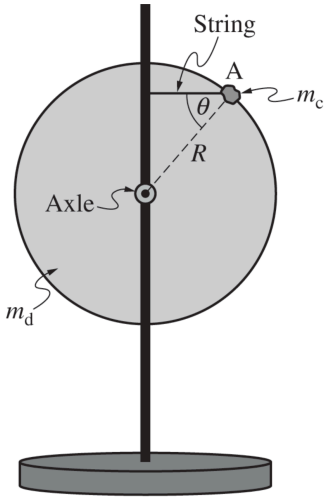


Figure 1

Note: Figure not drawn to scale.

3. A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

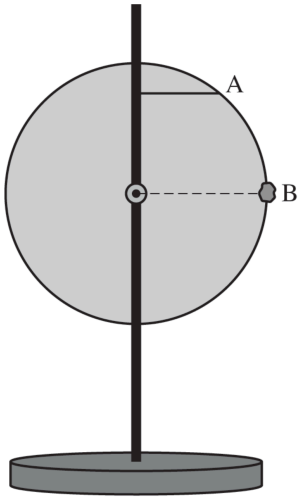


Figure 2

Note: Figure not drawn to scale.

(c) The string remains connected to the edge of the disk at Point A. The clay is moved to Point B, which is horizontally in line with the axle, as shown in Figure 2. How does the new tension $F_{\mathrm{T,\,new}}$ compare with tension F_{T} from part (b)? **Justify** your reasoning.

解：

绳仍在A，绳力臂不变。黏土移到与轴等高的B后，重力的力臂达到R，比原来大。为抵消更大重力矩，新张力必须增大。

官方评分

(c)	For indicating a direct relationship between the torque exerted by the clay and the tension in the string	**1 point**
	For correctly relating a greater torque exerted by the clay from at least **one** of the following: • An increase in the angle between the radial direction and the weight of the clay • A greater perpendicular component of the weight of the clay • A greater lever arm	**1 point**
	Example Response *The torque caused by the weight of the clay at Point* B *is greater than when the clay is at Point* A *because the component of the weight that is perpendicular to the lever arm is larger. To maintain equilibrium, the net torque on the system is still zero, therefore the tension* $F_{\text{T, new}}$ *must be greater.*	
	Total for part (c)	**2 points**

移动转轴：逐个外力重新看垂直力臂

比较不同转轴下的力矩时，每个力都单独找新的作用线与转轴距离。盘重从原来过轴变为不过轴，产生新的重力矩；右侧绳也有新的力臂。轴的作用力仍从轴点出发，它对轴本身的力矩始终为零。

例题 2022 Set 1 Q3 (e)

本问承接 2022 Set 1 Q3 (b)。

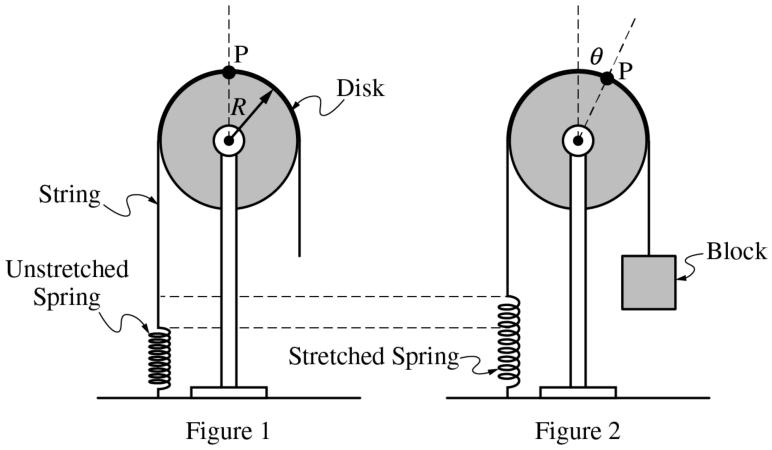


Figure 1 Figure 2

Note: Figures not drawn to scale.

3. A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_{d} and R, respectively. The rotational inertia of the disk is $I_{\mathrm{d}} = \frac{1}{2} M_{\mathrm{d}} R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

   A block of mass m_{B} is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

   Give all algebraic answers in terms of M_{d}, R, k, θ, and physical constants, as appropriate.

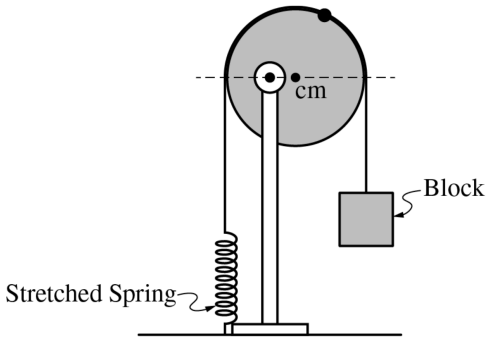


Figure 3

Note: Figure not drawn to scale.

(e) The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

解：

原(b)位置盘重过轴、力矩为零；移轴后盘重力臂非零，其力矩增大。新平衡位置右绳也在盘上，它的力臂更长，所引起力矩增大。为维持平衡，左侧弹簧张力矩也增大。轴力本身始终作用在轴上，其力矩仍为零。

官方评分

(e)	For indicating that the torque exerted by the force due to gravity on the disk increased **Example Response** *The force due to gravity on the disk now has a non-zero lever arm and hence it exerts a larger torque on the disk.*	**1 point**
	For indicating that the torque exerted by the tension caused by the force due to gravity on the block increased **Example Response** *The force exerted by the right side of the string (from the block) on the disk has a longer lever arm, hence the torque it exerts is larger.*	**1 point**
	For indicating that the torque exerted by the tension increased **Example Response** *The counterclockwise torque due to the tension caused by the spring must increase to counteract the increase in clockwise torques due to the force due to gravity of the disk and tension caused by the force of gravity due to block to keep the disk in equilibrium.*	**1 point**
	Scoring Notes: • A response that references the torque due to the force at the axle staying the same can earn all 3 points. • A response that references the torque due to the force on the axle changing, or any additional torques can earn a maximum of 2 points.	
		Total for part (e) 3 points
		Total for question 3 15 points

线密度变化：小段质量再累加

λ(x)表示距支点x处每单位长度的质量，dm表示一小段的质量。杆上每一米有多少质量随位置改变，不能用均匀杆公式。距支点x的一小段长度dx，质量是λ(x)dx；把从0到杆长L的各小段质量相加就是总质量。dx表示一个极短长度，积分把这些小段累积起来。

$$dm = \lambda(x)\,dx, \qquad m = \int_0^L \lambda(x)\,dx.$$

例题 2024 Set 1 Q3 (d)(i)

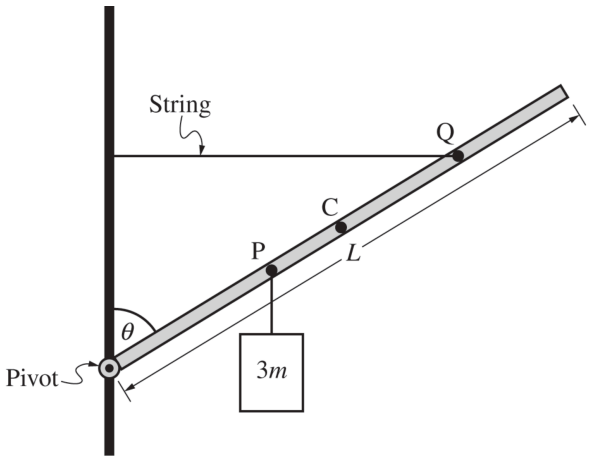


Figure 1

Note: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

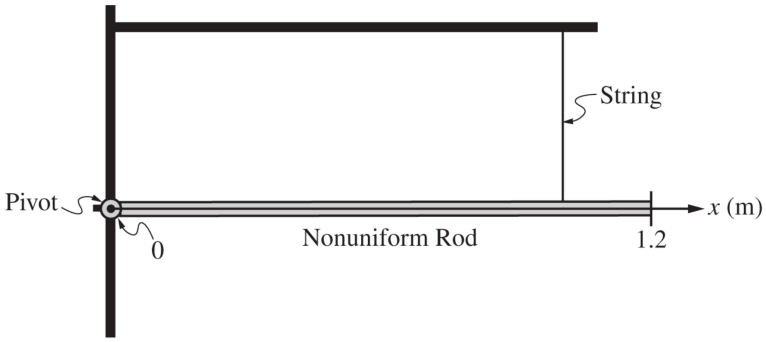


Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0\,\text{kg/m}$, and $B = 10.0\,\text{kg/m}^2$.

i. **Calculate** the mass of the rod.

解：

非均匀杆不能套用均匀杆的质量公式。距支点x处一小段长度dx，其质量是线密度乘这段长度，累加整根杆。

$$dm = (A + Bx)\,dx, \qquad M = \int_0^{1.2} (6 + 10x)\,dx = \left[6x + 5x^2\right]_0^{1.2} = 14.4\ \text{kg}.$$

官方评分

(d)(i)	For indicating the total mass is the sum of differentiable masses along the length of the rod	**1 point**
	Example Response $M = \int dm$	
	For correctly writing dm in terms of x	**1 point**
	Example Response $M = \int_0^{1.2} (6 + 10x)\,dx$	
	For a correct numeric answer with correct units	**1 point**
	Example Response $M = 14.4\ \text{kg}$	
	Example Solution $M = \int dm$ $M = \int \lambda\ dx$ $M = \int_0^{1.2} (6 + 10x)\,dx$ $M = \left(6x + \frac{10x^2}{2}\right)\Big\|_0^{1.2\ \text{m}}$ $M = 6\ \text{kg/m}(1.2\ \text{m}) + \frac{10\ \text{kg/m}^2\,(1.2\ \text{m})^2}{2}$ $M = 14.4\ \text{kg}$	

惯量积分：同一小段再乘距离平方

转动惯量用I表示，描述质量离轴远近对转动的影响。同一小段离轴距离为x，它贡献x^2dm；把刚求质量时的dm继续使用，再累加全杆。质量和惯量的积分不同，后者多乘了距离平方。

$$I = \int_0^L x^2 \lambda(x)\,dx.$$

例题 2024 Set 1 Q3 (d)(ii)

本问承接 2024 Set 1 Q3 (d)(i)。

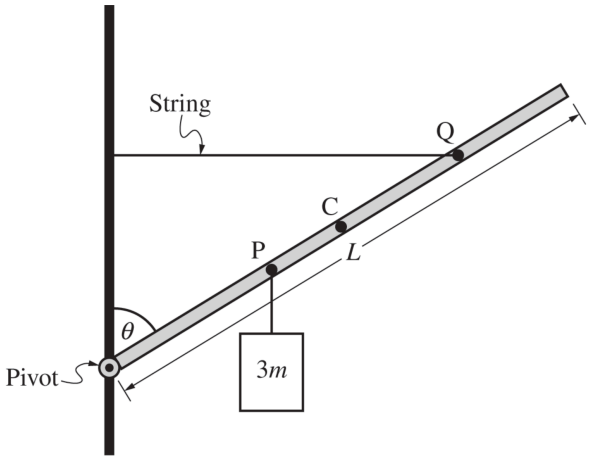


Figure 1

Note: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

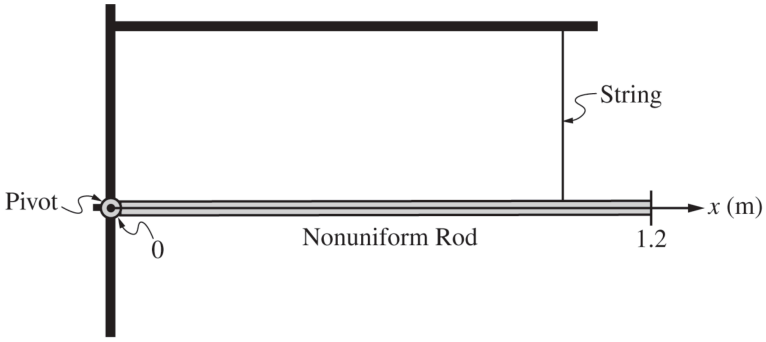


Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0\,\text{kg/m}$, and $B = 10.0\,\text{kg/m}^2$.

ii. **Calculate** the rotational inertia of the rod about the pivot.

解：

每小段对支点的转动惯量是质量乘距离平方，仍使用同一小段质量。

$$I = \int_0^{1.2} x^2(6 + 10x)\,dx = \left[2x^3 + 2.5x^4\right]_0^{1.2} = 8.64\ \text{kg}\,\text{m}^2.$$

官方评分

(d)(ii) For a correct substitution of λ into an integral expression of rotational inertia **1 point**

Example Response

$$I = \int_0^{1.2\,\text{m}} (A + Bx)x^2 dx$$

For a correct integration **1 point**

Example Response

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4} \right) \Bigg|_0^{1.2\,\text{m}}$$

For a correct numeric answer with correct units **1 point**

Example Response

$$I = 8.64\,\text{kg} \cdot \text{m}^2$$

Example Solution

$$I = \int r^2 dm \qquad dm = \lambda dr \text{ and } r = x$$

$$I = \int_0^{1.2\,\text{m}} \lambda x^2 dx$$

$$I = \int_0^{1.2\,\text{m}} (A + Bx)x^2 dx$$

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4} \right) \Bigg|_0^{1.2\,\text{m}}$$

$$I = \frac{(6.0\,\text{kg/m})(1.2\,\text{m})^3}{3} + \frac{\left(10.0\,\text{kg/m}^2\right)(1.2\,\text{m})^4}{4}$$

$$I = 8.64\,\text{kg} \cdot \text{m}^2$$

Total for part (d) 6 points

Total for question 3 15 points

面密度变化：用薄圆环建立惯量积分

盘的密度是每平方米的质量。ρ(r)表示半径r处每单位面积的质量。取半径r、宽度dr的薄圆环，面积约是周长2πr乘宽度dr；先把面密度乘这块面积得到dm，再乘r^2得到环的惯量。不能把面密度直接乘dr，那会缺少圆环周长。

$$dm = \rho(r)\, 2\pi r\, dr, \qquad I = \int_0^R r^2\, dm.$$

例题 2024 Set 2 Q3 (d)(i)

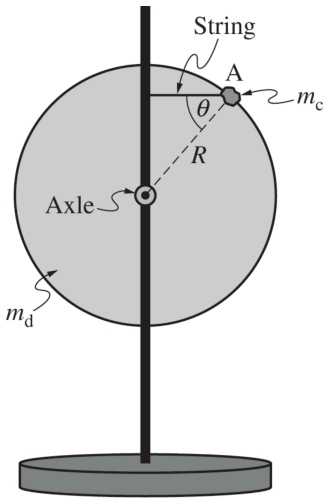


Figure 1

Note: Figure not drawn to scale.

3. A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

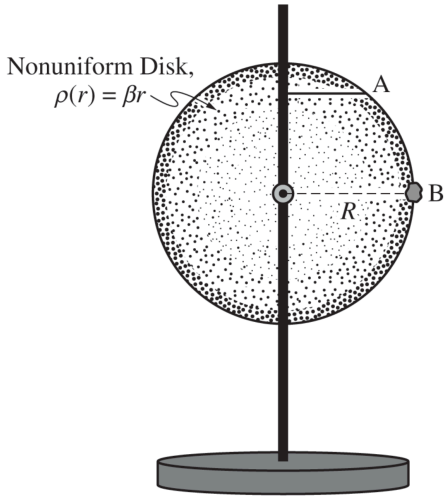


Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60\,\text{kg}$ and the disk has a radius $R = 0.30\,\text{m}$. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0\,\text{kg/m}^3$.

i. **Calculate** the rotational inertia of the disk about the axle.

解：

盘的面密度随半径变化，取半径r、宽dr的薄圆环：面积是周长乘宽度，质量是密度乘面积。不能把面密度直接乘dr。

$$dA = 2\pi r\,dr, \qquad dm = \rho(r)dA = (\beta r)(2\pi r\,dr).$$

$$I_d = \int_0^R r^2\,dm = \int_0^R 2\pi\beta r^4\,dr = \frac{2\pi\beta R^5}{5} = 0.0122\,\mathrm{kg\,m^2}.$$

官方评分

(d)(i)	For using integration to calculate rotational inertia	**1 point**
	For **one** of the following: • Substituting $\rho(2\pi r)dr$ for dm • Indicating the correct limits of integration	**1 point**
	Example Response $I = \int_0^{0.3\text{ m}} r^2\rho(2\pi r)dr$	
	For a correct answer of $I = 0.012\text{ kg}\cdot\text{m}^2$, including units	**1 point**
	Example Response $I = 0.012\text{ kg}\cdot\text{m}^2$	
	Example Solution $I = \int r^2 dm \qquad dm = \rho dA \quad \text{and} \quad dA = 2\pi r\,dr$ $I = \int_0^R r^2\rho dA$ $I = \int_0^{0.3\text{ m}} 2\pi\beta r^4 dr$ $I = \frac{2\pi\beta R^5}{5}\Big\vert_0^{0.3\text{ m}}$ $I = \frac{2\pi(4.0\,\text{kg}/\text{m}^3)(0.3\text{ m})^5}{5}$ $I = 0.012\text{ kg}\cdot\text{m}^2$	

已知质心惯量：先移轴，再相加

题目给叶片关于质心的惯量，但实际旋转轴在端点。先加上质量乘两平行轴间距平方，把每片叶片的惯量移到实际轴；三片一起转动，惯量相加。轴距是题给质心到端点的距离，不一定是半片长度。

$$I_{\text{axis}} = I_{\text{cm}} + Md^2.$$

例题 2023 Set 2 Q3 (a)

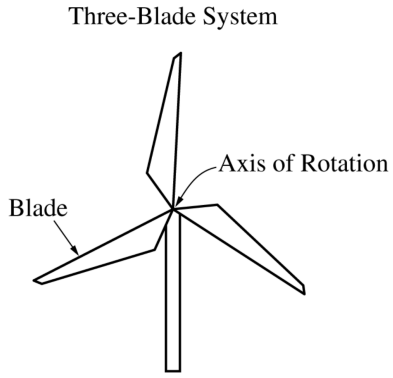


Figure 1

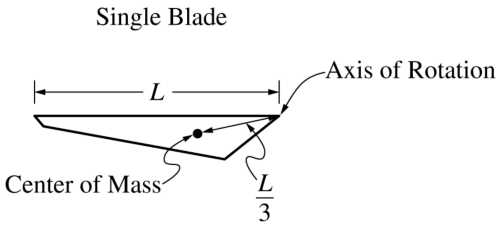


Figure 2

3. A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

(a) Derive an expression for the rotational inertia of the three-blade system. Express your answer in terms of M, L, and physical constants, as appropriate. The rotational inertia of each blade about an axis through its center of mass is given by the equation $I_{\text{cm}} = \frac{1}{18}ML^2$.

解：

给定惯量关于每片叶片质心，实际轴位于端点，轴距为L/3。先用平行轴定理移轴，再把三片叶片相加。

$$I_{\text{blade}} = I_{\text{CM}} + M\left(\frac{L}{3}\right)^2 = \frac{1}{18}ML^2 + \frac{1}{9}ML^2 = \frac{1}{6}ML^2.$$

$$I_{\text{sys}} = 3I_{\text{blade}} = \frac{1}{2}ML^2.$$

官方评分

(a)	For stating the parallel axis theorem	**1 point**
	Example Response	
	$I_{\text{blade}} = I_{\text{CM}} + Md^2$	
	For using correct substitutions of the rotational inertia of one blade about its center of mass and substituting the distance from the center of mass	**1 point**
	Example Response	
	$I_{\text{blade}} = \frac{1}{18}ML^2 + M\left(\frac{L}{3}\right)^2$	
	For multiplying the rotational inertia of one blade by 3	**1 point**
	Example Response	
	$I_{\text{rotor}} = 3I_{\text{blade}} = \frac{1}{2}ML^2$	
	Example Solution	
	$I_{\text{blade}} = I_{\text{CM}} + Md^2$ $I_{\text{blade}} = \frac{1}{18}ML^2 + M\left(\frac{L}{3}\right)^2$ $I_{\text{blade}} = \frac{1}{6}ML^2$ $I_{\text{rotor}} = 3I_{\text{blade}} = \frac{1}{2}ML^2$	
		Total for part (a) 3 points

剪绳瞬间：净力矩除以总惯量

角加速度α表示角速度每秒改变多少，总转动惯量记作I。剪断的绳不再施力，弹簧或黏土的位置却还没有来得及改变。只把仍存在、且不过轴的力计入力矩；重力或轴力过轴时力矩为零。若黏土随盘转，黏土的质量也有惯量，不能只使用盘的惯量。

$$\sum \tau = I\alpha.$$

例题 2022 Set 1 Q3 (c)

本问承接 2022 Set 1 Q3 (b)、2022 Set 1 Q3 (a)。

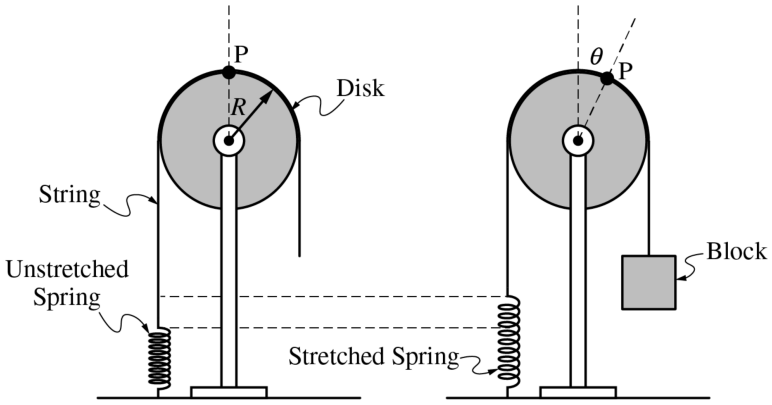


Figure 1　　Figure 2

Note: Figures not drawn to scale.

3. A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_{d} and R, respectively. The rotational inertia of the disk is $I_{\mathrm{d}} = \frac{1}{2} M_{\mathrm{d}} R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

   A block of mass m_{B} is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

   Give all algebraic answers in terms of M_{d}, R, k, θ, and physical constants, as appropriate.

(c) Derive an expression for the angular acceleration α of the disk immediately after the string is cut.

解：

取顺时针为正。剪绳瞬间弹簧仍保持原伸长量，只有左侧拉力对轴产生负力矩，盘重与轴力都过轴。

$$\sum \tau = I_d \alpha, \qquad -k(R\theta)R = \frac{1}{2} M_d R^2 \alpha, \qquad \alpha = -\frac{2k\theta}{M_d}.$$

官方评分

(c)	For indicating that the net torque is due only to the force exerted on the disk by the tension in the rotational form of Newton's second law $\tau_s = I_d\alpha$	**1 point**
	For correctly expressing the torque on the disk by the tension in terms of the spring force, which is equal to the tension, and the lever (moment) arm $F_sR = I_d\alpha$	**1 point**
	For correctly substituting for F_s $-k\Delta xR = I_d\alpha$	**1 point**
	For correctly substituting I_d and Δx, or an expression for Δx consistent with part (a) $-k(R\theta)R = \frac{1}{2}M_dR^2\alpha$ $\alpha = -\frac{2k\theta}{M_d}$ **Scoring Note:** The negative sign is not necessary to earn this point.	**1 point**
		Total for part (c) 4 points

例题 2024 Set 2 Q3 (d)(ii)

本问承接 2024 Set 2 Q3 (d)(i)。

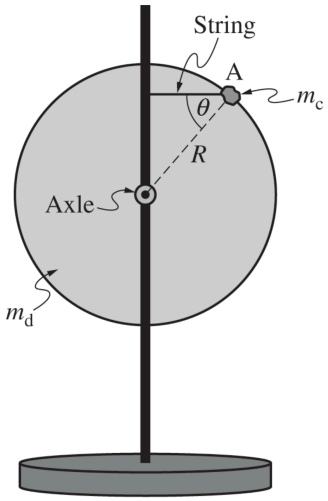


Figure 1

Note: Figure not drawn to scale.

3. A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

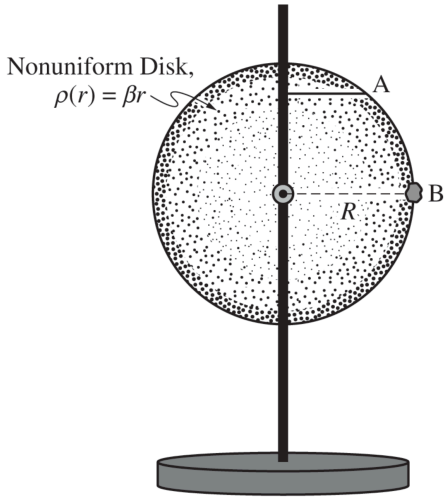


Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60\,\text{kg}$ and the disk has a radius $R = 0.30\,\text{m}$. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0\,\text{kg/m}^3$.

ii. The string connecting the disk to the pole is cut. **Calculate** the magnitude of the initial angular acceleration of the clay-disk system.

解：

剪绳后只有黏土重力对轴产生力矩，方向顺时针。黏土也随盘转动，必须加入它的惯量。

$$I_{\text{sys}} = I_d + m_c R^2 = 0.0122 + 0.60(0.30)^2 = 0.0662\,\text{kg m}^2.$$

$$\sum \tau = I_{\text{sys}}\alpha, \qquad |\alpha| = \frac{m_c g R}{I_{\text{sys}}} \approx 26.6\,\text{rad/s}^2.$$

官方评分

(d)(ii)	For using the rotational form of Newton's second law	**1 point**
	For a correct expression for the net torque exerted on the clay-disk system	**1 point**

Example Response

$\tau_{\text{net}} = Rm_{\text{c}}g$

For indicating that the rotational inertia of the clay-disk system is the sum of the rotational inertia of the disk and the rotational inertia of the lump of clay	**1 point**

Example Response

$I_{\text{system}} = I_{\text{disk}} + I_{\text{clay}}$

Example Solution

$\Sigma\tau = I\alpha$

$\alpha = \frac{\tau_{\text{net}}}{I}$

$\alpha = \frac{Rm_{\text{c}}g}{I_{\text{disk}} + I_{\text{clay}}}$

$\alpha = \frac{(0.3\text{ m})(0.60\text{ kg})\left(9.8\text{ m/s}^2\right)}{\left(\left(0.012\text{ kg}\cdot\text{m}^2\right) + (0.60\text{ kg})(0.3\text{m})^2\right)}$

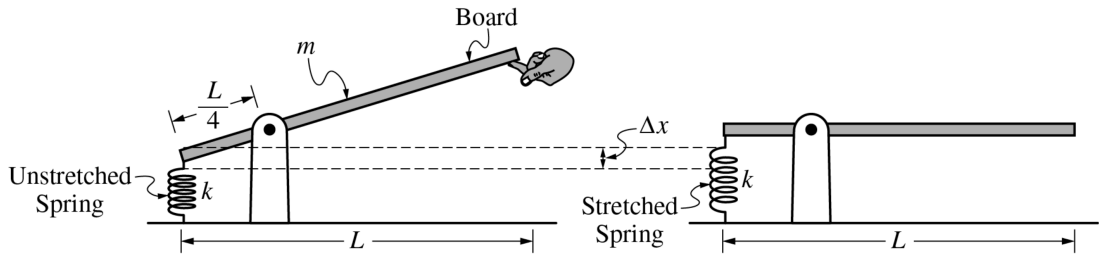
$\alpha = 26.7\text{ rad/s}^2$

Total for part (d) 6 points

Total for question 3 15 points

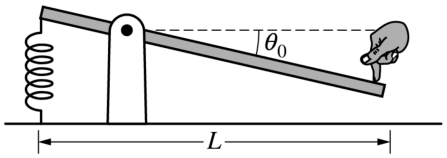
练习 2022 Set 2 Q3 (c)(ii)

本问承接 2022 Set 2 Q3 (b)。



Note: Figures not drawn to scale.

3. A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I.



Note: Figure not drawn to scale.

(c) A student pushes the board down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.

ii. Derive an expression for the angular acceleration α_0 of the board immediately after the board is released. Express your answer in terms of k, L, m, I, Δx_2, θ_0, and physical constants, as appropriate.

解：

此问求释放瞬间，取弹簧使板右端上升的方向为正。重力矩反向；倾斜后两竖直力的力臂都乘余弦。

$$I\alpha_0 = k\Delta x_2 \frac{L}{4}\cos\theta_0 - mg\frac{L}{4}\cos\theta_0.$$

$$\alpha_0 = \frac{L\cos\theta_0}{4I}(k\Delta x_2 - mg).$$

官方评分

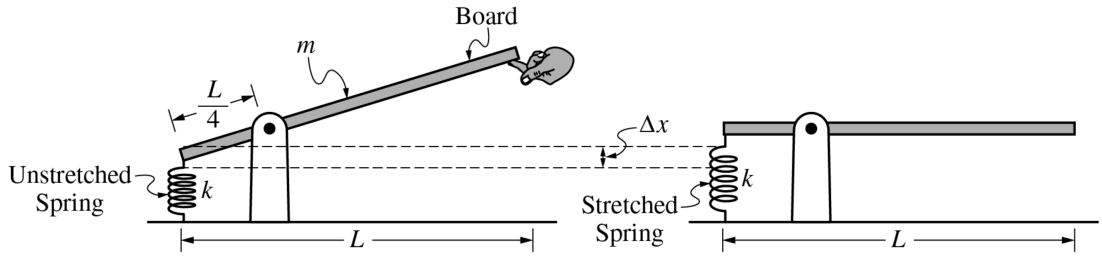
(c)(ii)	For indicating that the torques are in opposite directions in the rotational form of Newton's second law $\tau_{\text{spring}} - \tau_g = I\alpha$	**1 point**
	For including $\sin(90 - \theta_0)$, $\sin(90 + \theta_0)$, or $\cos\theta_0$ in an expression for the net torque exerted on the rod $F_{spring} d_{spring}(\cos\theta) - F_g d_g(\cos\theta) = I_B \alpha_0$	**1 point**
	For substituting mg for the force due to gravity and substituting $k\Delta x_2$ for the spring force in an expression for the net torque exerted on the rod $d_{spring} k\Delta x_2(\cos\theta_0) - d_g mg(\cos\theta_0) = I_B \alpha_0$	**1 point**
	For substituting correct lever arms in an expression for the net torque exerted on the rod $\frac{L}{4}(\cos\theta_0)k\Delta x_2 - \frac{L}{4}(\cos\theta_0)mg = I_B\alpha_0$	**1 point**
	Example Response $\tau_{\text{spring}} - \tau_g = I\alpha$ $F_{spring} d_{spring}(\cos\theta_0) - F_g d_g(\cos\theta_0) = I_B\alpha_0$ $l_{spring}(\cos\theta_0)k\Delta x_2 - l_g(\cos\theta_0)mg = I_B\alpha_0$ $\frac{L}{4}(\cos\theta_0)k\Delta x_2 - \frac{L}{4}(\cos\theta_0)mg = I_B\alpha_0$ $\alpha_0 = \frac{L}{4I_B}(k\Delta x_2\cos\theta_0 - mg\cos\theta_0)$	
		Total for part (c) 6 points

增加对称质量：力矩不变但惯量增大

两块关于支点对称放置，增加的重力矩相互抵消，弹簧的状态也没有改变，所以原净力矩不变。但两块离转轴的距离都非零，各增加正的惯量。相同净力矩作用于更大惯量，角加速度大小更小。

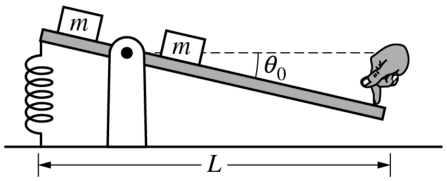
例题 2022 Set 2 Q3 (d)

本问承接 2022 Set 2 Q3 (c)(ii)。



Note: Figures not drawn to scale.

3. A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I.



Note: Figure not drawn to scale.

(d) Two blocks of equal mass m are attached to the board equal distances from the pivot point, as shown. The board is again pushed down on the right side so that the spring stretches the same distance Δx_2 as in part (c). The board is then released. How does the new angular acceleration α' when the blocks are attached compare to the angular acceleration α_0 from part (c) ?

___$\alpha' > \alpha_0$ ___$\alpha' < \alpha_0$ ___ $\alpha' = \alpha_0$

Justify your answer.

解：

选较小。新加的两块关于支点对称，增加的重力矩互相抵消，原弹簧和杆的净力矩不变；但二者都增加正的转动惯量。同一净力矩分给更大惯量，角加速度更小。

$$\alpha' = \frac{\tau_{\text{net}}}{I_{\text{new}}} < \frac{\tau_{\text{net}}}{I} = \alpha_0.$$

官方评分

(d)	For selecting "$\alpha' < \alpha_0$" with an attempt at a relevant justification	**1 point**
	For indicating that the net torque does not change	**1 point**
	For indicating the rotational inertia increases	**1 point**
	Example Response *The net torque on the system is the same (the spring and the person exert the same force and the additional torques from the gravitational forces on the masses cancel) but the rotational inertia of the system is now larger. This means that the angular acceleration of the system is now smaller than it was before.*	
	Total for part (d)	**3 points**
	Total for question 3	**15 points**

角速度图：斜率表示角加速度

图纵轴ω表示角速率，曲线斜率表示角加速度。盘从静止开始回转，角速率从零增加。弹簧逐渐恢复原长，驱动力矩与角加速度大小都减小；所以角速率图上升但越来越平，向下凹。弹簧回到原长的时刻，驱动力矩为零，图的切线水平。

例题 2022 Set 1 Q3 (d)

本问承接 2022 Set 1 Q3 (c)。

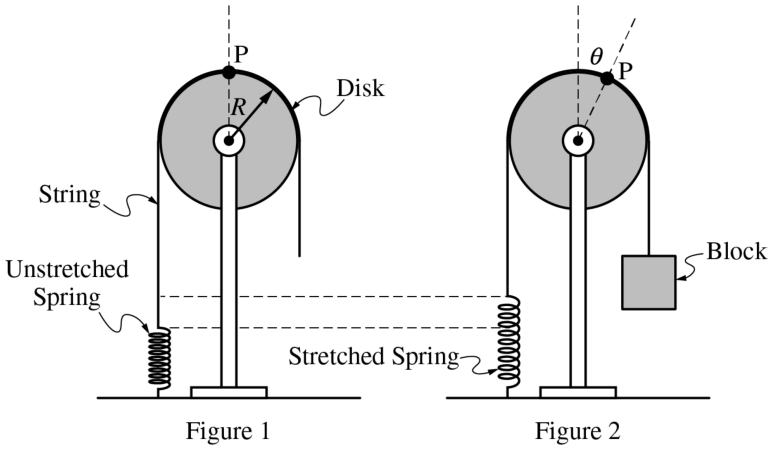


Figure 1 Figure 2

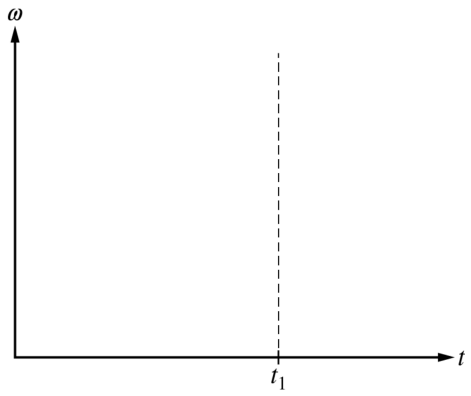
Note: Figures not drawn to scale.

3. A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_{d} and R, respectively. The rotational inertia of the disk is $I_{\mathrm{d}} = \frac{1}{2} M_{\mathrm{d}} R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

   A block of mass m_{B} is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

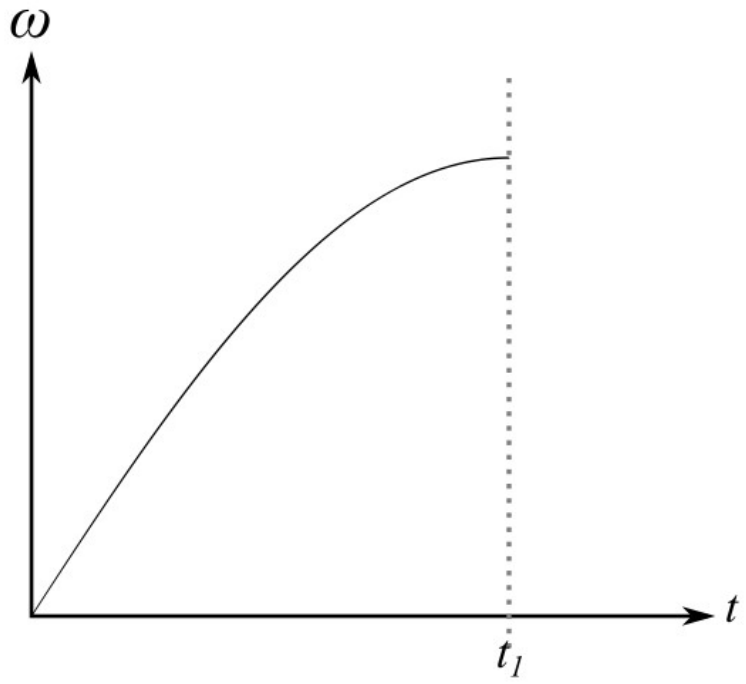
   Give all algebraic answers in terms of M_{d}, R, k, θ, and physical constants, as appropriate.

(d) At $t = t_1$, the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity ω of the disk as a function of time t from $t = 0$ to $t = t_1$.



解：

盘从静止回转，角速率大小从零增加；弹簧伸长越来越小，驱动的力矩和加速度大小也越来越小，所以角速率曲线增长而向下凹，回到原长时斜率为零。

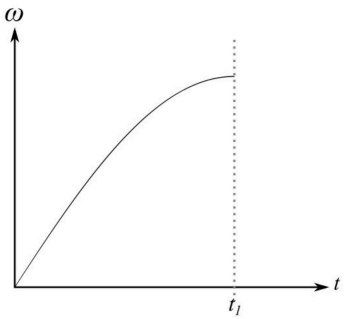


只画从释放到首次回到原长的区间。

官方评分

(d)	For a sketch that starts at zero and monotonically increases until time $t = t_1$	**1 point**
	For a sketch that is concave down between time $t = 0$ and $t = t_1$	**1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

Total for part (d) 2 points

匀角速率：一圈的时间由转角除速率

一整圈是2π弧度。角速率恒定时，每秒转过相同角度，所以一圈时间就是2π除以题给角速率；叶片边缘的半径不会改变一圈所用时间。

$$T = \frac{2\pi}{\omega_0}.$$

例题 2023 Set 2 Q3 (b)

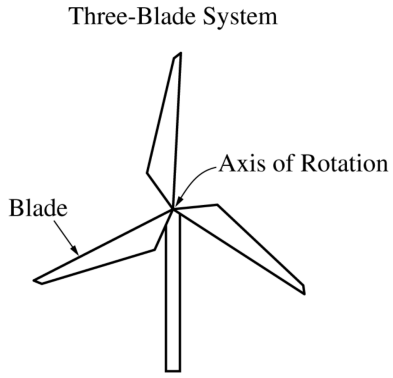


Figure 1

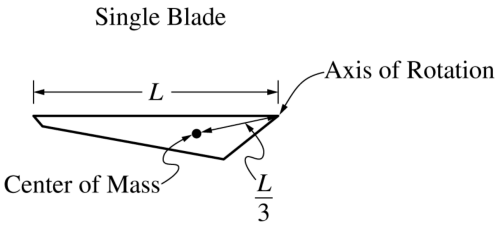


Figure 2

3. A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

(b) While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6$ rad/s. The length of one blade is $L = 36$ m. The numerical value of the rotational inertia of the system is $I_{sys} = 6.7 \times 10^6$ kg·m^2. Calculate the time T it takes the outer edge of a single blade to complete one revolution.

解：

一圈转角为2π，角速率恒定；直接用转角除角速率。

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{2.6} = 2.42\ \text{s}.$$

官方评分

(b)	For calculating the correct answer with correct units $(2.4\ \text{s})$	**1 point**

Example Solution

$v = r\omega$

$v = L\omega_0$

$\frac{d}{t} = L\omega_0$

$t = \frac{d}{L\omega_0} = \frac{2\pi L}{L\omega_0} = \frac{2\pi}{\omega_0}$

$t = \frac{2\pi}{(2.6\ \text{rad/s})}$

$\therefore t = 2.4\ \text{s}$

Total for part (b) **1 point**

转动停止：初始转动动能就是耗散量

物体最终静止，末转动动能为零。初动能是$I\omega_0^2/2$，题目问耗散量就报告这个正值；若写系统的动能变化，才是这个值的负数。惯量和初角速率采用该问情境里已经给定的数值。

例题 2023 Set 2 Q3 (c)(i)

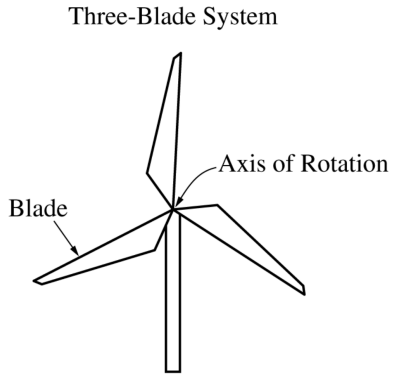


Figure 1

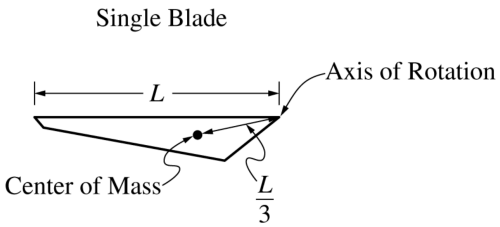


Figure 2

3. A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

(c) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

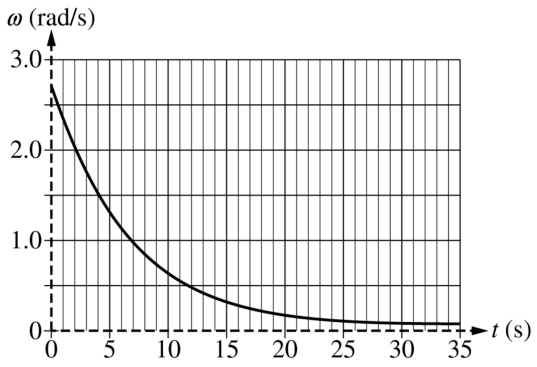


Figure 3

While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6$ rad/s. The length of one blade is $L = 36$ m. The numerical value of the rotational inertia of the system is

$$I_{\text{sys}} = 6.7 \times 10^6 \text{ kg·m}^2.$$

i. Calculate the amount of energy dissipated from $t = 0$, when the wind stops blowing, until the system comes to rest.

解：

末态静止，全部初始转动动能耗散。题问耗散量，报告正的能量大小；系统动能变化则是其负值。

$$E_{\text{diss}} = \frac{1}{2} I_{\text{sys}} \omega_0^2 = \frac{1}{2}(6.7 \times 10^6)(2.6)^2 = 2.26 \times 10^7 \text{ J}.$$

官方评分

(c)(i)	For indicating that the total initial rotational kinetic energy is dissipated	**1 point**

Example Response

$$\Delta K_{\text{rot}} = E_{\text{dis}}$$
$$0 - \frac{1}{2} I \omega_0^{\,2} = E_{\text{dis}}$$

For substituting correct values for the rotational inertia and initial angular speed of the system **1 point**

Example Response

$$E_{\text{dis}} = -\frac{1}{2}\left(6.7\times10^{6}\ \text{kg}\cdot\text{m}^2\right)\left(2.6\ \text{rad/s}\right)^2$$

For an answer consistent with substitutions above and with correct units **1 point**

Example Response

$$E_{\text{dis}} = -2.3\times10^{7}\ \text{J}$$

Example Solution

$$\Delta K_{\text{rot}} = E_{\text{dis}}$$
$$0 - \frac{1}{2} I \omega_0^{\,2} = E_{\text{dis}}$$
$$E_{\text{dis}} = -\frac{1}{2}\left(6.7\times10^{6}\ \text{kg}\cdot\text{m}^2\right)\left(2.6\ \text{rad/s}\right)^2$$
$$E_{\text{dis}} = -2.3\times10^{7}\ \text{J}$$

Scoring Note: A response may earn full credit for positive or negative values of dissipated energy.

给定角速度：求导得到净力矩

角加速度是角速度对时间的导数。先对指数衰减函数求导，再乘关于同一转轴的惯量。负号说明力矩反对原旋转方向；不要因为题目讨论减速就把角速度本身直接当作力矩。

$$\alpha = \frac{d\omega}{dt}, \qquad \tau = I\alpha.$$

例题 2023 Set 2 Q3 (c)(ii)

本问承接 2023 Set 2 Q3 (a)。

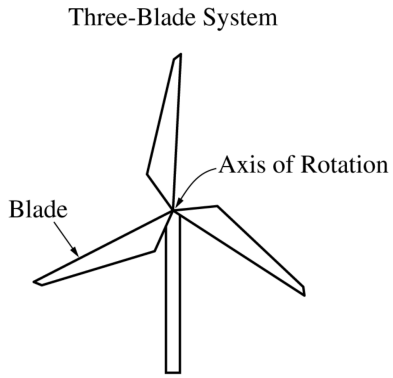


Figure 1

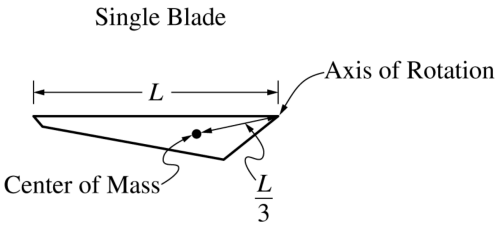


Figure 2

3. A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

(c) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

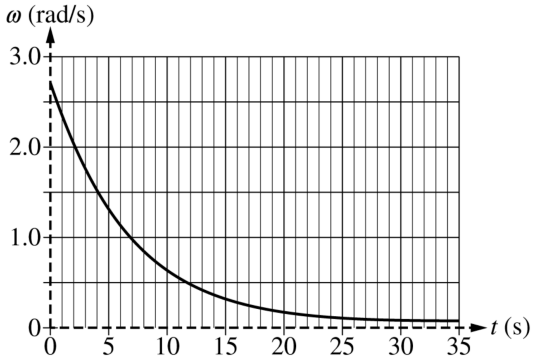


Figure 3

ii. Derive an expression for the net torque exerted on the system as a function of t as the system slows down. Express your answers in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

解：

先对角速度求导，再乘惯量得到净力矩。负号表示力矩反对原转动方向。

$$\alpha = \frac{d\omega}{dt} = -\beta_0 \omega_0 e^{-\beta_0 t}, \qquad \tau_{\text{net}} = I_{\text{sys}} \alpha = -\beta_0 I_{\text{sys}} \omega_0 e^{-\beta_0 t}.$$

官方评分

(c)(ii) For using Newton's second law in rotational form **1 point**

For attempting to differentiate the equation for ω **1 point**

Example Response

$$\tau = I_{\text{sys}} \frac{d}{dt}\left(\omega_0 e^{-\beta_0 t}\right)$$

For a correct expression for the torque on the system **1 point**

Example Response

$$\tau = -\beta_0 I_{\text{sys}} \omega_0 e^{-\beta_0 t}$$

Example Solution

$$\tau = I\alpha$$
$$\tau = I_{\text{sys}} \frac{d\omega}{dt}$$
$$\tau = I_{\text{sys}} \frac{d}{dt}\left(\omega_0 e^{-\beta_0 t}\right)$$
$$\therefore \tau = -\beta_0 I_{\text{sys}} \omega_0 e^{-\beta_0 t}$$

给定角速度：积分得到角位移

角速率告诉每秒转过多少角度，把它从开始时刻累积到t，就得到转角。指数函数积分时要除以指数内时间系数，并检查t=0时转角确实为零。

$$\Delta\theta = \int_0^t \omega(t')\,dt'$$

例题 2023 Set 2 Q3 (c)(iii)

本问承接 2023 Set 2 Q3 (c)(ii)。

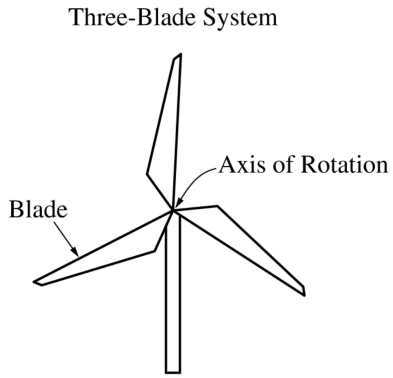


Figure 1

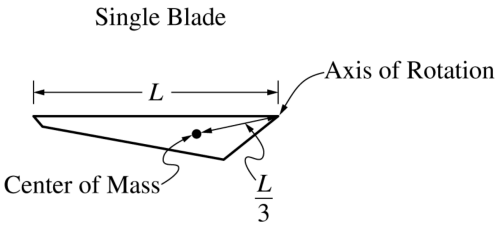


Figure 2

3. A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

(c) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

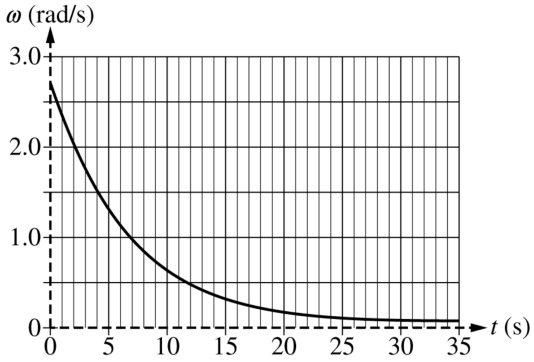


Figure 3

iii. Derive an expression for the angular displacement of the system $\Delta\theta$ as a function of t. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

解：

转过的角度是角速度—时间曲线下的面积，积分从开始时刻到所求时刻。

$$\Delta\theta = \int_0^t \omega_0 e^{-\beta_0 u}\, du = \left[-\frac{\omega_0}{\beta_0} e^{-\beta_0 u}\right]_0^t = \frac{\omega_0}{\beta_0}(1 - e^{-\beta_0 t}).$$

官方评分

(c)(iii) For attempting to integrate the expression for angular speed **1 point**

Example Response

$$\Delta\theta = \int \omega(t)\,dt$$

For using the correct limits of integration **1 point**

Example Response

$$\Delta\theta = \int_0^t \omega_0 e^{-\beta_0 t}\,dt$$

For a correct expression for angular displacement **1 point**

Example Response

$$\Delta\theta = \frac{\omega_0}{\beta_0}\left(1 - e^{-\beta_0 t}\right)$$

Example Solution

$$\Delta\theta = \int \omega(t)\,dt$$

$$\Delta\theta = \int_0^t \omega_0 e^{-\beta_0 t}\,dt$$

$$\Delta\theta = \left(-\frac{\omega_0}{\beta_0} e^{-\beta_0 t}\right)\Bigg|_0^t = -\frac{\omega_0}{\beta_0} e^{-\beta_0 t} + \frac{\omega_0}{\beta_0}(1)$$

$$\therefore \Delta\theta = \frac{\omega_0}{\beta_0}\left(1 - e^{-\beta_0 t}\right)$$

Total for part (c) 9 points

阻尼增大：保持初值，改变衰减速度

两系统初角速率相同，图在t=0起点相同。指数中的衰减常量更大，使每个正时间的角速率更小；新曲线仍为正、单调减小、向上凹并渐近于零。不能把更快衰减画成较低初值。

例题 2023 Set 2 Q3 (d)

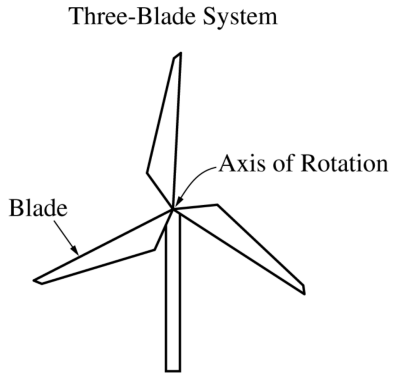


Figure 1

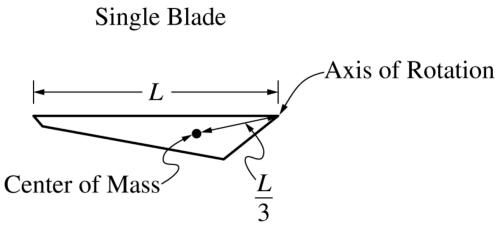


Figure 2

3. A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

The three-blade system is now replaced with a second three-blade system identical to the first, except that the second three-blade system slows down according to the equation $\omega = \omega_0 e^{-\beta t}$, where $\omega_0 = 2.6\,\text{rad/s}$ and $\beta > \beta_0$. The original angular speed function is shown as a dashed line in Figure 4.

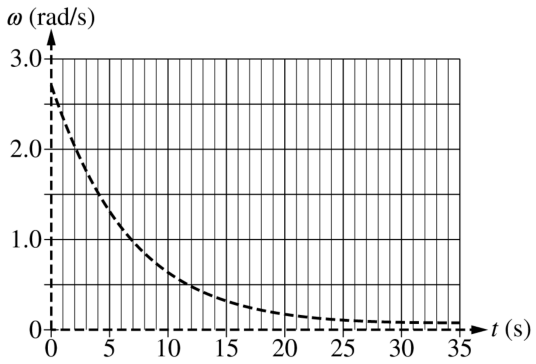
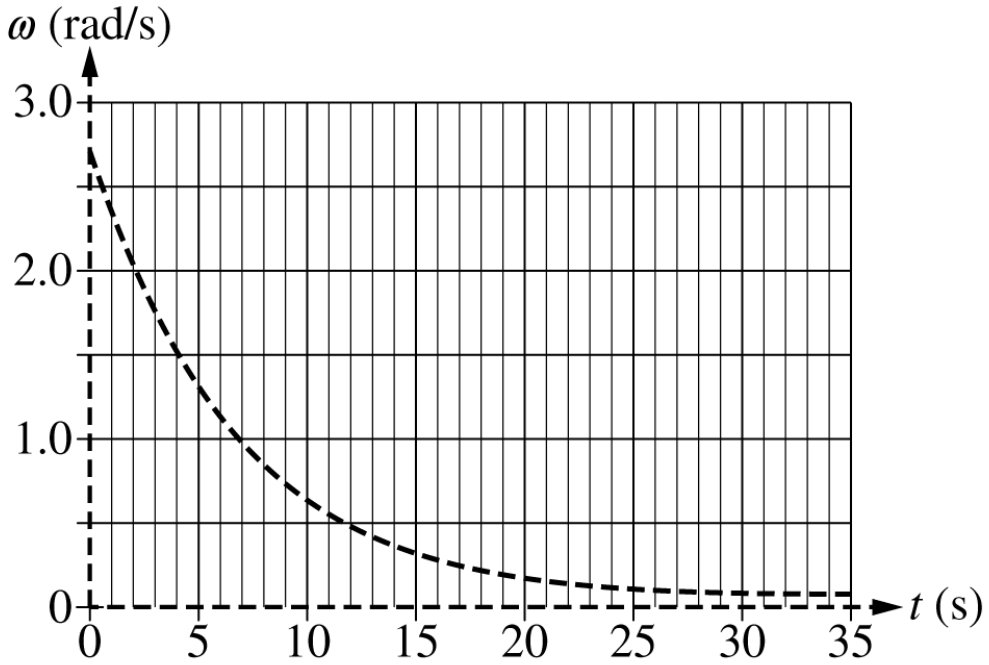


Figure 4

(d) On the graph in Figure 4, sketch the angular speed of the second three-blade system as a function of time t.

解：

两条曲线同从2.6弧度每秒出发；新阻尼常量更大，所以任意正时间的新角速率更小。新图仍连续、单调下降、向上凹，且渐近于零。

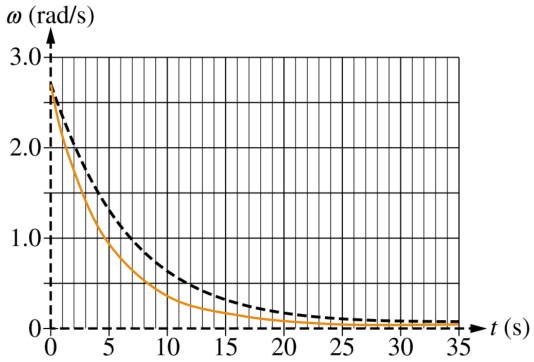


实线代表更大阻尼常量，虚线保留原衰减。

官方评分

(d)	For drawing a continuous curve showing an exponential decay	**1 point**
	For starting at $\omega = 2.6$ rad/s and drawing a curve below the original curve	**1 point**

Example Solution

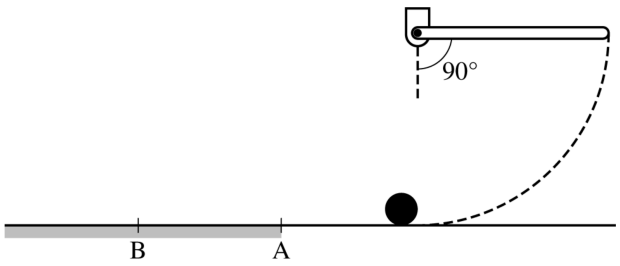


Total for part (d)	**2 points**
Total for question 3	**15 points**

杆下落：质心下降的势能变成转动动能

固定支点没有位移，支点力不做功。杆从水平转到竖直时，质心下降半根杆长；用这段重力势能降低量等于关于支点的转动动能。惯量必须对应支点轴。

例题 2023 Set 1 Q3 (a)



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

解：

杆质心由水平位置下降半杆长。支点不做功，杆的重力势能变成绕支点的转动动能。

$$Mg\frac{\ell}{2} = \frac{1}{2}\left(\frac{1}{3}M\ell^2\right)\omega^2, \qquad \omega = \sqrt{\frac{3g}{\ell}}.$$

官方评分

(a) For using a correct expression for conservation of energy of the rod-Earth system **1 point**

Example Response

$\Delta U + \Delta K = 0$

$\left(0 - Mgh_{\text{CM}}\right) + \left(\frac{1}{2} I\omega_f^2 - 0\right) = 0$

$Mgh_{\text{cm}} = \frac{1}{2} I\omega_f^2$

For correctly substituting h and I into the correct energy expression **1 point**

Example Response

$Mg\frac{\ell}{2} = \frac{1}{2}\left(\frac{1}{3} M\ell^2\right)\omega_f^2$

Example Solution

$\Delta U + \Delta K = 0$

$\left(0 - Mgh_{\text{CM}}\right) + \left(\frac{1}{2} I\omega_f^2 - 0\right) = 0$

$Mgh_{\text{cm}} = \frac{1}{2} I\omega_f^2$

$Mg\frac{\ell}{2} = \frac{1}{2}\left(\frac{1}{3} M\ell^2\right)\omega_f^2$

$\therefore \omega_f = \sqrt{\frac{3g}{\ell}}$

Alternate Solution **1 point**

For using a correct expression of Newton's second law in rotational form

Alternate Example Response

$$\tau_{net} = I\alpha$$

For correctly substituting expressions for net torque and rotational inertia of the rod **1 point**

Alternate Example Response

$$\frac{\ell}{2}Mg\cos\theta = \frac{1}{3}M\ell^2\alpha$$

Alternate Example Solution

$$\tau_{net} = I\alpha$$
$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{d\omega}{d\theta}\omega$$
$$\frac{\ell}{2}Mg\cos\theta = \frac{1}{3}M\ell^2\alpha$$
$$\frac{\ell}{2}Mg\cos\theta = \frac{1}{3}M\ell^2\frac{d\omega}{d\theta}\omega$$
$$\frac{\ell}{2}Mg\cos\theta d\theta = \frac{1}{3}M\ell^2\omega d\omega$$
$$\int_0^{\frac{\pi}{2}}\frac{\ell}{2}Mg\cos\theta d\theta = \int_0^{\omega_f}\frac{1}{3}M\ell^2\omega d\omega$$
$$\frac{\ell}{2}Mg\left[\sin\frac{\pi}{2} - \sin 0\right] = \frac{1}{3}M\ell^2\left[\frac{1}{2}\omega_f^2 - 0\right]$$
$$3\frac{g}{\ell}\left[1-0\right] = \left[\omega_f^2\right]$$
$$\omega_f = \sqrt{\frac{3g}{\ell}}$$

Scoring Note: *The full integration is not needed to earn points but is presented for clarity.*

Total for part (a) 2 points

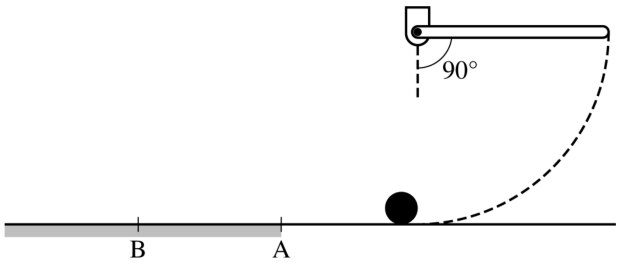
带支点碰撞：守恒关于支点的角动量

碰撞时支点给系统外冲量，系统线动量不能直接守恒。但这个力的作用线过支点，对支点力矩为零；碰撞极短，重力角冲量可忽略，于是可以守恒关于支点的角动量。球的平动角动量是它的动量乘到支点的垂直距离。

$$L_{\text{before}} = L_{\text{after}}, \qquad L_{\text{particle}} = mvr_{\perp}.$$

例题 2023 Set 1 Q3 (b)

本问承接 2023 Set 1 Q3 (a)。



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate.

解：

碰撞中支点有外冲量，不能守恒系统线动量；但支点力对该点的力矩为零，且碰撞短促，重力的角冲量可忽略，所以守恒关于支点的角动量。球心距支点近似为杆长，碰后杆停止。

$$I\omega = mv_0\ell, \qquad \frac{1}{3}(2m)\ell^2\sqrt{\frac{3g}{\ell}} = mv_0\ell.$$

$$v_0 = \sqrt{\frac{4g\ell}{3}}.$$

官方评分

(b)	For using a correct expression for conservation of angular momentum	**1 point**

Example Response

$L_{\mathrm{i}} = L_{\mathrm{f}}$

$I\omega = mvr$

For correctly substituting the expression for ω_f from part (a)	**1 point**

Example Response

$$\frac{2}{3}m\ell^2\sqrt{\frac{3g}{\ell}} = mv_0\ell$$

Example Solution

$L_{\mathrm{i}} = L_{\mathrm{f}}$

$I\omega = mvr$

$$\frac{2}{3}m\ell^2\sqrt{\frac{3g}{\ell}} = mv_0\ell$$

$$\therefore v_0 = \sqrt{\frac{4}{3}g\ell}$$

Scoring Note: The last equation is not needed for scoring the item but is presented for clarity.

Total for part (b)	**2 points**

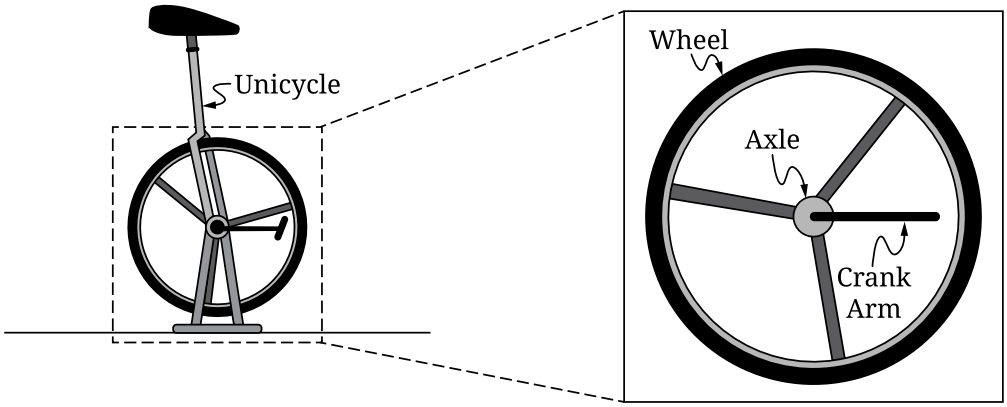
恒定垂直力转一圈：力矩做功转为动能

力始终垂直曲柄，力矩大小是力乘曲柄长，整个过程保持不变。力矩乘转角给转动做功；转一圈的转角是2π。从静止开始，这些功全部变成轮的转动动能。

$$W = \int \tau\, d\theta = \tau(2\pi) = \frac{1}{2}I\omega^2.$$

例题 2026 Q4 (B)

4. A unicycle is placed on a stand such that the wheel does not touch the ground. The rotational inertia of the wheel about the axle is I_{W}. One end of a crank arm can be attached to the wheel, as shown in the side view.



Side View

In Scenarios 1 and 2, crank arms of different lengths are connected to the wheel. In each scenario, the wheel is initially at rest. A force of magnitude F is then exerted at the end of each crank arm. The direction of the force remains perpendicular to each crank arm at all times. The mass of each crank arm is negligible. The scenarios differ as described.

- Scenario 1: The length of the crank arm is ℓ_1. The angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_1.
- Scenario 2: The length of the crank arm is ℓ_2, where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2.

Part B

Derive an expression for the angular speed ω_1 of the wheel in Scenario 1 immediately after the crank arm has been turned through one rotation. Express your answer in terms of I_{W}, F, ℓ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

从功—能定理开始。垂直力的力矩恒定，一整圈转角是2π，初始动能为零。

$$W = \Delta K_{\mathrm{rot}}, \qquad F\ell_1(2\pi) = \frac{1}{2} I_W \omega_1^2.$$

$$\omega_1 = \sqrt{\frac{4\pi F \ell_1}{I_W}}.$$

官方评分

B	For a multistep derivation that includes **one** of the following: • Conservation of energy that indicates work is done on the wheel • The work-energy theorem • Newton's second law in rotational form	**Point B1**
	For indicating **one** of the following: • The work done by F on the crank arm is $2\pi F\ell_1$. • The magnitude of the angular acceleration of the wheel is $\alpha = \frac{F\ell_1}{I_W}$.	**Point B2**
	For a correct expression for ω_1 in terms of given quantities	**Point B3**

Example Responses

Solution 1

$W = \Delta K$

$\tau\Delta\theta = \Delta K_{rot}$

$(F\ell_1)(2\pi) = \frac{1}{2} I\omega_1^2$

$\frac{4\pi F\ell_1}{I_W} = \omega_1^2$

$\omega_1 = \sqrt{\frac{4\pi F\ell_1}{I_W}}$

Solution 2

$\alpha_{sys} = \frac{\sum\tau}{I_{sys}}$

$\alpha = \frac{Fr}{I} = \frac{F\ell_1}{I_W}$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$\omega_1^2 = 2\alpha(2\pi)$

$\omega_1^2 = 4\pi\alpha$

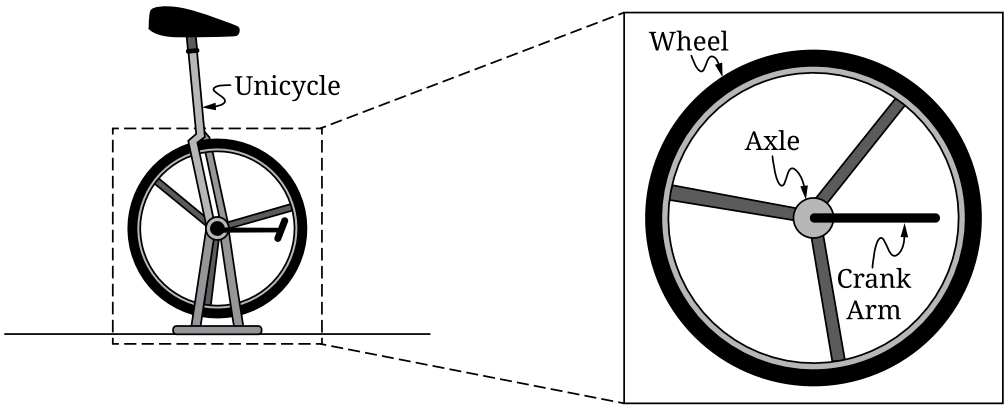
$\omega_1^2 = 4\pi\frac{F\ell_1}{I_W}$

$\omega_1 = \sqrt{\frac{4\pi F\ell_1}{I_W}}$

练习 2026 Q4 (A)

本问承接 2026 Q4 (B)。

4. A unicycle is placed on a stand such that the wheel does not touch the ground. The rotational inertia of the wheel about the axle is I_W. One end of a crank arm can be attached to the wheel, as shown in the side view.



Side View

In Scenarios 1 and 2, crank arms of different lengths are connected to the wheel. In each scenario, the wheel is initially at rest. A force of magnitude F is then exerted at the end of each crank arm. The direction of the force remains perpendicular to each crank arm at all times. The mass of each crank arm is negligible. The scenarios differ as described.

- Scenario 1: The length of the crank arm is ℓ_1. The angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_1.
- Scenario 2: The length of the crank arm is ℓ_2, where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2.

Part A

Indicate whether ω_2 is greater than, less than, or equal to ω_1 by writing one of the following.

- $\omega_2 > \omega_1$
- $\omega_2 < \omega_1$
- $\omega_2 = \omega_1$

Justify your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

解：

选长曲柄转得更快。同样大小的力始终垂直曲柄，长曲柄的力矩较大；都转一圈，较大力矩做功更多。轮的转动惯量相同，所得转动动能越多，末角速率越大。

$$\omega_2 > \omega_1.$$

官方评分

A	For indicating $\omega_2 > \omega_1$	**Point A1**
	For a justification that correctly relates force and crank arm length to the torque exerted on the crank arm	**Point A2**
	For a justification that correctly relates torque exerted over a single rotation to **one** of the following: • The rotational kinetic energy of the wheel • The angular acceleration	**Point A3**
	Example Response $\omega_2 > \omega_1$. *The torque exerted on the crank arm will be greater at the end of the crank arm with a greater length, and a greater torque exerted over one rotation means a greater change in rotational kinetic energy of the wheel. A greater rotational kinetic energy for the same wheel with the same rotational inertia means the wheel has a greater angular speed.*	

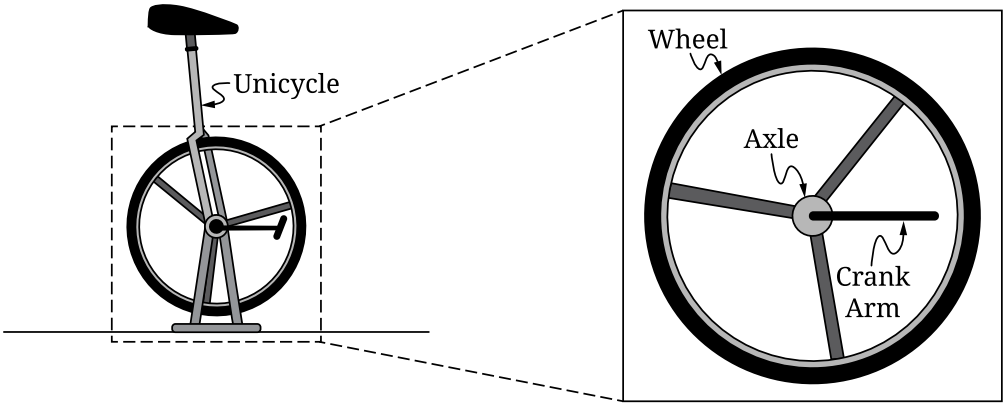
增加平动：同样能量需要分给两种运动

轮固定在架上时，输入能量只增加转动动能；放在地上纯滚动时，还要增加系统向前的平动动能。在同样输入功与同一轮的惯量下，可用于转动的份额减小，末角速率更小。

例题 2026 Q4 (C)

本问承接 2026 Q4 (B)。

4. A unicycle is placed on a stand such that the wheel does not touch the ground. The rotational inertia of the wheel about the axle is I_{W}. One end of a crank arm can be attached to the wheel, as shown in the side view.



Side View

In Scenarios 1 and 2, crank arms of different lengths are connected to the wheel. In each scenario, the wheel is initially at rest. A force of magnitude F is then exerted at the end of each crank arm. The direction of the force remains perpendicular to each crank arm at all times. The mass of each crank arm is negligible. The scenarios differ as described.

- Scenario 1: The length of the crank arm is ℓ_1. The angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_1.
- Scenario 2: The length of the crank arm is ℓ_2, where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2.

Part C

In Scenario 3, the unicycle is placed on the ground such that the unicycle can move forward without the wheel slipping on the ground. A force of the same magnitude F is exerted at the end of the crank arm of length ℓ_1, as in Scenario 1. The direction of the force remains perpendicular to the crank arm at all times. The resulting angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_3.

Indicate whether ω_3 is greater than, less than, or equal to ω_1 by writing one of the following.

- $\omega_3 > \omega_1$
- $\omega_3 < \omega_1$
- $\omega_3 = \omega_1$

Briefly **justify** your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

解：

选放到地上后轮的角速率较小。转同一曲柄一圈所输入的能量现在既产生整体向前的平动动能，也产生轮的转动动能。原来固定在架上时只增加转动动能，所以能分给转动的份额减少，末角速率较小。

$$\omega_3 < \omega_1.$$

官方评分

C	For indicating $\omega_3 < \omega_1$	**Point C1**
	For a justification that indicates **one** of the following: • The work done to turn the crank arm is converted into translational and rotational kinetic energies. • The torque exerted on the wheel by friction changes the net torque on the wheel.	**Point C2**
	Example Response $\omega_3 < \omega_1$. *When the unicycle is stationary, the work done by the force of magnitude F to turn the crank arm one rotation is converted into only rotational kinetic energy. When the unicycle can move forward, the same amount of work done is converted into translational and rotational kinetic energies. This means the wheel has to turn more slowly when the unicycle can move forward.*	

滑动到滚动与振动

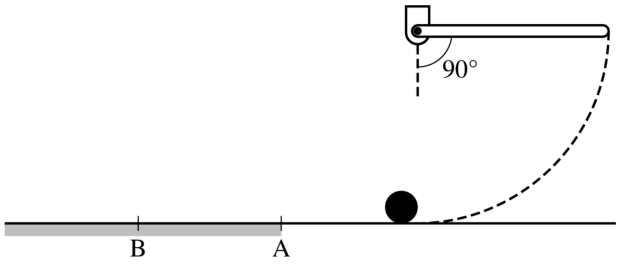
滑动摩擦的力矩：角速度从零增长

摩擦从球底向右，关于球心产生逆时针力矩。先取这个转动方向为正，用摩擦力乘半径得到力矩，再除以球的惯量得到角加速度。球最初不转，角速度从零开始累积。

$$\tau = f_k R = I\alpha, \qquad \omega(t) = \omega(0) + \int_0^t \alpha\, dt.$$

例题 2023 Set 1 Q3 (d)(ii)

本问承接 2023 Set 1 Q3 (c)。



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

(d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

ii. The angular velocity ω of the sphere as a function of time t

解：

取逆时针为角量正方向。摩擦关于球心的力矩为正，球的初角速度为零。

$$\mu mgR = \left(\frac{2}{5}mR^2\right)\alpha, \qquad \alpha = \frac{5\mu g}{2R}, \qquad \omega(t) = \frac{5\mu g}{2R}t.$$

官方评分

(d)(ii) For using Newton's second law in rotational form with substitutions for rotational inertia and torque **1 point**

Example Response

$\tau = I\alpha$

$F_f R = \frac{2}{5} mR^2 \alpha$

For correctly substituting for friction and solving for α **1 point**

Example Response

$F_f = \mu mg$

$\alpha = \frac{5\mu g}{2R}$

For correctly substituting α into a rotational kinematic equation and solving for ω **1 point**

Example Response

$\omega = \omega_0 + \alpha t$

$\therefore \omega = \frac{5\mu g}{2R} t$

Example Solution

$\tau = I\alpha$

$F_f R = \frac{2}{5} mR^2 \alpha$

$F_f = \mu mg$

$\alpha = \frac{5\mu g}{2R}$

$\omega = \omega_0 + \alpha t$

$\therefore \omega = \frac{5\mu g}{2R} t$

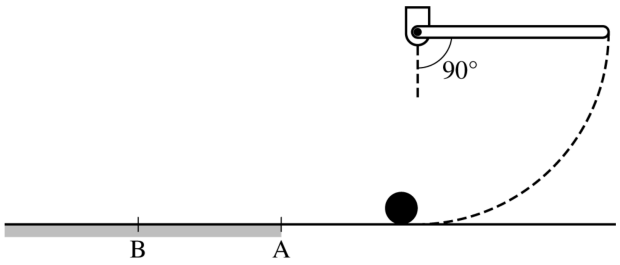
Total for part (d) 6 points

开始纯滚动：接触点相对地面速度为零

滑动过程中，质心速率在减小，边缘转动速率在增加。接触点恰好不再相对地面滑动的瞬间，两种速率大小相等，即v=Rω。先把已求的两函数代入这个条件求时间，再代回求那一刻的线速度；在此前不能提前使用纯滚动约束。

例题 2023 Set 1 Q3 (e)(i)

本问承接 2023 Set 1 Q3 (d)(i)、2023 Set 1 Q3 (d)(ii)。



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

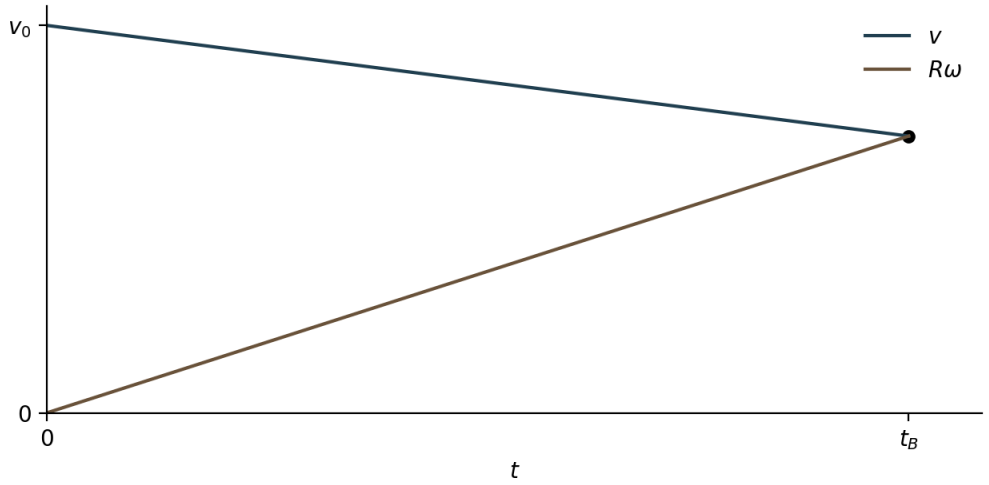
After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

解：

滑动结束时，接触点相对地面速度为零，即质心速率等于边缘转动速率。此前不能提前使用纯滚动关系。

$$v = R\omega, \qquad v_0 - \mu g t = \frac{5}{2}\mu g t, \qquad t_B = \frac{2v_0}{7\mu g}.$$



质心速率减小、边缘转动速率增大；交点的时刻是开始纯滚动的时刻t_B。

官方评分

(e)(i) For indicating the linear speed is equal to $R\omega$ when slipping stops at Point B **1 point**

Example Response

$v = R\omega$

For correctly substituting v and ω from parts (d)(i) and (d)(ii) **1 point**

Scoring Note: Substituting the acceleration from part (d)(i) into a valid kinematic equation that includes time can earn this point.

Example Response

$$v_0 - \mu g t = R\frac{5\mu g}{2R}t$$

Example Solution

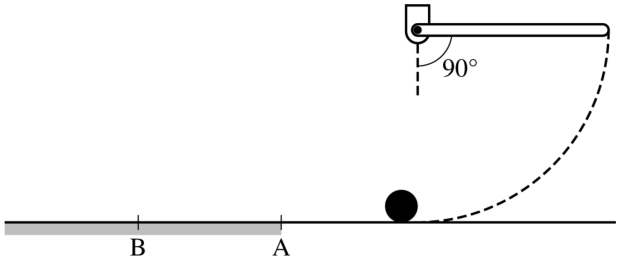
$v = R\omega$

$$v_0 - \mu g t = R\frac{5\mu g}{2R}t$$

$$\therefore t = \frac{2v_0}{7\mu g}$$

练习 2023 Set 1 Q3 (e)(ii)

本问承接 2023 Set 1 Q3 (e)(i)、2023 Set 1 Q3 (d)(i)。



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

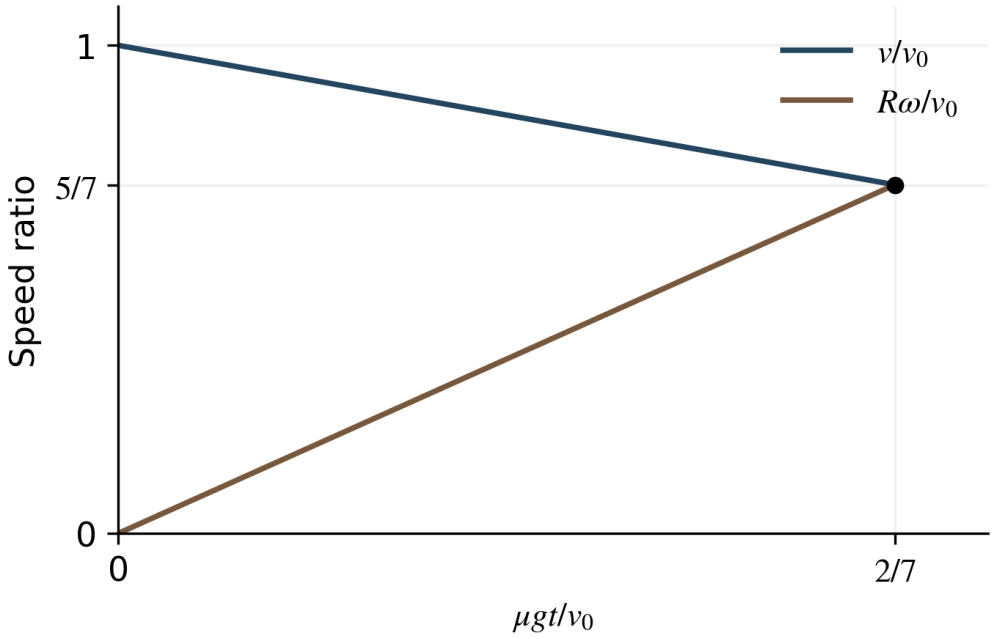
After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

解：

将这个时刻代回平动速度。

$$v_B = v_0 - \mu g \frac{2v_0}{7\mu g} = \frac{5}{7}v_0.$$



平动速率下降、边缘转动速率上升，交点才开始无滑动滚动。

官方评分

(e)(ii) For correctly substituting the expression for time from (e)(i) into the expression for velocity in (d)(i) **1 point**

Example Solution

$$v = v_0 - \mu g t$$

$$v = v_0 - \mu g \frac{2v_0}{7\mu g}$$

$$\therefore v = \frac{5}{7}v_0$$

Scoring Note: The last equation is not needed for scoring the item but is presented for clarity.

Total for part (e) 3 points

Total for question 3 15 points

纯滚动上坡：平动与转动方程一起求静摩擦

纯滚动时，摩擦既影响质心减速，也提供使角速率减小的力矩。取上坡为平动正方向、上滚方向为角量正方向，两个加速度都为负。分别列牛顿第二定律和转动定律，再用a=Rα消去加速度；实际静摩擦由运动条件决定，不预先等于μsN。

$$f - Mg\sin\theta = Ma, \qquad -fR = I\alpha, \qquad a = R\alpha.$$

例题 2025 Q4 (B)

4. A uniform disk and ring, each of mass M and radius R, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v.

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

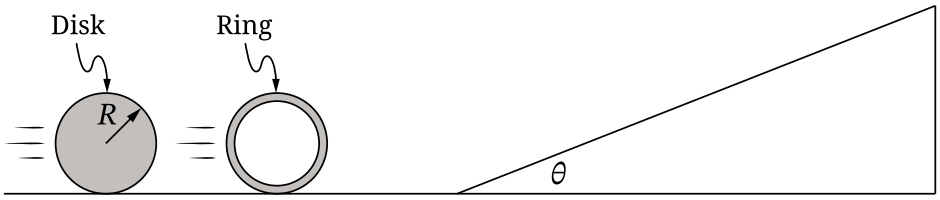


Figure 1

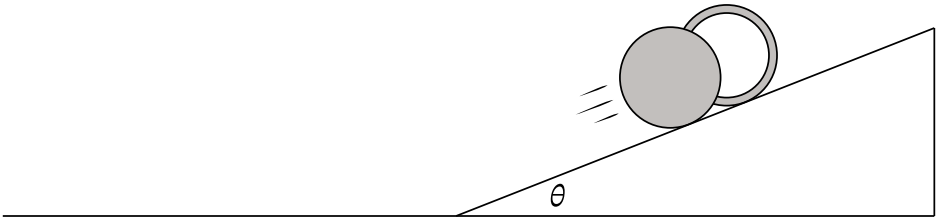
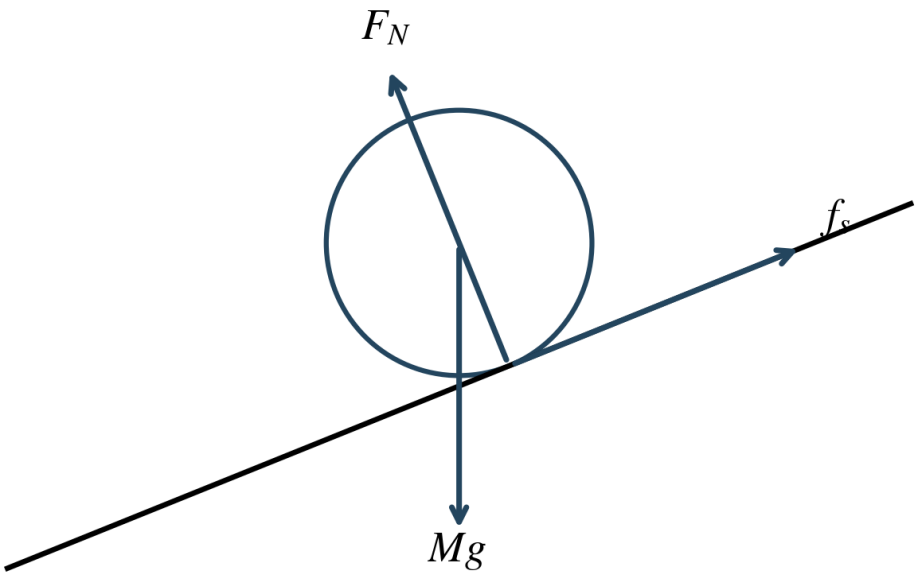


Figure 2

B. A cylinder has mass M, radius R, and rotational inertia I about its central axis. The cylinder rolls without slipping up a ramp that is inclined at an angle θ above the horizontal.
Derive an expression for the magnitude of the static frictional force f exerted on the cylinder by the ramp. Express your answer in terms of M, R, I, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

解：

取沿斜面向上为平动正方向，取上滚时的转动方向为角量正方向。上滚正在减速，平动与角加速度均为负；向上的摩擦提供反向转矩，使角速率减小。两个方程都保留方向。



上滚且无滑动时，静摩擦沿斜面向上。

$$f - Mg\sin\theta = Ma, \qquad -fR = I\alpha, \qquad a = R\alpha.$$

由第二式和滚动约束消去加速度，再解出摩擦大小。

$$a = -\frac{fR^2}{I}, \qquad f - Mg\sin\theta = -\frac{MfR^2}{I}.$$

$$f = \frac{IMg\sin\theta}{I + MR^2}.$$

官方评分

B	For a multistep derivation that includes Newton's second law in both translational and rotational forms	**Point B1**
	For indicating opposite signs for the gravitational force and frictional force in an expression for Newton's second law	**Point B2**
	For using the relationship $a = r\alpha$ in an attempt to solve a system of equations **Scoring Note:** Responses that include conservation of energy may earn full credit.	**Point B3**

Example Response

$$\alpha_{\text{sys}} = \frac{\sum \tau}{I_{\text{sys}}}$$

$$\left(\frac{a}{R}\right) = \frac{fR}{I}$$

$$a = \frac{fR^2}{I}$$

$$\vec{a}_{\text{sys}} = \frac{\sum \vec{F}}{m_{\text{sys}}}$$

$$f - Mg\sin\theta = -Ma$$

$$f - Mg\sin\theta = -M\frac{fR^2}{I}$$

$$f + M\frac{fR^2}{I} = Mg\sin\theta$$

$$f\left(1 + \frac{MR^2}{I}\right) = Mg\sin\theta$$

$$f = \frac{Mg\sin\theta}{1 + \frac{MR^2}{I}}$$

练习 2025 Q4 (A)

本问承接 2025 Q4 (B)。

4. A uniform disk and ring, each of mass M and radius R, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v.

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

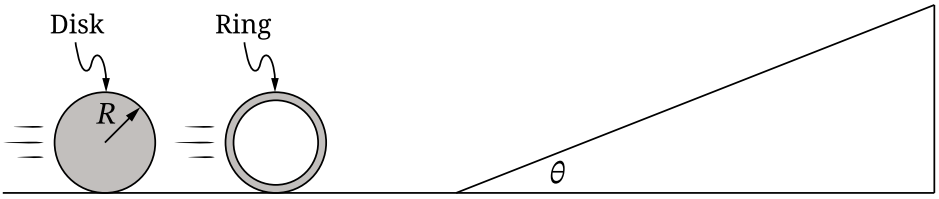


Figure 1

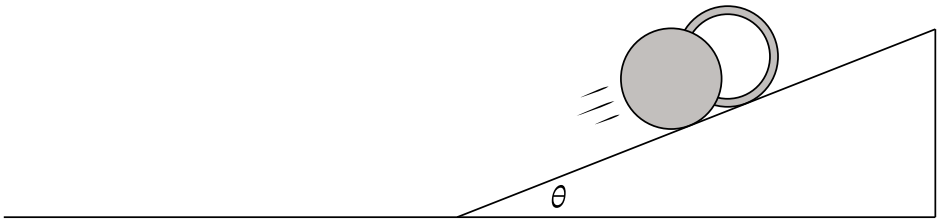


Figure 2

A. While the disk and the ring are rolling on the ramp without slipping, the magnitudes of the static frictional force exerted on the disk and on the ring by the ramp are f_{D} and f_{R}, respectively.

Indicate whether f_{D} is greater than, less than, or equal to f_{R} by writing one of the following.

- $f_{\mathrm{D}} > f_{\mathrm{R}}$
- $f_{\mathrm{D}} < f_{\mathrm{R}}$
- $f_{\mathrm{D}} = f_{\mathrm{R}}$

Justify your answer using qualitative reasoning beyond referencing equations.

解：

选盘的静摩擦较小。两物体的质量、重力沿斜面分量相同；环的质量更靠外，转动惯量较大，因此在同一质心速度下有更多转动动能，能滚得更远，向下的减速度较小。要让相同向下重力留下更小的向下合力，环需要更大的向上静摩擦。

$$f_D < f_R.$$

官方评分

A	For indicating $f_D < f_R$	**Point A1**
	For a justification that compares **one** of the following: • The motions of the disk and the ring using translational kinematics • The motions of the disk and the ring using rotational kinematics • The rotational inertias of the disk and the ring	**Point A2**
	For a justification that includes **one** of the following: • Reasoning that attempts Newton's second law in translational form • Reasoning that attempts Newton's second law in rotational form • Reasoning that attempts conservation of energy	**Point A3**
	Example Response *The ring has a greater rotational inertia because it has more mass distributed towards the edge. So the ring travels farther and has less acceleration down the ramp. Because the ring and the disk have the same mass, the gravitational forces exerted on the shapes are the same. From Newton's second law, the ring must have less net force down the ramp and more friction up the ramp.*	

已发生打滑：同质量同材料的动摩擦

打滑时使用f=μkN，与纯滚动时求实际静摩擦的动作不同。两物体质量、斜面角度和接触材料相同，动摩擦系数和实际支持力都相同，所以动摩擦大小相同；转动惯量的差异不出现在这条动摩擦定律里。

例题 2025 Q4 (C)

4. A uniform disk and ring, each of mass M and radius R, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v.

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

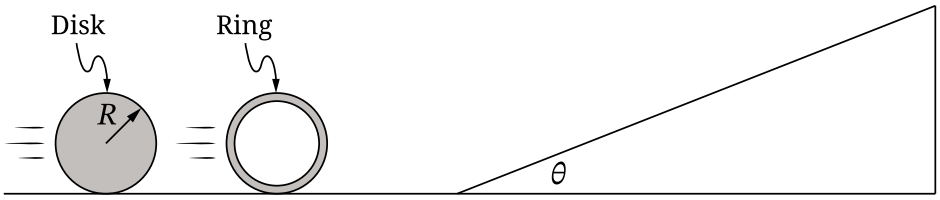


Figure 1

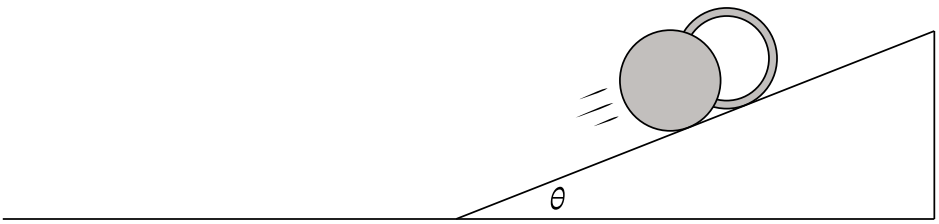


Figure 2

C. In a different scenario, the centers of mass of the original disk and ring each have the same initial speed v as they did in the original scenario. The ramp is replaced by a new ramp on which the disk and the ring initially slip as they roll up the new ramp.

Indicate whether the magnitude of the kinetic frictional force exerted on the disk by the new ramp is greater than, less than, or equal to the magnitude of the kinetic frictional force exerted on the ring by the new ramp while both are slipping.

Briefly **justify** your answer.

解：

选Equal to。滑动时使用动摩擦模型。两者边缘材料相同，动摩擦系数相同；质量、斜面角度也相同，支持力相同，因此动摩擦大小相同。纯滚动时实际静摩擦由运动条件决定，不能照搬这个结论。

$$f_k = \mu_k Mg\cos\theta.$$

官方评分

C	For indicating “Equal to”	**Point C1**
	For a justification that includes that kinetic friction is only dependent on the surfaces (or equivalently, the coefficient of kinetic friction between those surfaces) and normal force	**Point C2**
	Example Response *The forces of kinetic friction are equal. If slipping, the friction is kinetic, and the force of kinetic friction on the hoop and the disk are the same because the coefficient of kinetic friction is only dependent on the surfaces, and the normal forces on both are the same.*	

并联弹簧：同位移的回复力相加

几根并联弹簧都伸长同样的距离，各自提供回复力，合回复力相加。N根相同弹簧的等效常量是Nk；质量不变时，周期由质量与等效回复常量之比的平方根决定。重力只改变竖直平衡位置，不改变围绕该位置的回复常量。

$$k_{\text{eff}} = Nk, \qquad T = 2\pi\sqrt{\frac{m}{Nk}}.$$

例题 2023 Set 2 Q2 (a)

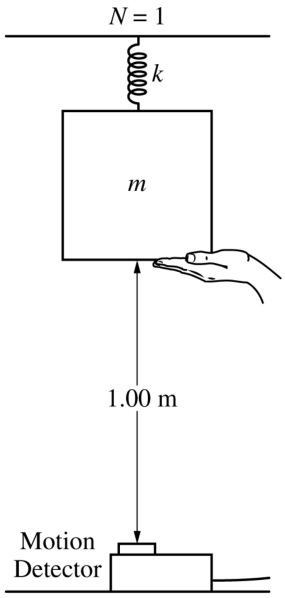


Figure 1

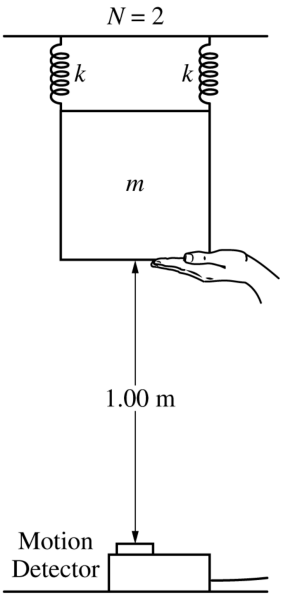


Figure 2

2. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height $1.00\ \text{m}$ above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

(a) Derive an expression for T as a function of N. Express your answer in terms of m, k, N, and physical constants as appropriate.

解：

并联弹簧都伸长同一距离，每根回复力相加，所以等效弹簧常量随根数相加。

$$k_{\text{eq}} = Nk, \qquad T = 2\pi\sqrt{\frac{m}{Nk}}.$$

官方评分

(a)	For indicating the equivalent spring constant k_{eq} is the sum of the spring constants of all the springs arranged in parallel	**1 point**
	Example Response $k_{eq} = Nk$	
	For a correct expression for period consistent with k_{eq} above	**1 point**
	Example Response $T = 2\pi\sqrt{\frac{m}{Nk}}$	
	Example Solution $k_{eq} = \sum_{i=1}^{N} k_i$ $k_{eq} = Nk$ $T = 2\pi\sqrt{\frac{m}{Nk}}$	
	Total for part (a)	**2 points**

周期随根数：按幂次判断曲线

周期正比于根数的负二分之一次方。弹簧越多，周期越小，但每多一根带来的减小量逐渐变小，所以图单调下降、向上凹。根数只能为正整数，可以画这些整数位置的点，不延伸到N=0。

例题 2023 Set 2 Q2 (b)

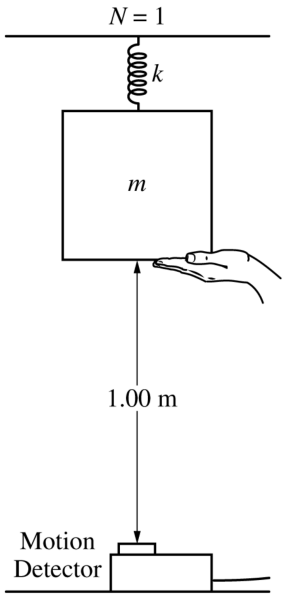


Figure 1

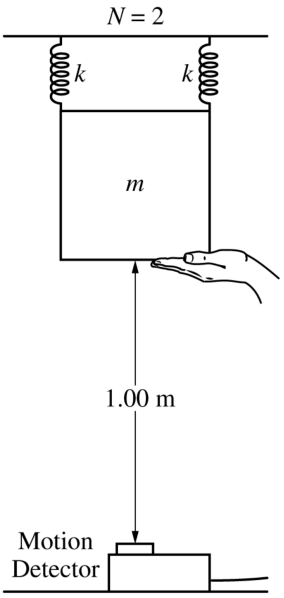


Figure 2

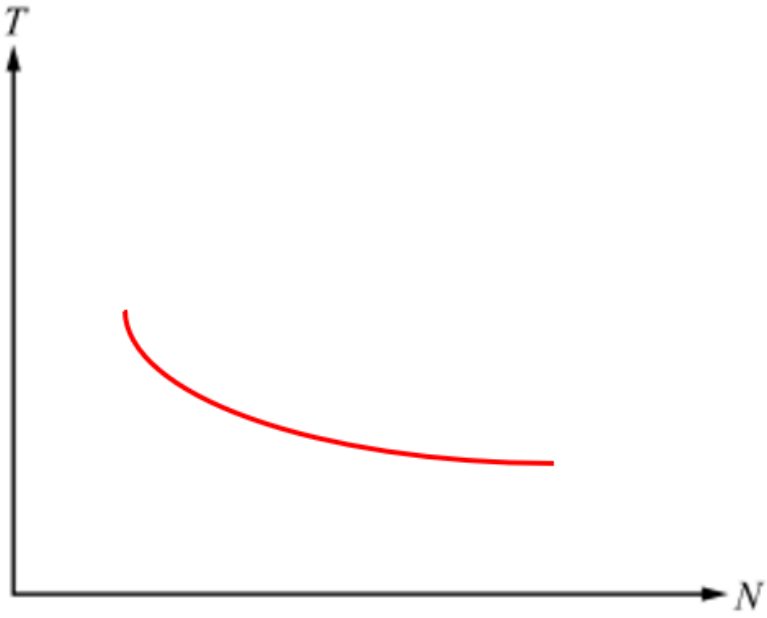
2. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height $1.00\ \text{m}$ above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

(b) On the following axes, sketch a graph of T as a function of N for $N \geq 1$.



解：

根数增加，周期按根数平方根的倒数减小；越往右，减小越慢，曲线向上凹。N是正整数，也可以画沿该曲线排列的离散点。

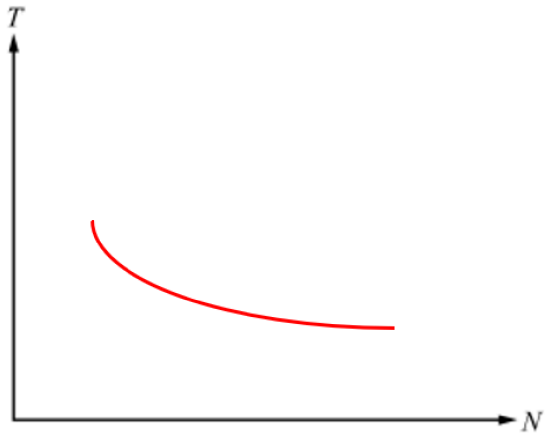


周期随根数减小，曲线向上凹。

官方评分

(b)	For a graph that increases or decreases with N consistent with the expression derived in part (a)	**1 point**
	For a concavity that is consistent with the expression derived in part (a)	**1 point**

Example Solution



Scoring Note: The student is not required to label $N = 1$ on the horizontal axis, so the curve can start at the vertical axis, implying that $N = 1$ is at the origin.

Scoring Note: Discrete points following the appropriate curve will earn this point.

Total for part (b) 2 points

扭转振动：惯量相加，回复常量不变

同轴盘一起转动，惯量按盘数相加；同一根金属丝的扭转常量不因盘数改变。题给周期关系是2π乘惯量与扭转常量之比的平方根，把N个盘的总惯量放入这条关系。扭转常量描述单位转角引起多少回复力矩。角位移用弧度计量，盘缘的弧长等于半径乘转角。

例题 2023 Set 1 Q2 (a)

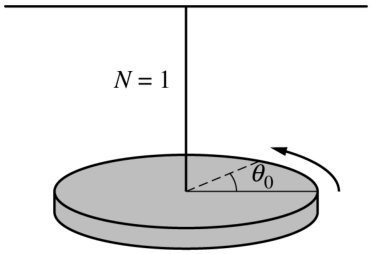


Figure 1

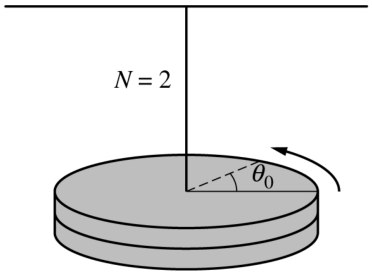


Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

   The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

   (a) Using $T = 2\pi\sqrt{\frac{I}{\kappa}}$, derive an expression for T as a function of N. Express your answer in terms of M, R, κ, N, and physical constants, as appropriate.

解：

多个盘同轴一起转动，惯量相加；扭转常量不随盘数变化。

$$I_{\text{sys}} = \frac{1}{2}NMR^2, \qquad T = 2\pi\sqrt{\frac{NMR^2}{2\kappa}}.$$

官方评分

(a)	For indicating the rotational inertia is the sum of the rotational inertia for all the stacked disks	**1 point**

Example Response

$$I_{\text{eq}} = \sum_{i=1}^{N} I_i$$

$$I_{\text{eq}} = N\left(\frac{1}{2}MR^2\right)$$

	For an expression for the period consistent with the previous rotational inertia expression	**1 point**

Example Response

$$T = 2\pi\sqrt{\frac{N\left(\frac{1}{2}MR^2\right)}{\kappa}}$$

Example Solution

$$I_{\text{eq}} = \sum_{i=1}^{N} I_i$$

$$I_{\text{eq}} = N\left(\frac{1}{2}MR^2\right)$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

$$T = 2\pi\sqrt{\frac{N\left(MR^2\right)}{2\kappa}}$$

Total for part (a) 2 points

相同初位移：储能相同，最大动能不变

初角位移和扭转常量相同，扭转储能相同。到平衡位置时，全部储能成为动能；增加盘数虽改变角速率与周期，不改变这份能量。因此最大动能是随N不变的正值，图不是零线。

例题 2023 Set 1 Q2 (b)

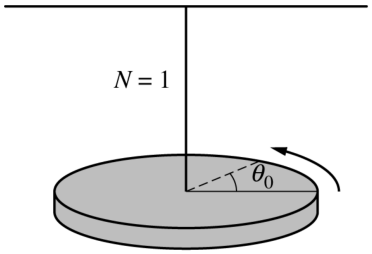


Figure 1

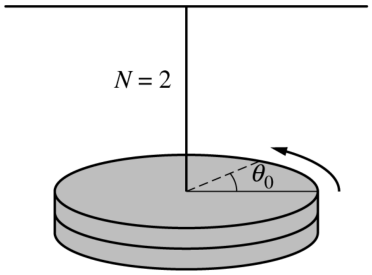
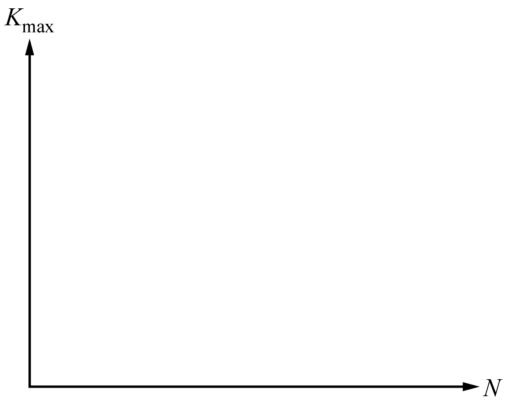


Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

   The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

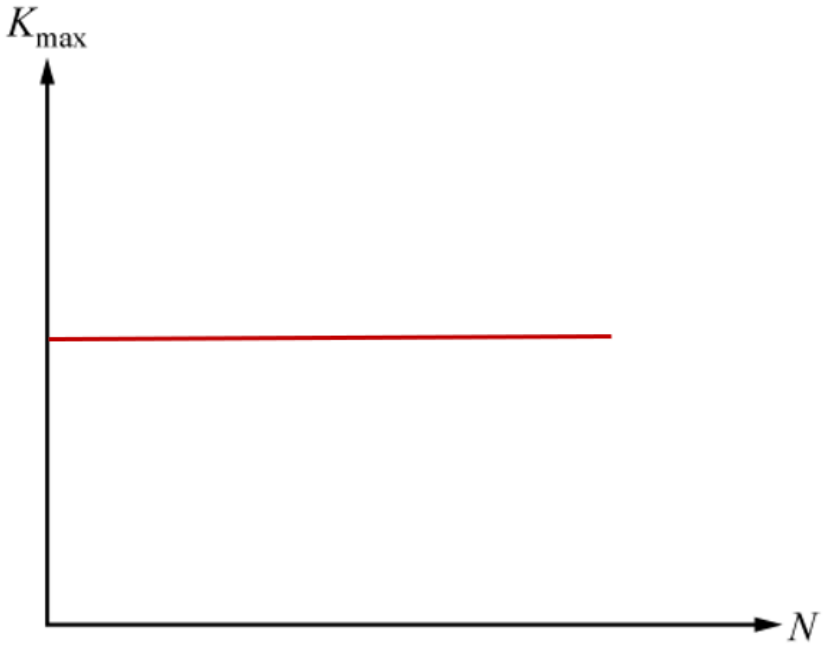
(b) The potential energy stored in the torsional pendulum when the disks are displaced is $U = \frac{1}{2}\kappa(\Delta\theta)^2$. On the following axes, sketch a graph of the maximum kinetic energy K_{max} of the torsional pendulum as a function of N for $N \geq 1$.



解:

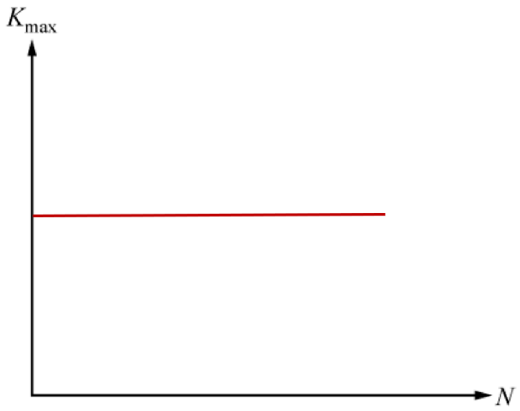
相同初角位移下，金属丝初始扭转储能相同，过平衡位置时全变为动能。因此最大动能为正且与盘数无关，画非零水平线。

$$K_{\text{max}} = \frac{1}{2}\kappa\theta_0^2.$$



增加盘数不增加相同初始扭角下的储能。

官方评分

(b)	For a sketch that begins at a non-zero value	**1 point**
	For a sketch that is constant with slope equal to zero	**1 point**
	Example Solution	
		
	Total for part (b)	**2 points**

质量向外分布：惯量增加，峰速下降

同样总质量和半径下，更多质量分布在边缘，每部分的距离平方更大，惯量因此较大。同一扭转常量的周期随惯量平方根增大；同样储能变成动能时，惯量较大需要的角速率较小。分别按题目问图线斜率还是最大角速率给出结论。

例题 2023 Set 1 Q2 (d)(i)

本问承接 2023 Set 1 Q2 (a)。

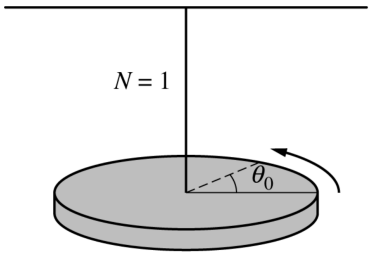


Figure 1

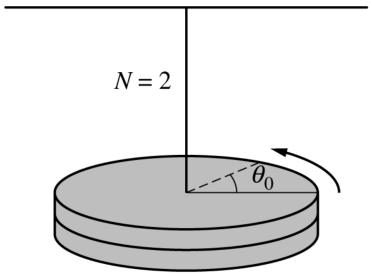


Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(d) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$.

i. Would the slope of the best-fit line for this new data be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than ___less than ___the same as

Justify your answer.

解：

同半径、同质量的比较中，密度朝边缘增大使更多质量远离转轴，惯量比均匀盘更大。周期对盘数平方根的斜率正比于单盘惯量的平方根，因此斜率更大。

官方评分

(d)(i)	For indicating that the slope would be greater with an attempt at a relevant justification	**1 point**
	For indicating that using disks with densities that increase with r will increase the rotational inertia	**1 point**
	For indicating the functional relationship between slope and rotational inertia: $\sqrt{I} \propto slope$	**1 point**

Example Solution

The disks with a density that increases towards the edge of the disk will have a greater proportion of their mass farther from the axis of rotation, so their rotational inertia will be larger than that of a uniform disk. Therefore, the slope of the line will be greater because the slope is proportional to $\sqrt{I}$.

练习 2023 Set 1 Q2 (d)(ii)

本问承接 2023 Set 1 Q2 (b)。

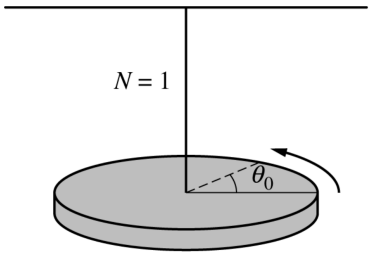


Figure 1

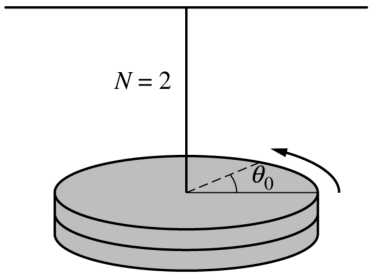


Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(d) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$.

ii. When $N = 1$, the maximum angular speed of the torsional pendulum with a uniform disk is found to be ω_{U}. When $N = 1$, the maximum angular speed of the torsional pendulum with a nonuniform disk is $\omega_{\text{non-U}}$. Which of the following correctly compares ω_{U} and $\omega_{\text{non-U}}$?

____$\omega_{\text{U}} > \omega_{\text{non-U}}$ ____$\omega_{\text{U}} < \omega_{\text{non-U}}$ ____ $\omega_{\text{U}} = \omega_{\text{non-U}}$

Briefly justify your answer.

解：

同一扭转常量、同一初始角位移储存相同能量。非均匀盘惯量更大，获得这份动能需要的角速率较小。

$$\frac{1}{2}I\omega_{\text{max}}^2 = \frac{1}{2}\kappa\theta_0^2, \qquad \omega_U > \omega_{\text{non-U}}.$$

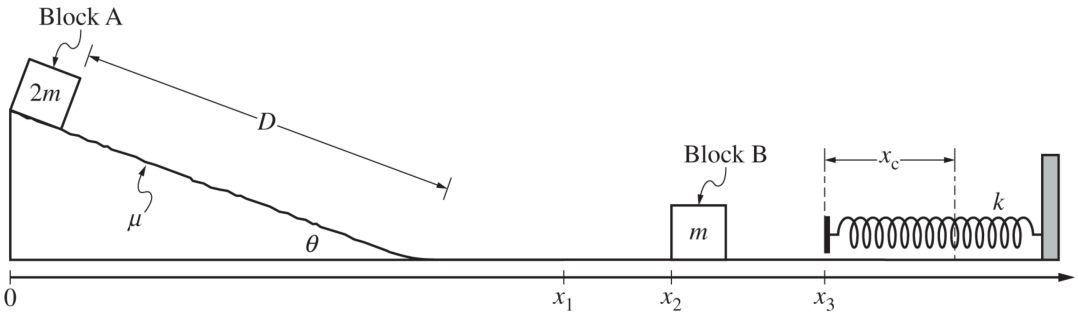
官方评分

(d)(ii)	For selecting $\omega_U > \omega_{non\text{-}U}$ with an attempt at a relevant justification, or a selection consistent with part (d)(i)	**1 point**
	For using energy conservation to justify the relationship	**1 point**
	Example Solution *Energy in a torsion pendulum is conserved,* $\frac{1}{2}I\omega^2 = \frac{1}{2}\kappa\theta^2$*, where* $\frac{1}{2}\kappa\theta^2$ *is a constant.* *Therefore, since* $I_U < I_{non\text{-}U}$, $\omega_U > \omega_{non\text{-}U}$ *to keep energy conserved.*	
	Total for part (d)	**5 points**
	Total for question 2	**15 points**

改变振幅或摆放方向：周期看惯性与回复关系

理想线性弹簧的周期只取决于振动质量和回复常量，改变振幅不改变周期。把竖直弹簧改成水平，消除了重力造成的平衡位移，但质量和围绕平衡位置的回复常量没有改变。因此与周期关系相连的图线斜率也不变。

例题 2024 Set 2 Q1 (c)



Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

For times $t > t_4$, the two-block-spring system oscillates with period T_O. The procedure is then repeated using a new ramp, where there is negligible friction between Block A and the ramp.

(c) **Indicate** how the new period of oscillation T_N in the procedure that uses the new ramp compares with the period of oscillation T_O from the original procedure.

_____ $T_N > T_O$ _____ $T_N < T_O$ _____ $T_N = T_O$

Briefly **justify** your answer.

解：

选周期相同。移除斜面摩擦会增加进入弹簧时的速度和振幅；振动质量仍是3m，弹簧常量仍为同一个k。理想线性弹簧的周期不随振幅变。

$$T = 2\pi\sqrt{\frac{3m}{k}}, \qquad T_N = T_O.$$

官方评分

(c)	For selecting $T_N = T_O$ with an attempt at a relevant justification	**1 point**
	For a correct justification that includes **one** of the following: • The period of a spring-block oscillator is only dependent on the mass on the spring and spring constant, which do not change. • The period of a spring-block oscillator is not dependent on increasing amplitude, velocity, or compression distance.	**1 point**
	Example Response *Repeating the experiment on a smooth ramp will only affect the compression distance of the spring. The period of oscillation of a spring-block system depends only on mass and the spring constant, therefore the period of oscillation will not change.*	
	Total for part (c)	**2 points**
	Total for question 1	**15 points**

练习 2023 Set 2 Q2 (d)(i)

本问承接 2023 Set 2 Q2 (a)。

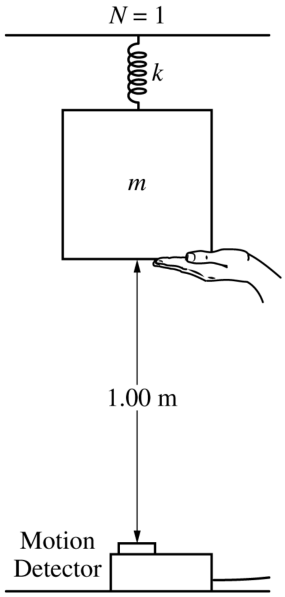


Figure 1

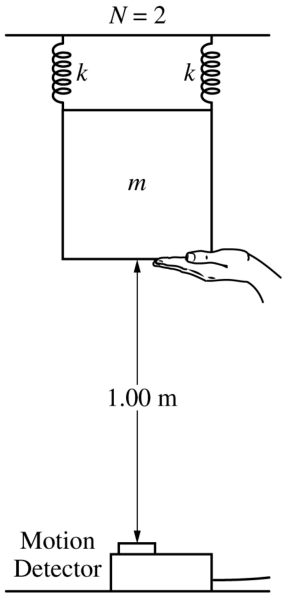


Figure 2

2. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height $1.00\ \text{m}$ above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

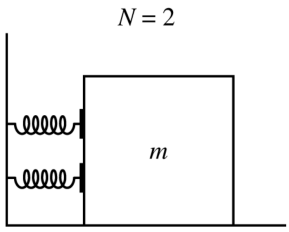


Figure 3

(d) The student conducts a similar investigation to determine the relationship between the period of oscillation T of a system consisting of a block and a number N of identical springs but arranges the block-spring system horizontally on a table, as shown in Figure 3. Frictional forces between the table and the block are negligible. In each trial, the block is displaced the same horizontal distance from equilibrium and released from rest.

i. The student plots T^2 as a function of N^{-1} for this new data. Would the slope of the best-fit line from this new investigation be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than ___ less than ___the same as

Briefly justify your answer.

解：

选the same。改放水平只消除平衡位置的重力位移，不改变围绕平衡位置的回复常量和惯性质量，所以周期和此图斜率都不变。

官方评分

(d)(i)	For indicating that the slope would not change with an attempt at a relevant justification	**1 point**
	For a justification that the period depends on the mass and spring constant which remain unchanged **OR** For a justification that indicates that the period is independent of gravitational force	**1 point**

Example Solution

The period of oscillation of the spring-block system depends on the mass and the effective spring constant, $T = 2\pi\sqrt{\frac{m}{Nk}}$ *. The slope is equal to the square of the period over the inverse number of identical springs,* $\text{Slope} = \frac{T^2}{N^{-1}}$*. This means that the slope is proportional to the mass divided by the spring constant. Since neither the mass nor the spring constant change, the slope will remain unchanged.*

OR

The period of oscillation of the spring-block system depends on the mass and the effective spring constant, $T = 2\pi\sqrt{\frac{m}{Nk}}$ *. The slope is equal to the square of the period over the inverse number of identical springs,* $\text{Slope} = \frac{T^2}{N^{-1}}$*. Since the period is independent of the gravitational force, and the only change was arranging the block-spring system horizontally instead of vertically, the period is not affected by the change in the orientation of the system. Therefore, the slope will remain unchanged.*

并联弹簧增加：相同振幅储能增加

每次横向拉开同样距离，N根并联弹簧总共储存$NkA^2/2$能量。过平衡点时势能最低、速度最大，全部初始储能变成同一块的动能。质量不变，储能随N增加使最大速度随根数平方根增加。

例题 2023 Set 2 Q2 (d)(ii)

本问承接 2023 Set 2 Q2 (a)。

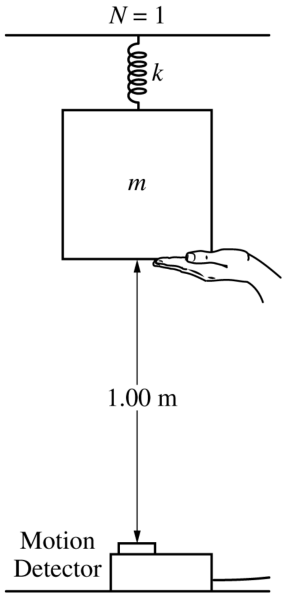


Figure 1

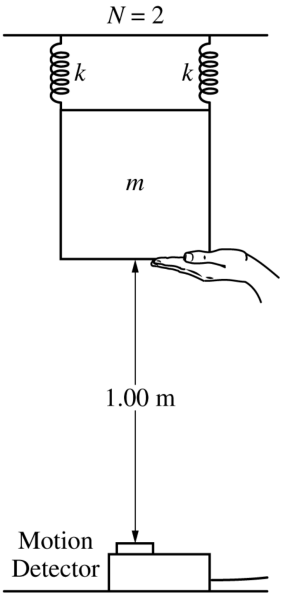


Figure 2

2. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height $1.00\ \text{m}$ above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

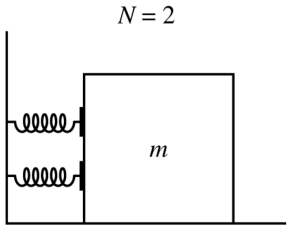


Figure 3

(d) The student conducts a similar investigation to determine the relationship between the period of oscillation T of a system consisting of a block and a number N of identical springs but arranges the block-spring system horizontally on a table, as shown in Figure 3. Frictional forces between the table and the block are negligible. In each trial, the block is displaced the same horizontal distance from equilibrium and released from rest.

ii. When $N = 1$, the maximum speed of the block is found to be v_{max}. When N increases, will v_{max} increase, decrease, or stay the same?

____ increase ____ decrease ____ stay the same

Justify your answer.

解：

峰速增加。每次水平位移振幅相同，更多并联弹簧储存更多初始弹性势能；过平衡点时都变为同一块的动能。

$$\frac{1}{2}NkA^2 = \frac{1}{2}mv_{\max}^2, \qquad v_{\max} = A\sqrt{\frac{Nk}{m}}.$$

官方评分

(d)(ii) For a relationship between v_{max} and N that is consistent with the expression from part (a) with an attempt at relevant justification — **1 point**

For using energy conservation to justify the relationship — **1 point**

OR

For using the relationship between v_{max} and ω to justify the relationship

For indicating that the effective spring constant changes the elastic potential energy of the spring block system U_s therefore changing v_{max} in a manner consistent with the expression from part (a) — **1 point**

OR

For an inverse relationship between ω and period, indicating a change in v_{max} consistent with the expression from part (a)

Example Solution

An increase in the number of identical springs attached in parallel to each other causes the effective spring constant to increase, $k_{eff} = \sum_{i=1}^{N} k_i$. Since the effective spring constant is proportional to potential energy of the block-spring system, $U_s = \frac{1}{2}k_{eff}x^2$, an increase in the effective spring constant causes the potential energy of the block-spring system to increase for a given displacement. Therefore, based on conservation of energy, $\frac{1}{2}k_{eff}x^2 = \frac{1}{2}mv_{max}^2$, an increase in the potential energy will cause an increase in the kinetic energy, resulting in a greater maximum velocity. Therefore, the maximum velocity will increase with an increase in the number of springs added.

OR

An increase in the number of identical springs attached in parallel to each other causes the effective spring constant to increase, $k_{eff} = \sum_{i=1}^{N} k_i$. Since the effective spring constant is inversely proportional to the period of oscillation of the block-spring system, $T = 2\pi\sqrt{\frac{m}{k_{eff}}}$, an increase in the effective spring constant causes the period to decrease. Since the period is inversely proportional to the angular frequency of the block-spring system, $\omega = \frac{2\pi}{T}$, the angular frequency increases. Since the maximum velocity is proportional to the angular frequency, $v_{max} = x_0\omega$, this results in an increase in the maximum velocity. Therefore, the maximum velocity will increase with an increase in the number of springs added.

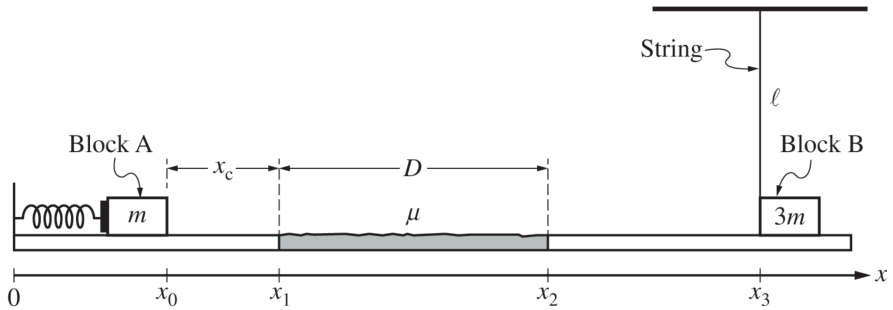
Total for part (d) 5 points

Total for question 2 15 points

小角摆：绳长改变周期与频率

小角摆的回复与重力和绳长有关，理想周期为2π√(ℓ/g)，频率是每秒往返多少次，等于周期的倒数。绳长加倍，周期增为√2倍，频率减为原来的1/√2；质量和小振幅不进入这个关系。

例题 2024 Set 1 Q1 (c)



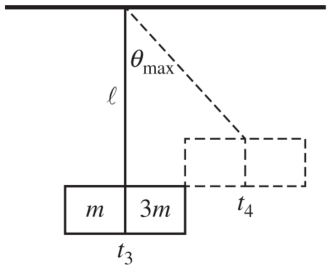
Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.



Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ. The support holding the string is raised, and the procedure is then repeated using a new string of length $2\,\ell$.

(c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length $2\,\ell$ will compare to the frequency of oscillation f_ℓ from the original procedure.

_____ $f_{2\ell} > f_\ell$ _____ $f_{2\ell} < f_\ell$ _____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

解：

小角摆的频率由绳长和重力加速度决定。绳长加倍，周期增为根号二倍，所以频率减小；球质量和初速不改变这个理想小角周期。

$$T = 2\pi\sqrt{\frac{\ell}{g}}, \qquad f_{2\ell} = \frac{f_\ell}{\sqrt{2}} < f_\ell.$$

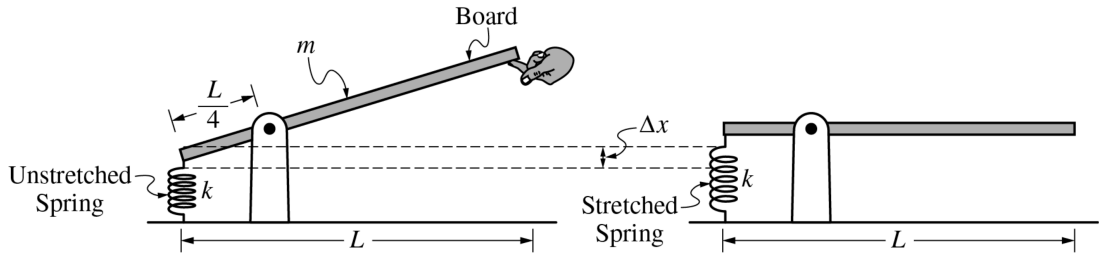
官方评分

(c)	For selecting $f_{2\ell} < f_{\ell}$ with an attempt at a relevant justification	**1 point**
	For correctly applying an equation that relates the length of a pendulum to the period or frequency of the pendulum	**1 point**
	Example Response *The period of a pendulum is calculated by using* $T = 2\pi\sqrt{\frac{l}{g}}$ *. Therefore, as the length is increased, the period will also increase. Because frequency and period are inversely related, an increase in period will result in a decrease in frequency.*	
	Total for part (c)	**2 points**
	Total for question 1	**15 points**

小角回复：加速度图从最大值降至零

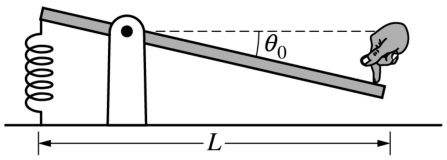
板在平衡位置附近的小角位移给回复力矩，角加速度大小与偏离平衡位置的角度大小成正比。从最大位移静止释放，到第一次过平衡位置，是余弦曲线的前四分之一段：开始最大、切线水平，随后向下凹地减到零。

例题 2022 Set 2 Q3 (c)(i)



Note: Figures not drawn to scale.

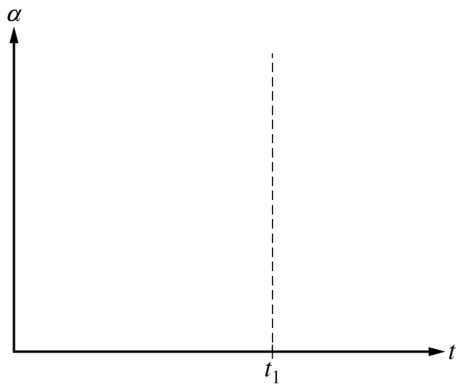
3. A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I.



Note: Figure not drawn to scale.

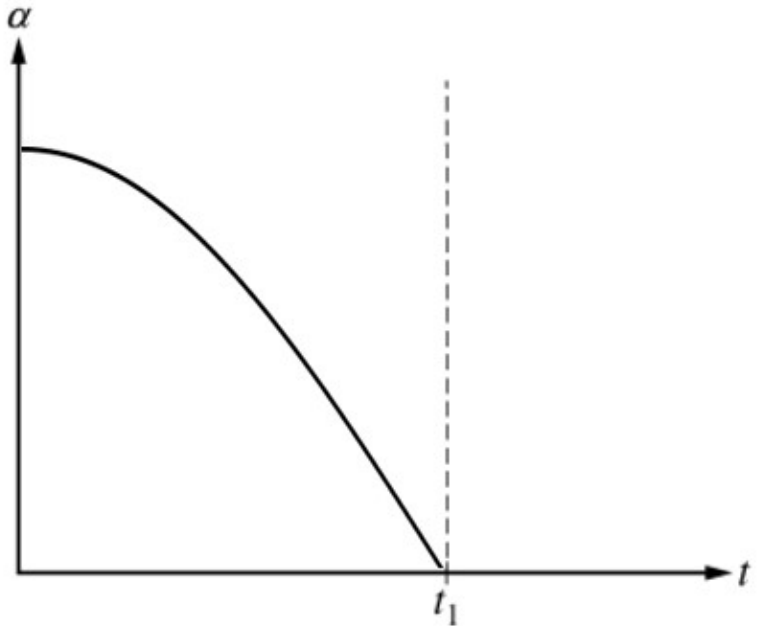
(c) A student pushes the board down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.

i. At time $t = 0$, the board is released. At $t = t_1$, the board first crosses the horizontal. Sketch a graph of the magnitude of the angular acceleration α of the board as a function of time t from $t = 0$ to $t = t_1$.



解：

被压低后从静止释放，回复力矩大小初始最大；回到水平平衡位置时降到零。小角振动的位移从最大按余弦减小，加速度大小与位移大小正比，故曲线单调下降、向下凹，初始切线水平。

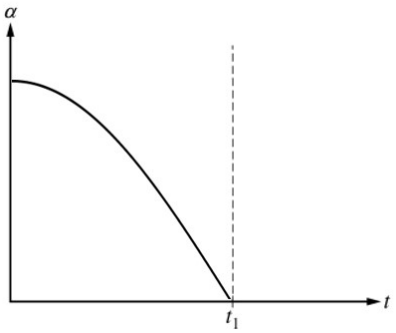


角加速度大小：从最大值向下凹地减到零。

官方评分

(c)(i)	For a sketch that begins at a maximum value and monotonically decreases to a minimum of zero at $t = t_1$	**1 point**
	For a sketch that is concave down between $t = 0$ and $t = t_1$	**1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

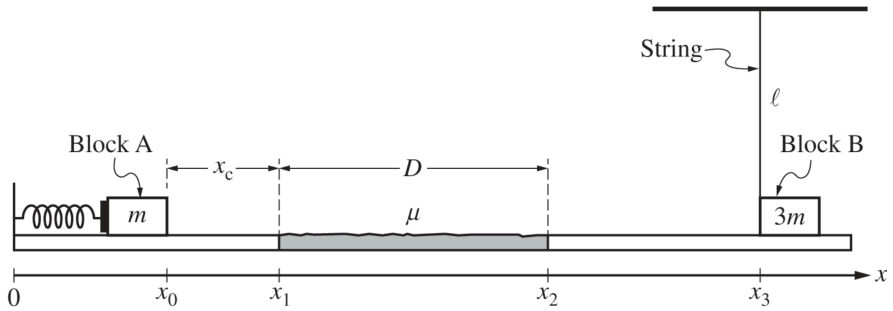
动能图分段：弹力做正功但功率也看速度

弹簧释放阶段，动能增加；每秒增加多少取决于力乘速度，起初速度零、最后弹力零，两端斜率都为零，中间斜率为正。粗糙段恒定摩擦使速度线性下降，动能是速度平方，向上凹地减小；光滑段动能为正常数。

$$\frac{dK}{dt} = Fv.$$

例题 2024 Set 1 Q1 (b)(i)

本问承接 2024 Set 1 Q1 (a)(i)、2024 Set 1 Q1 (a)(ii)。



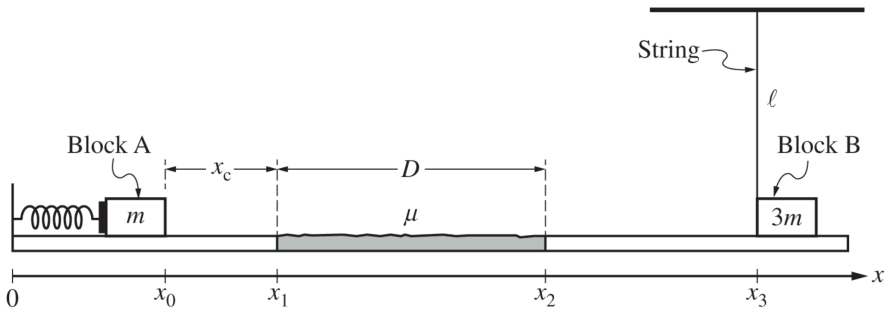
Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

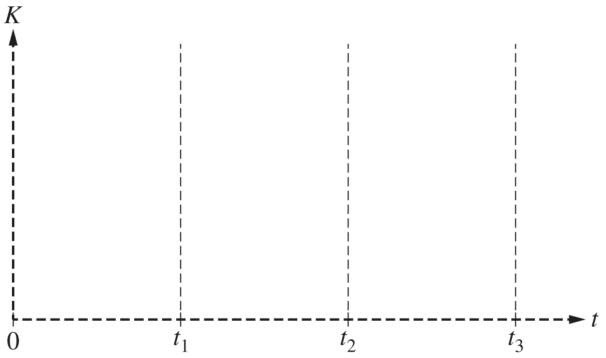


Note: Figure not drawn to scale.

Figure 1

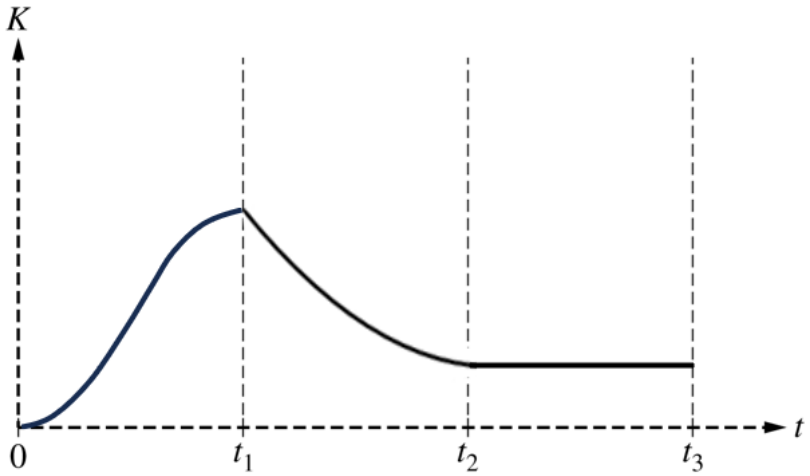
(b)

i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3.



解：

0到第一时刻：弹簧释放让动能非线性增加，起末斜率均为零，中间先向上凹再向下凹。粗糙段摩擦恒定、速度线性下降，动能是速度的平方，因此向上凹地减小。之后光滑段匀速，动能保持正常数；此图到碰撞前结束。

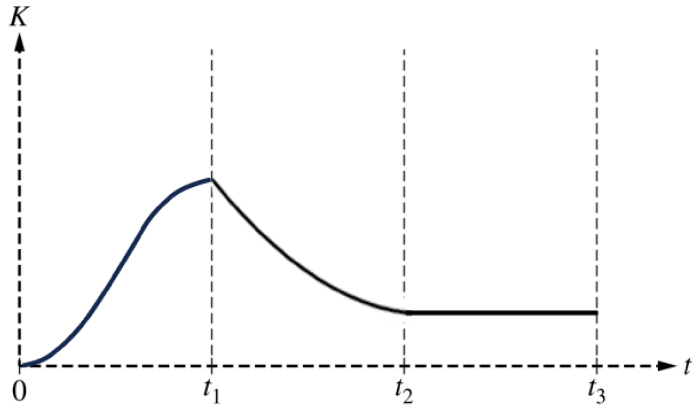


弹簧释放、粗糙减速、光滑匀速对应三种图形。

官方评分

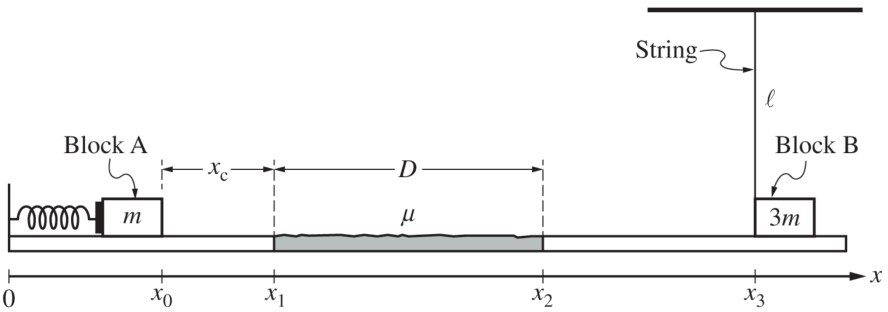
(b)(i)	For a nonlinear sketch that begins at zero and increases for the entire time interval $0 \le t \le t_1$	**1 point**
	For a sketch that decreases for the entire time interval $t_1 \le t \le t_2$ but does not go to zero	**1 point**
	For a sketch that is concave up for the time interval $t_1 \le t \le t_2$	**1 point**
	For a continuous function for the time interval $t_1 \le t \le t_3$ that has a horizontal line that is greater than zero for the time interval $t_2 \le t \le t_3$	**1 point**

Example Response



练习 2024 Set 1 Q1 (b)(ii)

本问承接 2024 Set 1 Q1 (b)(i)。



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

解：

弹力一直做正功，所以动能增加；但每秒增加多少还同时取决于速度，不能只凭弹力减小判整个图向下凹。

$$\frac{dK}{dt} = F_s v.$$

开始速度为零，动能图斜率零；中途力与速度均非零，斜率正；弹簧到原长时弹力为零，斜率又为零。

官方评分

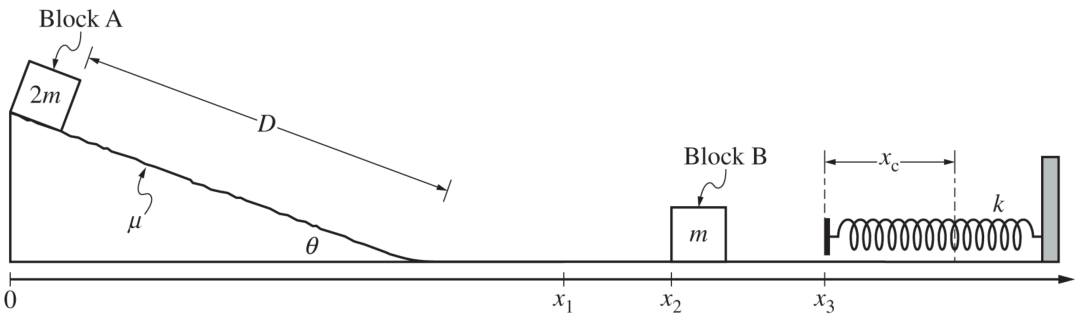
(b)(ii)	For a statement about the change in kinetic energy that is consistent with the graph drawn in the response for part (b)(i)	**1 point**
	For a correct explanation for why the kinetic energy is increasing, such as **one** of the following: • An increasing graph means positive work is being done on the block. • An external force is exerted on Block A, causing the velocity of the block to increase and the kinetic energy of the block to increase. • Mechanical energy is conserved and/or there is no work done for the block-spring system, and the potential energy decreases.	**1 point**
	For a correct explanation for why the graph is nonlinear, such as **one** of the following: • The rate at which the slope of the graph changes is related to the rate at which work is being done on the block. • The external force exerted on Block A is changing, which causes a nonuniform change in the velocity of Block A, which results in a nonuniform change in kinetic energy.	**1 point**
	Example Response *From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate, the kinetic energy of the block does not increase at a constant rate.*	
	Total for part (b)	**7 points**

单块动量图：粘连跳变与弹簧段凹向

先分清图画的是A单独的动量还是两块总动量。下坡恒力给直线，光滑段给水平线，粘连使A自己的动量跳变；进入弹簧后，压缩量增加使反向弹力增强，动量的负斜率绝对值不断增大，曲线向下凹。

例题 2024 Set 2 Q1 (b)(i)

本问承接 2024 Set 2 Q1 (a)(i)、2024 Set 2 Q1 (a)(ii)。

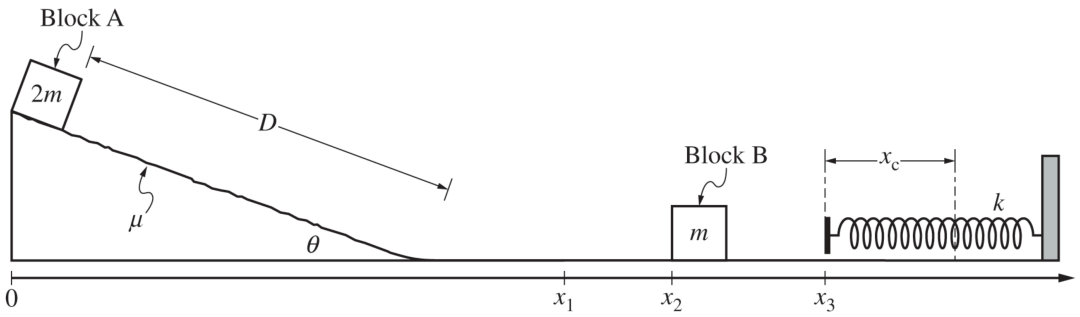


Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

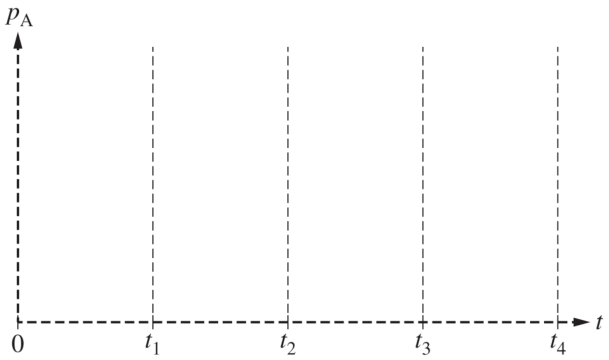
- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.



Note: Figure not drawn to scale.

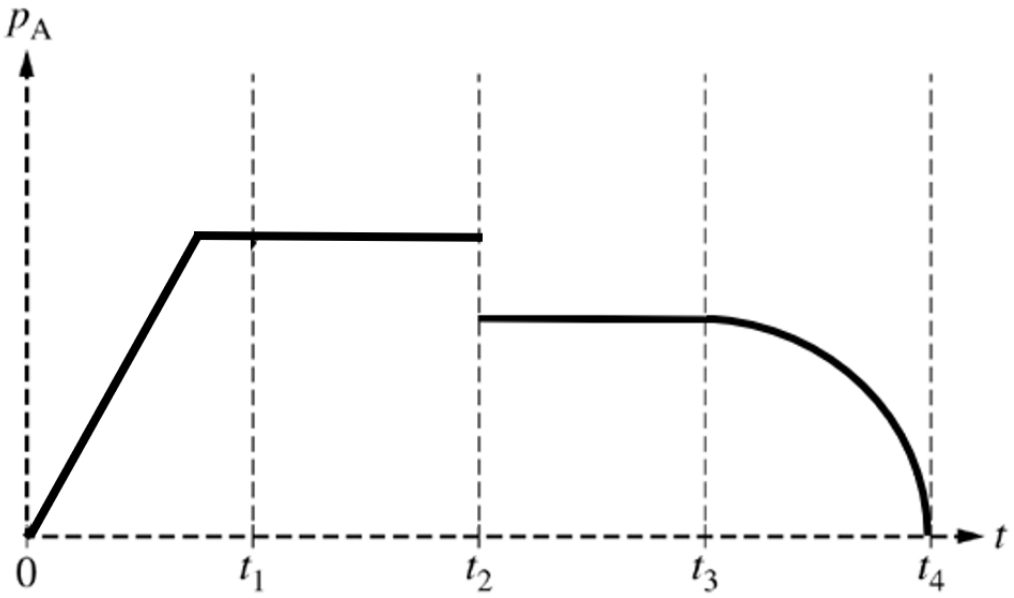
(b)

i. On the following axes, **sketch** a graph of the magnitude of the momentum p_A of Block A as a function of time t from $t = 0$ to t_4.



解:

图画的是A单独的动量，不是两块总动量。下坡恒加速，所以线性增加至P；第一光滑段恒为P；粘连时A速度变成原来的三分之二，故动量跳降到2P/3；下一光滑段保持该值；压弹簧时向下凹地降至零。

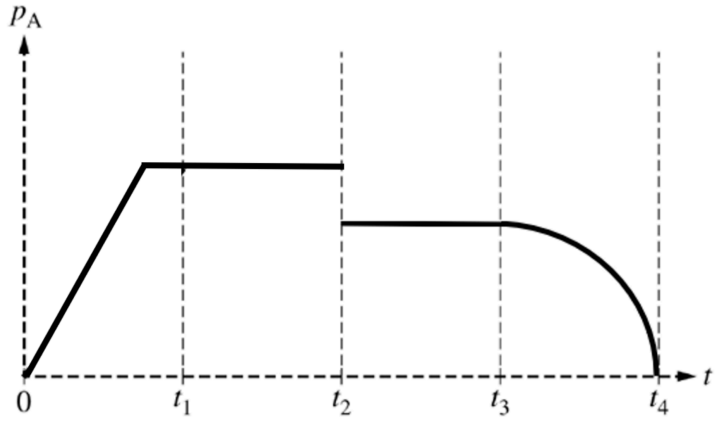


A单独的动量在碰撞中减少；系统总动量仍守恒。

官方评分

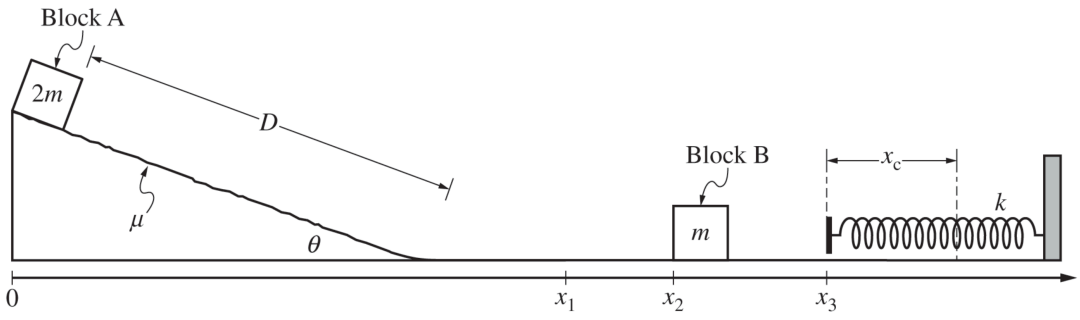
(b)(i)	For a sketch that increases linearly during the time interval $0 \le t < t_1$ **Scoring Note:** A sketch that only increases linearly from $t = 0$ to $t = t_1$ earns this point.	**1 point**
	For a horizontal line for the time interval $t_1 \le t \le t_2$ that is continuous at t_1	**1 point**
	For a horizontal line for the time interval $t_2 \le t \le t_3$ that has a smaller magnitude than the previous time interval	**1 point**
	For drawing a concave down curve, continuous at t_3, in the interval $t_3 \le t < t_4$ that reaches zero at t_4	**1 point**

Example Response



练习 2024 Set 2 Q1 (b)(ii)

本问承接 2024 Set 2 Q1 (b)(i)。



Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

ii. Use principles of forces to **justify** the graph drawn in part (b)(i) for the time interval $t = t_3$ to $t = t_4$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

解:

弹簧段两块质量共为3m，压得越多，反向弹力越大，速度下降得越快。A动量的变化率由它的质量乘共同加速度给出。

$$3ma = -kx, \qquad \frac{dp_A}{dt} = 2ma = -\frac{2}{3}kx.$$

刚接触原长弹簧时斜率为零，后来负斜率绝对值增大，因此曲线向下凹。

官方评分

(b)(ii)	For a statement about the change in momentum that is consistent with the graph drawn in the response for part (b)(i)	**1 point**
	For indicating that a decreasing graph means that the force exerted on Block A is in a direction opposite to the motion of Block A **Scoring Note:** A response that indicates that an increasing graph means that the force exerted on Block A is in the same direction as the motion of Block A also earns this point.	**1 point**
	For relating the change in momentum to the magnitude of the force exerted on Block A	**1 point**
	Example Response *The momentum of Block A decreases between t_3 and t_4 because the spring exerts a force on the blocks in the opposite direction of the velocity of the blocks, causing the blocks to slow to a stop. The spring force increases the more the spring compresses, so the momentum of Block A decreases at an increasing rate, which is shown in the slope of the curve becoming steeper with time.*	
	Total for part (b)	**7 points**

有摩擦的振动：动能非负，峰值逐次降低

动能是速度平方的正倍数，每次转向速度为零，所以图每半个位置周期回到零。摩擦持续耗能，各次峰值降低；图不会穿过零线成为负值。题目给定完成多次振动且周期不变，按这个模型确定相邻零点和峰的间隔。

例题 2025 Q2 (C)

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

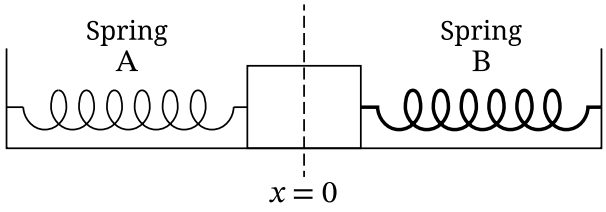


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

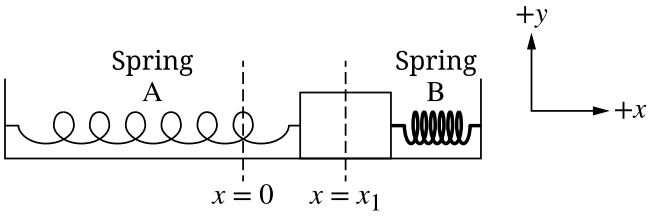
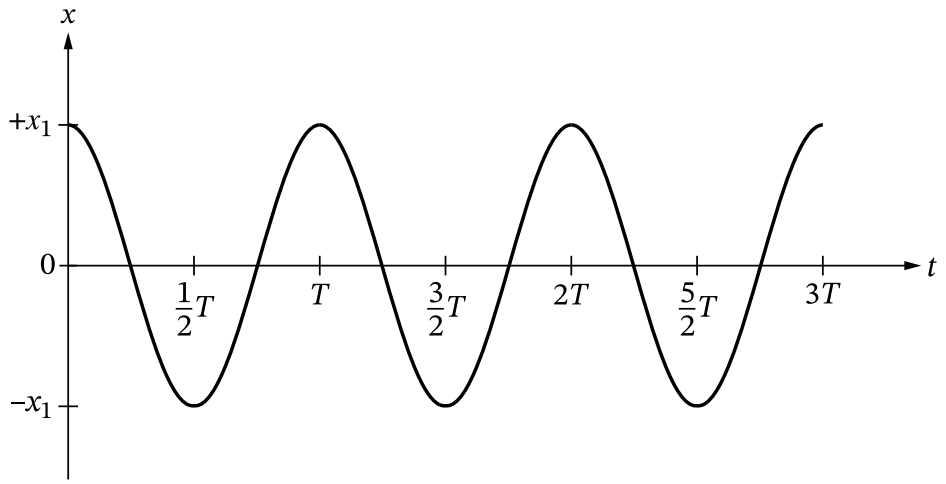


Figure 2

C. In Scenario 1, the block oscillates with period T. The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.

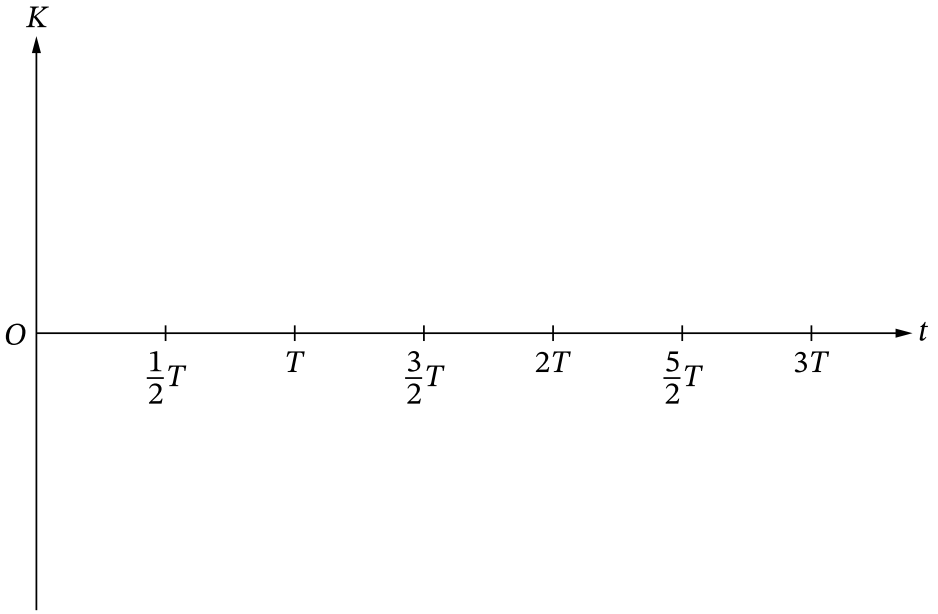


Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.

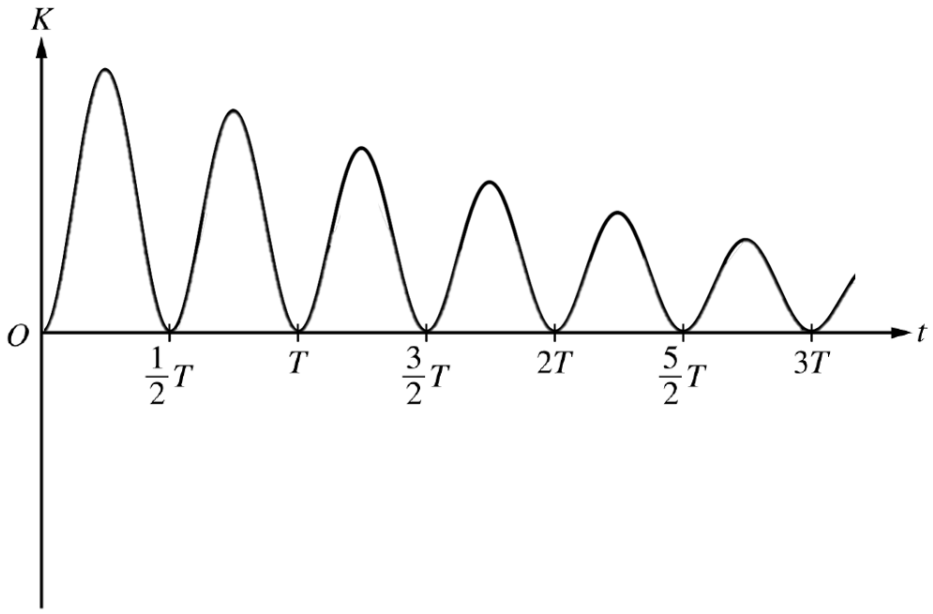


Scenario 2

Figure 5

解：

动能不会为负；每到一侧转向点速度为零，故每半个位置周期就出现一个零点。摩擦不断耗散机械能，各次动能峰值依次下降。



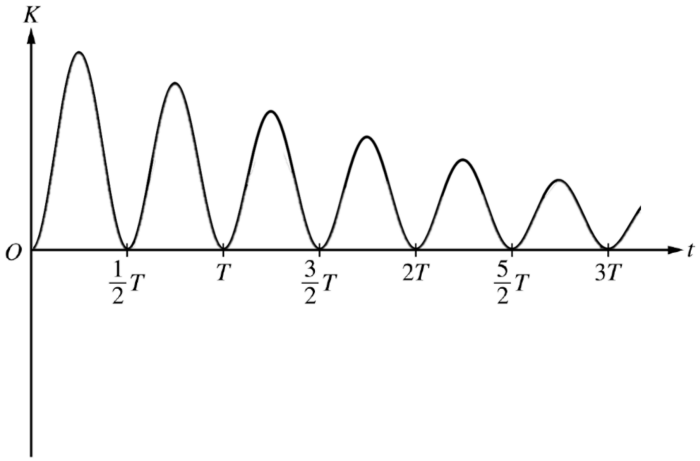
Scenario 2

Figure 5

零点相隔半个位置周期；纵轴始终非负，峰值递减。

官方评分

C	For sketching a curve that starts at zero and is always positive or zero	**Point C1**
	For sketching a periodic curve with zeros that have a period of $\frac{1}{2}T$	**Point C2**
	For sketching a periodic curve with a decreasing amplitude	**Point C3**
	Example Response	



Scenario 2

Figure 5

质量增大：振动时间尺度变长

同一组弹簧的回复常量不变，质量增大使周期按质量平方根增大。即使有摩擦，题设仍采用多次振动的模型；比较动能图的一个可辨认变化，可以选相邻峰或零点的时间间隔变大，并说明惯性增大的原因。

例题 2025 Q2 (D)

本问承接 2025 Q2 (C)。

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

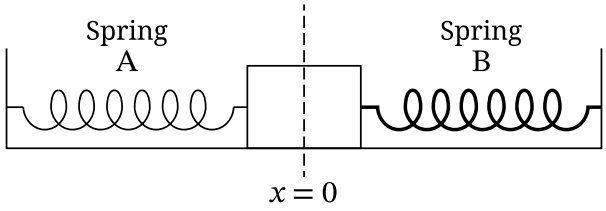


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

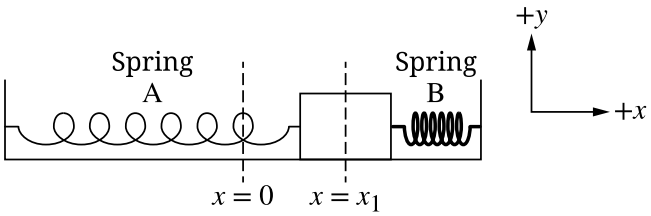
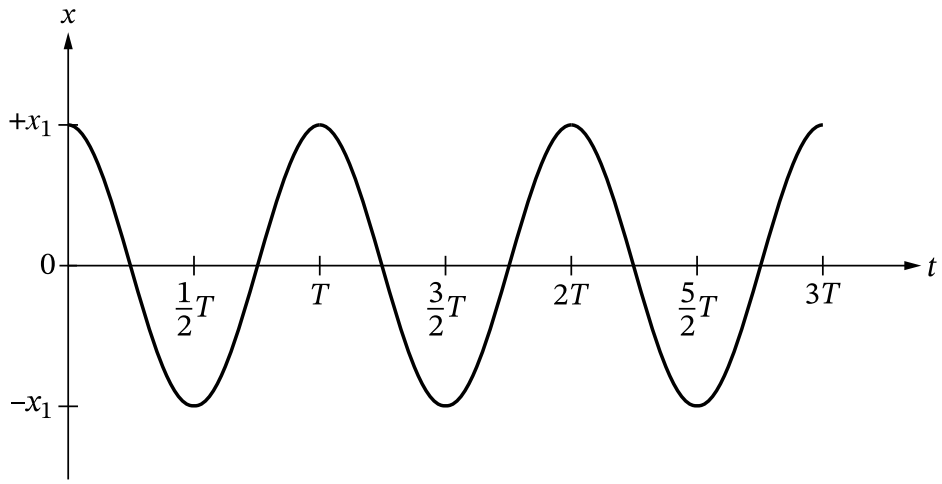


Figure 2

C. In Scenario 1, the block oscillates with period T. The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.



Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

D. In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

解：

选择说明周期增大。两弹簧的常量没有变，而新块质量较大；惯性增大使每次往返更慢，所以动能图相邻峰和零点间隔增大。

$$T = 2\pi\sqrt{\frac{m}{3k}}.$$

这是题设采用的多次振动模型，不把动摩擦的耗能曲线误写成持续等幅正弦。

官方评分

D	For indicating **one** of the following: • The period increases. • The rate at which the graph approaches zero increases. • The maximum value of K over each period is less than the graph drawn.	**Point D1**
	For a correct justification relevant to the feature indicated	**Point D2**
	Example Responses *The period of the graph increases. The period of a simple harmonic oscillator increases with increasing mass.* OR *The rate at which the graph approaches zero amplitude increases. Because the mass of the block is larger, the force of friction is greater, which causes the block to lose energy at a greater rate.*	

实验：测量、线性图与参数反求

只测高度和速率：设计重复测量

要用直线求重力加速度，先确定释放位置相对最低点的质心高度差，以及被推出后的水平速率。对多个高度收集一组组数据；同一高度重复，降低随机散差。每次从静止释放，保持同一装置，不把箱边的任意高度当作质心高度差。

例题 2025 Q3 (A)

3. A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance h, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

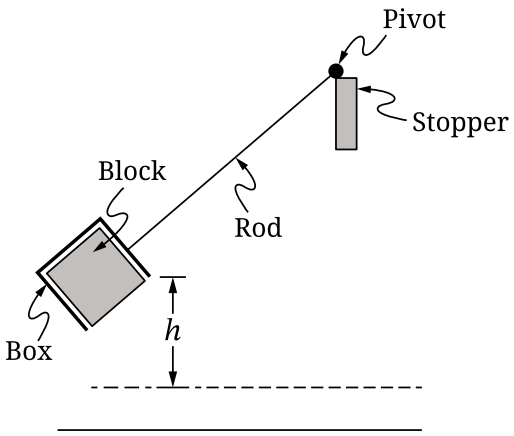


Figure 1

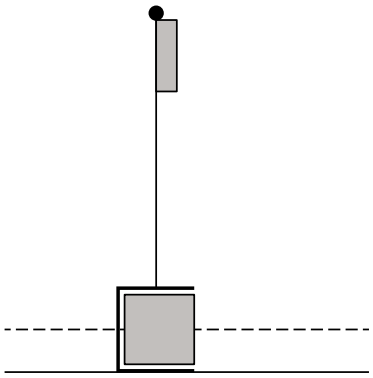


Figure 2

A. Students are asked to experimentally determine the acceleration due to gravity g using a linear graph. To determine g, the students are permitted to use measurements from only a meterstick and the motion sensor.

Describe an experimental procedure using the described setup to collect data that would allow the students to determine an experimental value of g using a linear graph. Include any steps necessary to reduce experimental uncertainty.

解：

用米尺测物块质心相对最低位置的释放高度h，静止释放，用运动探测器测物块被水平推出后的速率v。对同一高度重复测量并取平均，再用多个不同高度重复实验。应测质心高度差，而不是随意取箱边的高度。

官方评分

A	For describing a procedure that includes measuring h and the speed of the block as the block moves across the surface	**Point A1**
	For a procedure that indicates a reasonable method of reducing experimental uncertainty Examples of acceptable responses may include the following: • Repeating the experiment multiple times for the same value of h • Repeating the experiment for multiple values of h	**Point A2**
	Example Response *Measure the height h at which the block-box system is released. Measure the speed of the block as the block slides across the horizontal surface. Repeat the measurement of the speed of the block for varying release heights.*	

只用米尺：压缩量与粗糙段距离要分开

米尺能测长度，所以选择弹簧压缩量与物块在粗糙区的停止距离。先确定x=0是弹簧原长的位置；压缩弹簧时的光滑路程不属于摩擦耗能路段。测量要在同一零点、同一方向进行。

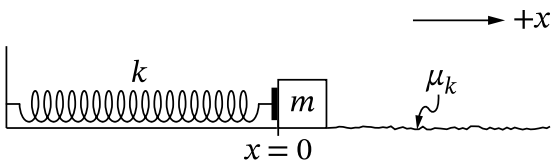
例题 2026 Q3 (A)(i)

The following information applies to parts A and B.

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3. A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

Figure 1



A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

i. **Indicate** quantities that could be measured by the student that would allow them to determine μ_k using a graph.

解：

测弹簧相对原长的压缩量c，以及从原长位置x=0起算的粗糙段停止距离d。不要把光滑的弹簧释放路程加入粗糙段距离。

官方评分

A	**(i)**	For indicating **both** of the following quantities: • The spring compression, x • The distance the block slides to the right of $x = 0$, d **Scoring Note:** The variables given are chosen only for illustrating algebraic relationships in parts A and B. Responses may earn this point by using any clearly defined quantities or variables.	**Point A1**

减小测量不确定度：重复同一条件再取平均

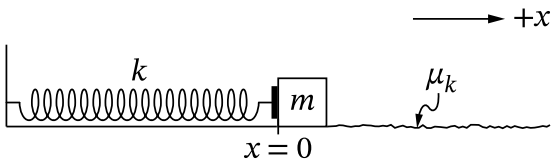
每一个压缩量都重复释放并记录停止距离，取平均以降低随机误差；统一米尺零点和读数方向可以减少不一致的系统读数。先保持条件相同做重复，再改变压缩量收集不同数据点，两种重复用途不同。

例题 2026 Q3 (A)(ii)

The following information applies to parts A and B.

3. A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

Figure 1



A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

ii. Briefly **describe** a method to reduce experimental uncertainty for the measured quantities.

解：

对每个压缩量重复释放并取平均，再使用多个压缩量收集数据，固定量尺的零点与读数方向。

官方评分

(ii)	For describing a reasonable method of reducing experimental uncertainty Examples of acceptable responses include the following: • Completing multiple trials using different values for the spring compression • Repeat measurements for each trial • Improved measurement technology to measure length more precisely	**Point A2**

Example Response

Measure the distance the spring has been compressed from equilibrium. Release the block and measure the distance the block traveled from the point at which the block loses contact with the spring. Perform multiple trials using different initial spring compressions.

检验尺寸的影响：固定质量，只改变尺寸

要判断直径或长度是否影响终速，选质量相同而尺寸不同的一组物体。质量改变会同时改变重力，把两种影响混在一起。用运动探测器取得各物体的速度平台，把终速作为纵轴、尺寸作为横轴。

例题 2024 Set 2 Q2 (e)(i)

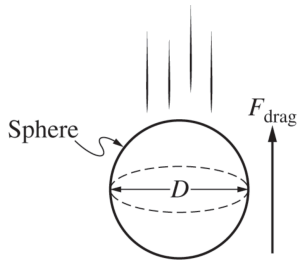


Figure 1

2. A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

A student claims that the terminal speed v_T of the sphere depends on the diameter D of the sphere. The student designs an experiment to collect data that can be used to provide evidence to support the claim.

(e) The student has access to but does not have to use all of the following equipment.

- Sphere Set 1: spheres of the same known mass with different known diameters
- Sphere Set 2: spheres of the same known diameter with different known masses
- A motion detector that can measure velocity as a function of time

i. **Indicate** two quantities that when graphed could be used to determine whether the diameter of the sphere affects the terminal speed.

Vertical axis: _____ Horizontal axis: _____

解：

纵轴取终速，横轴取球直径。选球组1：球的质量相同而直径不同。

官方评分

(e)(i)	For indicating the diameter of the sphere should be graphed	**1 point**
	For indicating the terminal velocity of the sphere should be graphed	**1 point**

练习 2024 Set 1 Q2 (e)(i)

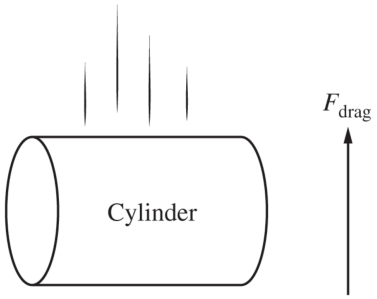


Figure 1

2. A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

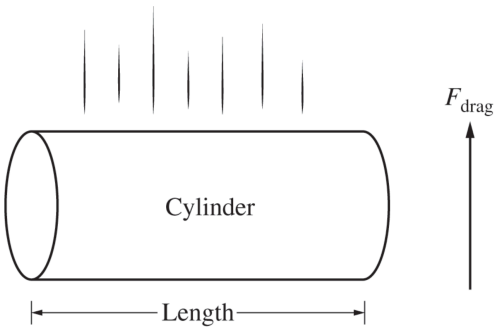


Figure 3

A student claims that the magnitude of the maximum speed of a cylinder dropped from rest depends on the length of the cylinder. The student designs an experiment to collect data that can be used to provide evidence to support the claim. The student drops cylinders with the orientation shown in Figure 3.

(e) The student has access to but does not have to use all of the following equipment.

- Cylinder Set 1: cylinders of the same known length with different known masses
- Cylinder Set 2: cylinders of the same known mass with different known lengths
- A motion detector that can measure velocity as a function of time

i. **Indicate** two quantities that when graphed could be used to determine whether the length of the cylinder affects the maximum speed.

Vertical axis: _____ Horizontal axis: _____

解：

纵轴取最大下落速率、横轴取圆柱长度。选质量相同、长度不同的圆柱组2。

官方评分

(e)(i)	For indicating that the length of the cylinder should be graphed	**1 point**
	For indicating that the maximum velocity of the cylinder should be graphed	**1 point**

依据测量趋势判断关系：变化必须超过散差

每个尺寸重复释放、测终速并取平均。将终速与尺寸作图：若终速有超过重复测量散差的变化，数据支持尺寸影响终速；若近似不变，数据未显示影响。保持质量和释放姿态相同，才使比较能针对尺寸。

例题 2024 Set 2 Q2 (e)(ii)

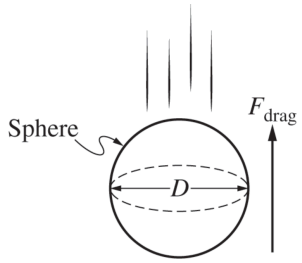


Figure 1

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A student claims that the terminal speed v_T of the sphere depends on the diameter D of the sphere. The student designs an experiment to collect data that can be used to provide evidence to support the claim.

(e) The student has access to but does not have to use all of the following equipment.

- Sphere Set 1: spheres of the same known mass with different known diameters
- Sphere Set 2: spheres of the same known diameter with different known masses
- A motion detector that can measure velocity as a function of time

ii. Briefly **describe** how the quantities graphed could be used to determine the relationship between sphere diameter and terminal speed.

解：

对每个直径从静止释放，用运动探测器记录速度随时间的曲线，读取平台速率，并重复测量。把终速对直径作图，若终速出现超过测量散差的变化，数据支持直径影响终速；若近似不变，数据未显示这一影响。保持质量不变是为了避免把重力差异误当作直径效应。

官方评分

(e)(ii)	For describing how the quantities graphed are related to the conclusion of the experiment	**1 point**
	Example Response *The slope of the diameter vs terminal velocity graph can be used to determine if sphere diameter affects terminal velocity.*	
	Total for part (e)	**3 points**
	Total for question 2	**15 points**

练习 2024 Set 1 Q2 (e)(ii)

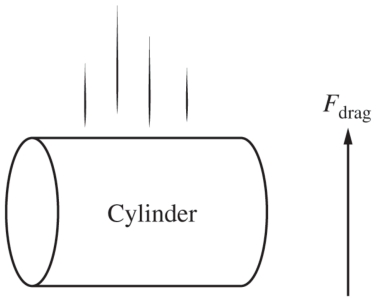


Figure 1

2. A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

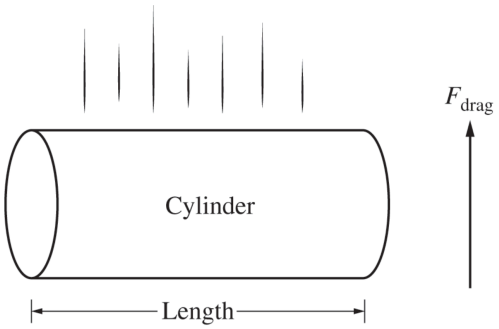


Figure 3

A student claims that the magnitude of the maximum speed of a cylinder dropped from rest depends on the length of the cylinder. The student designs an experiment to collect data that can be used to provide evidence to support the claim. The student drops cylinders with the orientation shown in Figure 3.

(e) The student has access to but does not have to use all of the following equipment.

- Cylinder Set 1: cylinders of the same known length with different known masses
- Cylinder Set 2: cylinders of the same known mass with different known lengths
- A motion detector that can measure velocity as a function of time

ii. Briefly **describe** how the quantities graphed could be used to determine the relationship between cylinder length and maximum speed.

解：

逐个圆柱按原图姿态释放，用探测器读速度平台，重复并取平均。把终速对长度作图，通过趋势是否超过重复测量的散差判断长度的影响。不能同时改变质量再把终速差异归给长度。

官方评分

(e)(ii)	For describing how the quantities graphed are related to the conclusions of the experiment	**1 point**
	Example Response *The slope of the length vs. maximum velocity graph can be used to determine if length affects terminal velocity.*	
		Total for part (e) 3 points
		Total for question 2 15 points

重力转动能：平方速度对两倍高度

无耗散时，mgh等于$mv^2/2$。质量约去后得到v^2=g(2h)，这正是纵量等于斜率乘横量的直线形式。纵轴选v^2、横轴选2h，斜率直接给g；不能把v对h的弯曲关系硬画成直线。

$$v^2 = g(2h).$$

例题 2025 Q3 (B)

本问承接 2025 Q3 (A)。

3. A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance *h*, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

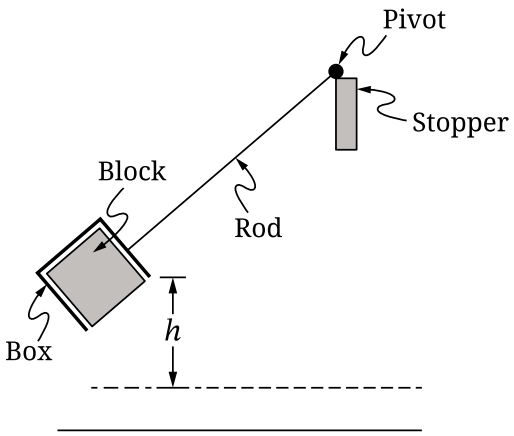


Figure 1

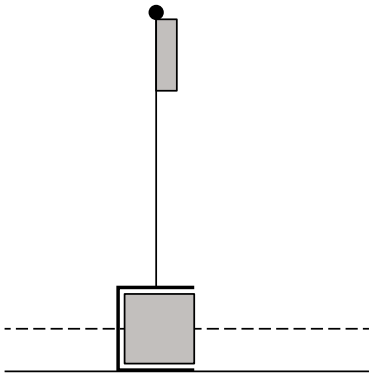


Figure 2

B. **Describe** how the data collected in part A could be graphed and how that graph would be analyzed to determine the value of g.

解：

转轴摩擦、杆与箱质量以及滑动摩擦都可忽略。释放时的重力势能变成物块的动能。

$$mgh = \frac{1}{2}mv^2, \qquad v^2 = g(2h).$$

横轴取2h、纵轴取速度平方，拟合直线斜率就是重力加速度。这样不需要测质量，也不把弯曲的速率—高度图当作直线。

官方评分

B For describing a graph that has linear trend that can be used to find g **Point B1**

Examples of acceptable responses include the following:

- v^2 vs. $2h$
- $\frac{1}{2}v^2$ vs. h
- v^2 vs. h
- v vs. $\sqrt{h}$

Scoring Notes:

- Reponses that include the reciprocals of the preceding examples, in addition to other equivalent graphs, also earn this point.
- This point may be earned independently of the response in part A.

For correctly relating the slope of the best-fit line to g **Point B2**

Examples of acceptable responses include the following:

Graph	Analysis
v^2 vs. $2h$	slope = g
$\frac{1}{2}v^2$ vs. h	slope = g
v^2 vs. h	slope = $2g$
v vs. $\sqrt{h}$	slope = $\sqrt{2g}$

Example Response

Plot v^2 on the vertical axis and $2h$ on the horizontal axis. The slope of the best-fit line is equal to g.

摩擦耗尽重力势能：高度对停止距离

物块最终停下，初始mgh全部转为粗糙面摩擦耗能μmgxmax。约去mg，得到h=μxmax，所以高度对停止位置的直线斜率就是摩擦系数。先用关系决定轴，再处理表中数据。

$$h = \mu x_{\text{max}}.$$

例题 2025 Q3 (C)(i)

3. A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance h, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

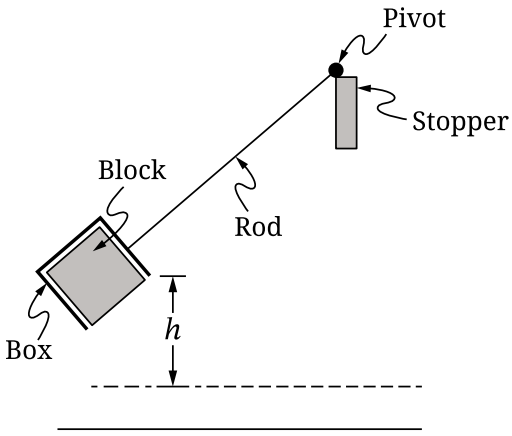


Figure 1

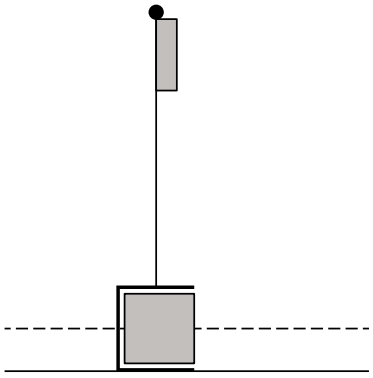


Figure 2

C. The experiment is repeated, but the horizontal surface on which the block slides is replaced with a new rough surface, as shown in Figure 3. The coefficient of kinetic friction between the block and the new surface is μ.

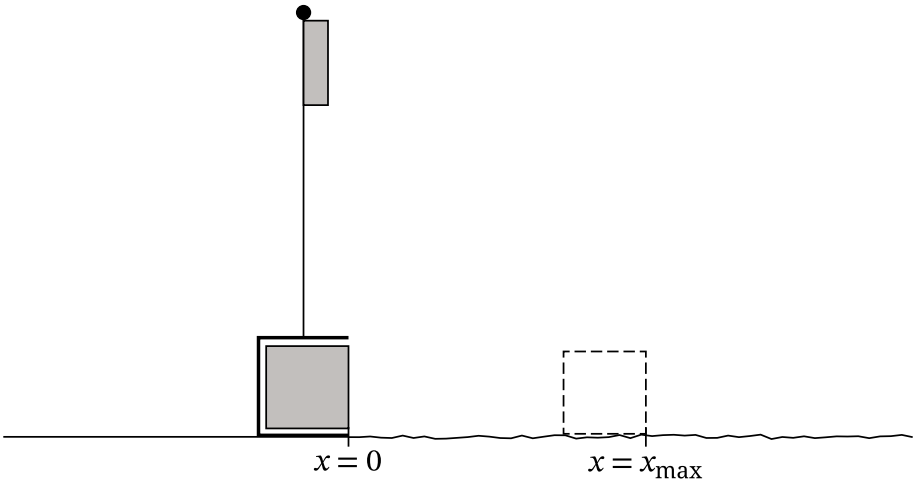


Figure 3

The block-box system is pulled aside so that the center of mass of the block is displaced various vertical distances h and then released from rest. For each vertical distance, students measure the position $x = x_{\max}$ at which the block comes to rest.

The students' measurements of h and $x_{\max}$ are shown in Table 1.

Table 1

h (m)	$x_{\max}$ (m)
0.30	0.76
0.45	1.10
0.60	1.40
0.75	1.90
0.90	2.30

i. **Indicate** two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine μ.

Vertical axis: _____ Horizontal axis: _____

解：

粗糙面使物块最后停下。起初重力势能最终全部转为摩擦耗散，支持力为重力大小。

$$mgh = \mu mgx_{\max}, \qquad h = \mu x_{\max}.$$

故横轴取停止位置、纵轴取释放高度，两轴单位均为m。

官方评分

C	(i)	For indicating appropriate quantities that could be plotted to produce a linear graph that can be used to determine μ, such as h **vs.** $x_{\max}$ **Scoring Note:** Reponses that include the reciprocal of the preceding example, in addition to other equivalent graphs, also earn this point.	**Point C1**

弹簧储能被摩擦耗尽：距离对压缩量平方

初始弹簧能量$kc^2/2$最终被粗糙段摩擦μmgd耗尽。整理成d对c^2的直线，就能用斜率反求摩擦系数。压缩量c和停止距离d都是长度，但c需要平方，横轴单位也随之变成m^2。

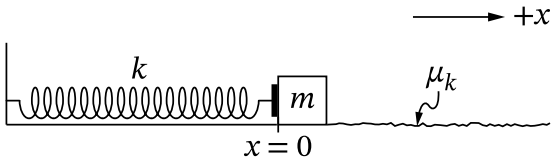
$$d = \frac{k}{2\mu_k mg}\, c^2.$$

例题 2026 Q3 (B)(i)

The following information applies to parts A and B.

3. A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

Figure 1



A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

i. **Indicate** what quantities the student could graph on the horizontal and vertical axes to create a graph that can be used to determine μ_k.

解：

起初的弹性势能最后全部由粗糙段的摩擦耗散。横轴取压缩量的平方、纵轴取停止距离。

$$\frac{1}{2}kc^2 = \mu_k mgd, \qquad d = \frac{k}{2\mu_k mg}c^2.$$

官方评分

B	**(i)**	For indicating quantities that produce a graph that has a linear trend that can be used to determine μ_k Examples of acceptable responses include the following: • d as a function of x^2 • $\sqrt{d}$ as a function of x **Scoring Notes:** • Responses that include the reciprocals of the preceding examples, in addition to other equivalent graphs also earn this point. • This point may be earned independently of the response in part A.	**Point B1**

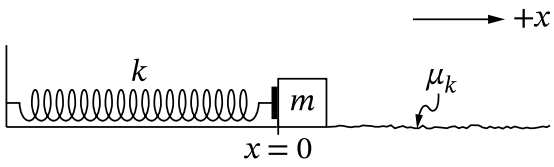
练习 2026 Q3 (B)(ii)

本问承接 2026 Q3 (B)(i)。

The following information applies to parts A and B.

3. A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

Figure 1



A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

ii. Briefly **describe** the relationship between μ_k and a feature of the graph from part B (i). Your answer may include an equation that relates μ_k and the chosen feature of the graph.

解：

把拟合线斜率记为S，先对应直线系数，再解出摩擦系数。

$$S = \frac{k}{2\mu_k mg}, \qquad \mu_k = \frac{k}{2mgS}.$$

官方评分

(ii) For correctly describing the relationship of the slope of the best-fit line to μ_k **Point B2**

Examples of acceptable responses include the following:

Graph	Analysis
d as a function of x^2	$\text{slope} = \frac{k}{2\mu_k mg}$
$\sqrt{d}$ as a function of x	$\text{slope} = \sqrt{\frac{k}{2\mu_k mg}}$

Example Response

Plotting sliding distance on the vertical axis and the square of the compression distance on the horizontal axis will yield a linear graph. The slope of the graph will be equal to $\frac{k}{2\mu_k mg}$, *so* $\mu_k = \frac{k}{2mg(\text{slope})}$.

弹簧储能升高：高度对压缩量平方

第二实验没有摩擦，弹簧能量变成最高点的重力势能。整理为h=(knew/2mg)s^2，纵轴是高度、横轴是压缩量平方。本实验物块的质量为2.0kg。

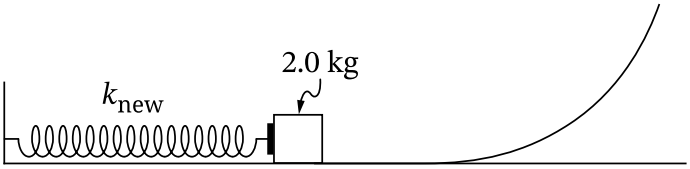
$$h = \frac{k_{\text{new}}}{2mg}\, s^2.$$

例题 2026 Q3 (C)(i)

The following information applies to parts C and D.

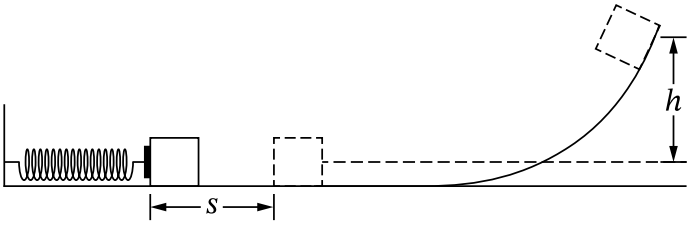
In a different experiment, the student is asked to determine the spring constant k_{new} of a new spring that is attached to a wall. A block of mass $2.0\ \text{kg}$ is placed next to the spring on a surface where frictional forces are negligible. The surface is horizontal near the spring and slopes upward into a ramp to the right of the initial position of the block, as shown in Figure 2.

Figure 2



The student pushes the block, compressing the spring a distance s, as shown in Figure 3. The block is released from rest.

Figure 3



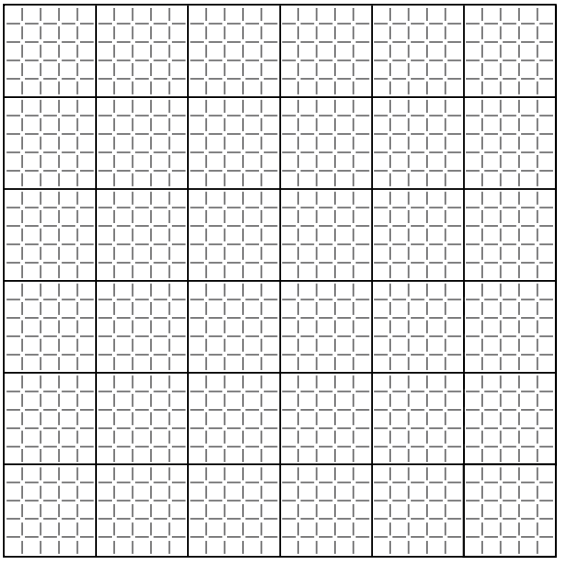
The student uses a meterstick to measure the maximum height h of the block on the ramp. The experiment is repeated several times with different compression distances s. The student's measurements of s and h are shown in Table 1.

Table 1

s (m)	h (m)
0.020	0.02
0.040	0.04
0.060	0.13
0.080	0.18
0.100	0.33

Quantity (units, if appropriate)

_____ _____



Quantity (units, if appropriate)

_____ _____

i. **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine k_{new}.

解：

第二个实验没有摩擦，弹簧储能在最高位置全部变成重力势能。横轴取s平方，纵轴取高度h。

$$\frac{1}{2}k_{\text{new}}s^2 = mgh, \qquad h = \frac{k_{\text{new}}}{2mg}s^2.$$

官方评分

C **(i)** For labeling the axes with appropriate quantities (including units) that could be plotted to produce a linear graph that can be used to determine k_{new}, such as: **Point C1**

- h as a function of s^2
- $\sqrt{h}$ as a function of s
- mgh as a function of $\frac{1}{2}s^2$

Scoring Note: Responses that include the reciprocals of the preceding examples, in addition to other equivalent graphs, also earn this point.

线性图点：先换算坐标，再设刻度与单位

选好轴后，对每行原数据算出对应横、纵坐标。需要平方的量要逐行平方；在轴上用均匀数字刻度，写清实际单位，再把所有原数据点画出。横轴刻度相同的距离表示相同数值增量。

例题 2025 Q3 (C)(ii)

本问承接 2025 Q3 (C)(i)。

3. A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance h, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

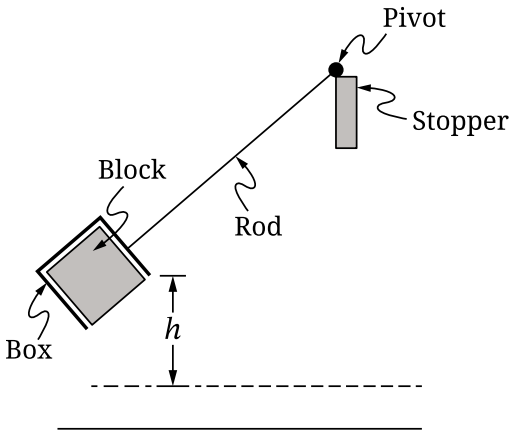


Figure 1

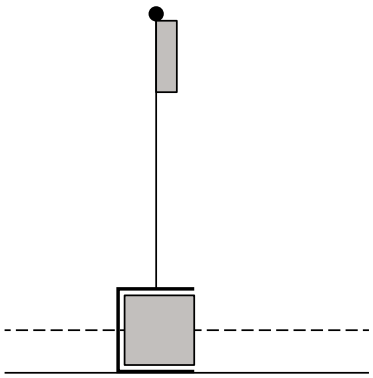


Figure 2

C. The experiment is repeated, but the horizontal surface on which the block slides is replaced with a new rough surface, as shown in Figure 3. The coefficient of kinetic friction between the block and the new surface is μ.

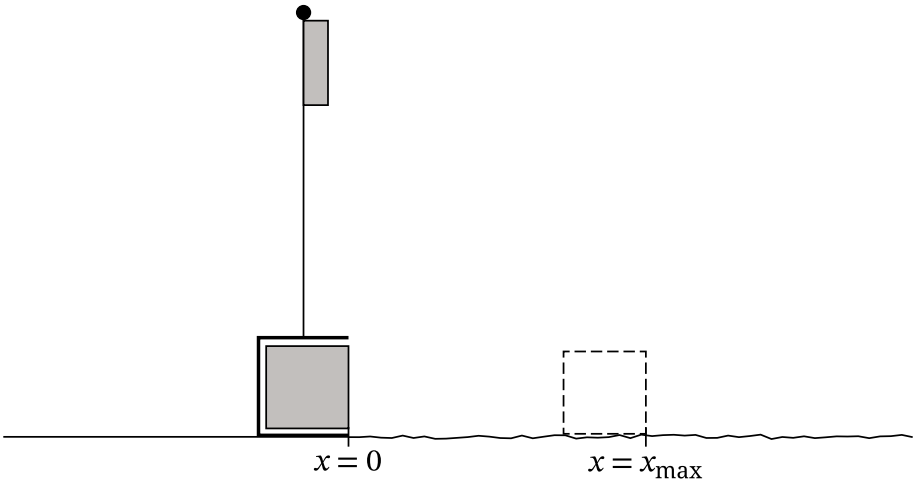


Figure 3

The block-box system is pulled aside so that the center of mass of the block is displaced various vertical distances h and then released from rest. For each vertical distance, students measure the position $x = x_{\max}$ at which the block comes to rest.

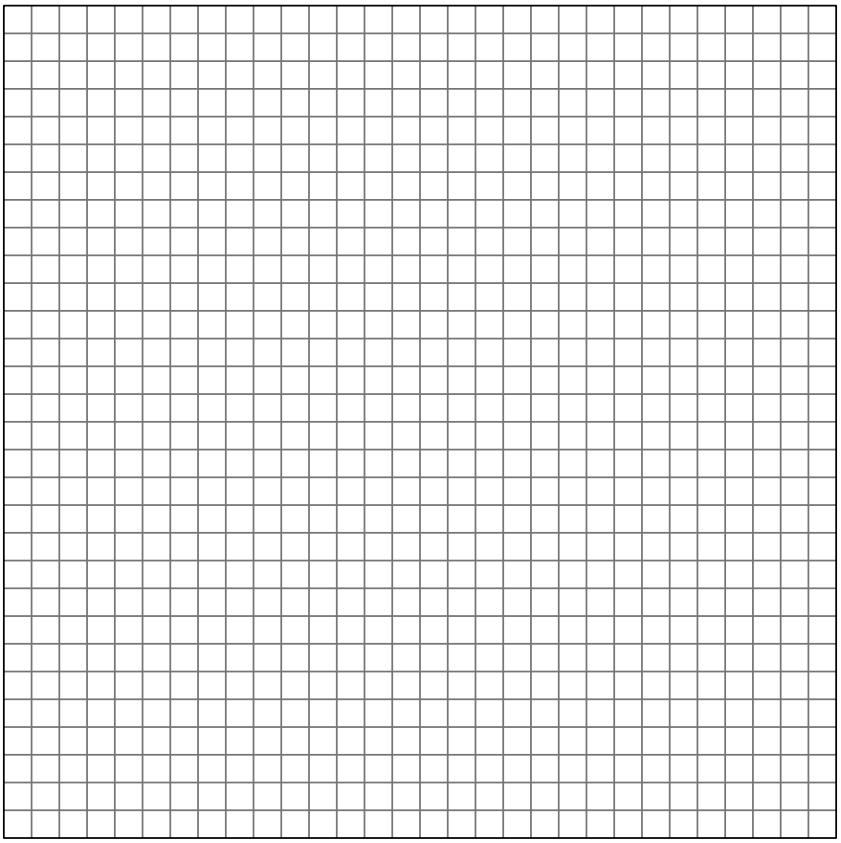
The students' measurements of h and $x_{\max}$ are shown in Table 1.

Table 1

h (m)	$x_{\max}$ (m)
0.30	0.76
0.45	1.10
0.60	1.40
0.75	1.90
0.90	2.30

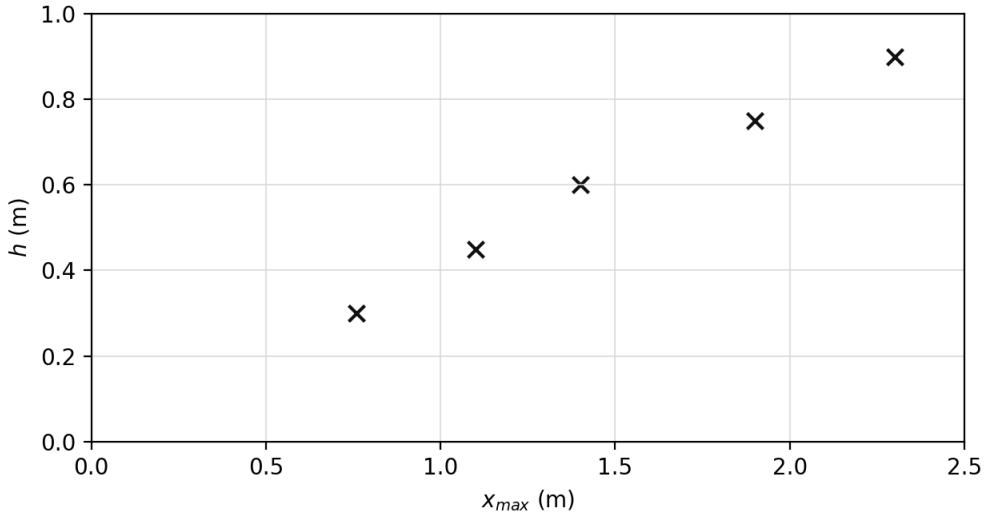
ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.
- **Plot** the points you recorded in Table 2.



解：

把原表中的五组高度与停止位置全部转成坐标。纵轴h的单位为m，横轴xmax的单位为m；两轴用等间隔线性刻度，把全部五点标出。



C(ii)：全部五个数据点；本问只完成坐标与描点。

官方评分

(ii)	For labeling the axes (including units) with a linear scale	**Point C2**
	For plotting data points consistent with **one** of the following: • The quantities indicated in part C (i) • The quantities provided in Table 2 • The axes indicated on the grid	**Point C3**

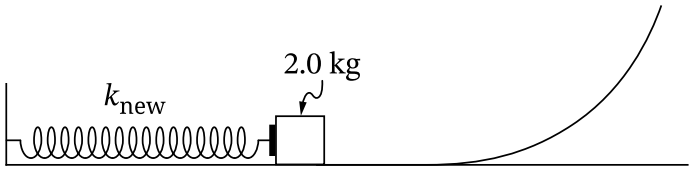
练习 2026 Q3 (C)(ii)

本问承接 2026 Q3 (C)(i)。

The following information applies to parts C and D.

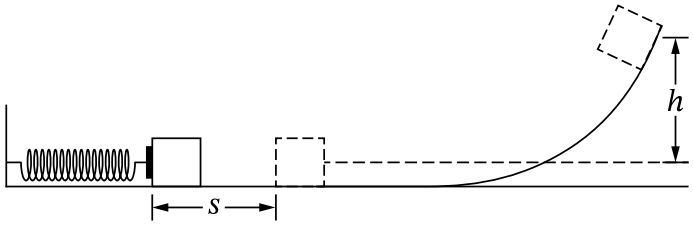
In a different experiment, the student is asked to determine the spring constant k_{new} of a new spring that is attached to a wall. A block of mass $2.0\ \text{kg}$ is placed next to the spring on a surface where frictional forces are negligible. The surface is horizontal near the spring and slopes upward into a ramp to the right of the initial position of the block, as shown in Figure 2.

Figure 2



The student pushes the block, compressing the spring a distance s, as shown in Figure 3. The block is released from rest.

Figure 3



The student uses a meterstick to measure the maximum height h of the block on the ramp. The experiment is repeated several times with different compression distances s. The student's measurements of s and h are shown in Table 1.

Table 1

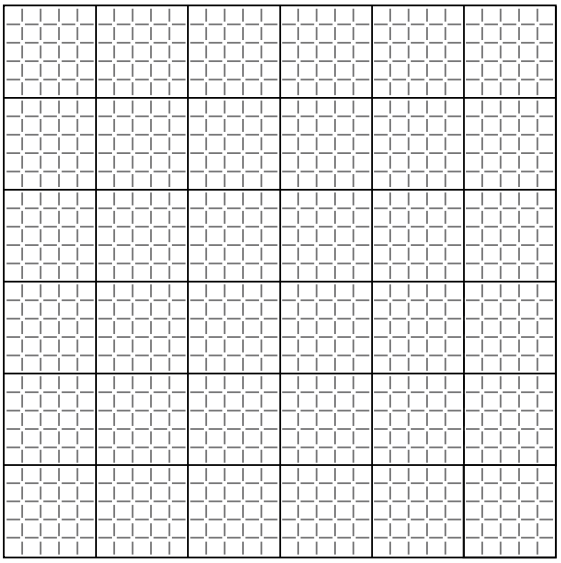
s (m)	h (m)
0.020	0.02
0.040	0.04
0.060	0.13
0.080	0.18
0.100	0.33

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Clearly **label** the vertical and horizontal axes with numerical scales.
- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work and will not be scored.

Quantity (units, if appropriate)

_____ _____



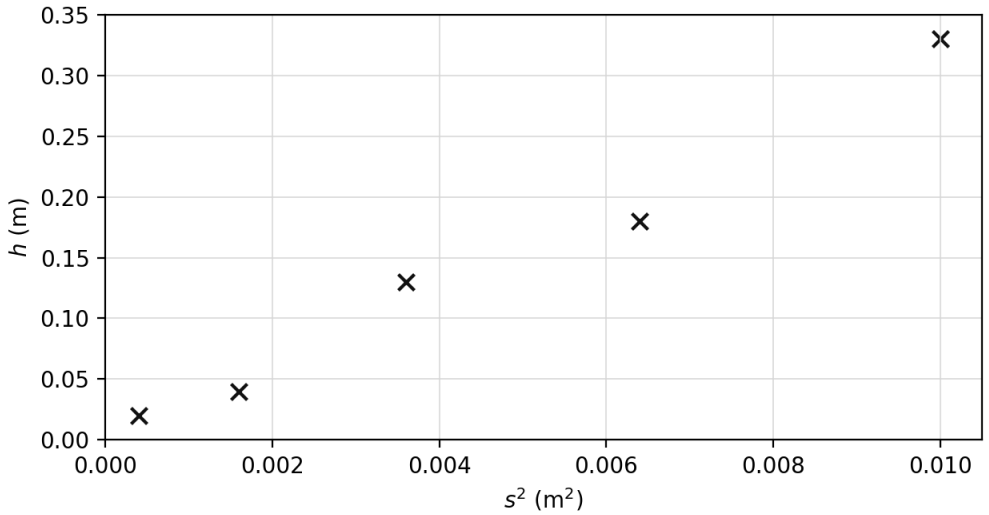
Quantity (units, if appropriate)

_____ _____

解：

由原数据计算五个横坐标，纵坐标保持原高度；两轴标清数值刻度和单位。

$$s^2 = 0.0004,\ 0.0016,\ 0.0036,\ 0.0064,\ 0.0100\ \mathrm{m}^2.$$



C(ii)：压缩量平方与高度的全部五个数据点。

官方评分

(ii)	For labeling **both** axes with a linear scale	**Point C2**
	For plotting data points consistent with the quantities labeled in part C (i)	**Point C3**

最佳拟合线：通过点群中央而非逐点连接

测量点有散差，直线不应依次经过每个点。画一条反映整体趋势的线，让两侧散点大致均衡，并覆盖测量范围。用原题给出的散点或自己上一问刚画出的全体点；不得只选其中两个点连线冒充拟合。

例题 2024 Set 2 Q2 (d)(i)

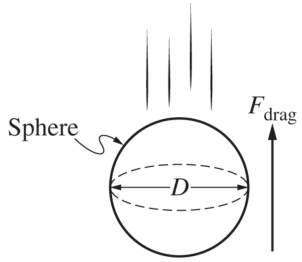
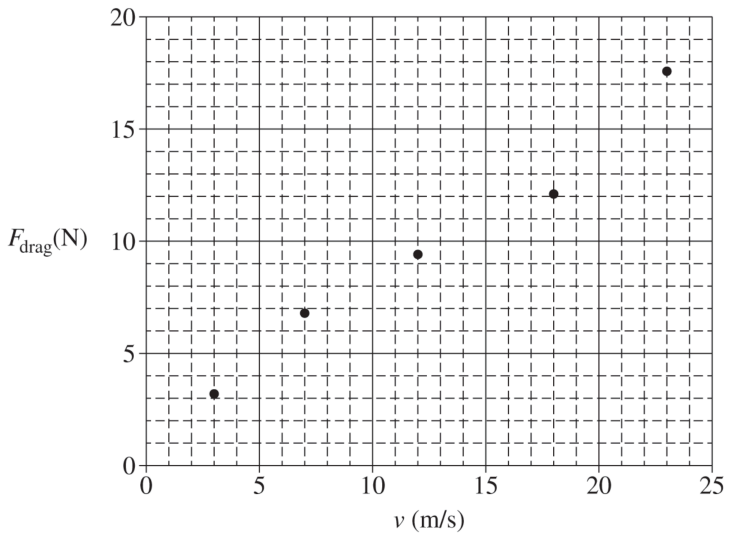


Figure 1

2. A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

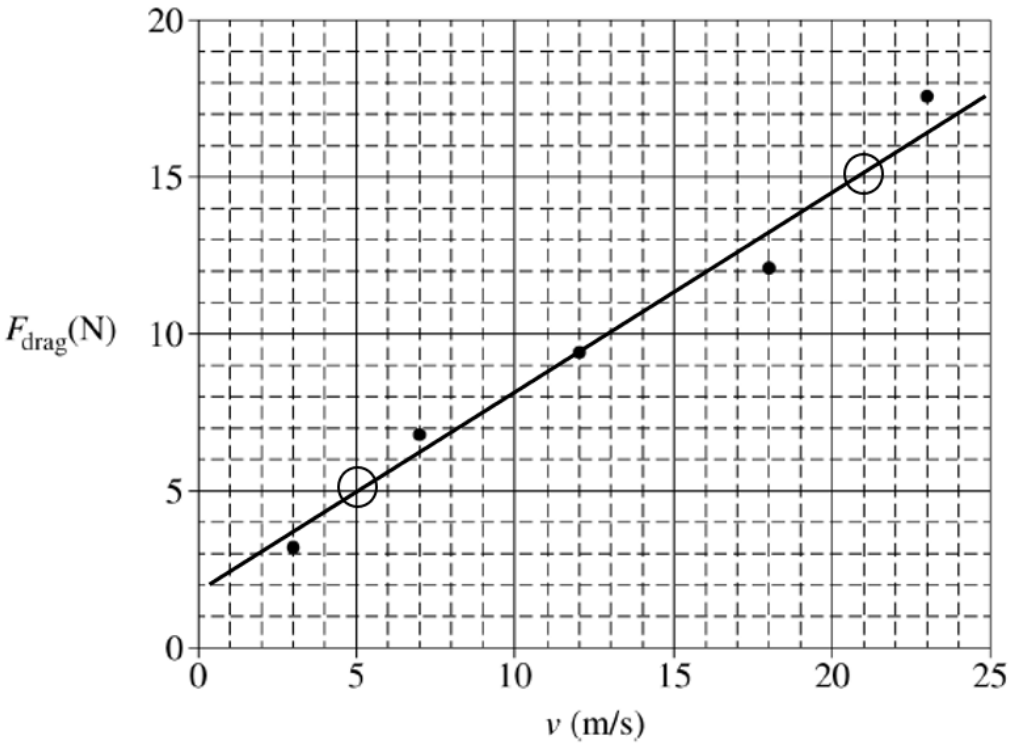
(d) The student conducts an experiment to better understand the relationship between F_{drag} and v. The student makes measurements to calculate and graph the magnitude of F_{drag} as a function of v for the falling sphere.



i. **Draw** the best-fit line for the data.

解：

拟合线通过点群中央，两侧点数大致相当，不逐点折线连接。

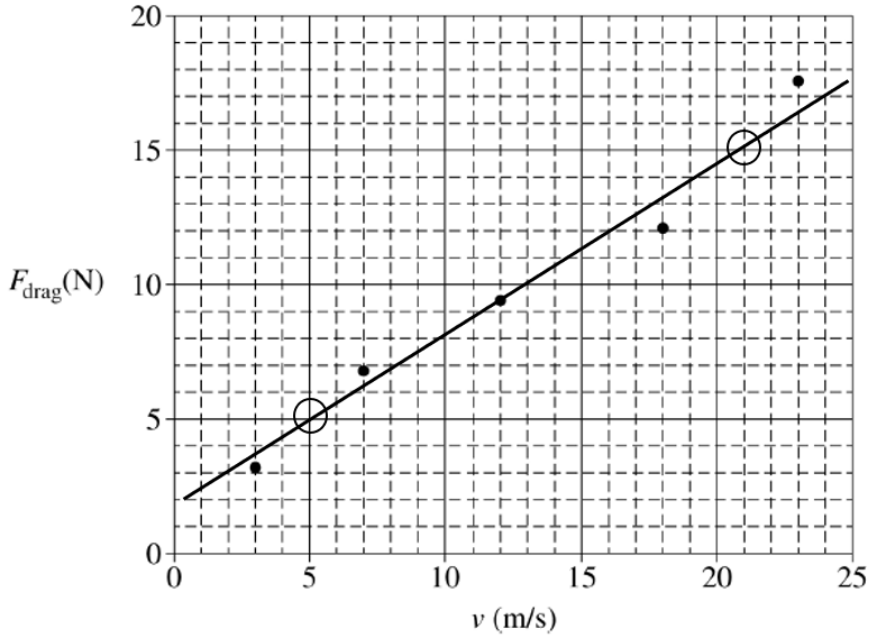


原图数据对应的官方拟合示例。

官方评分

(d)(i)	For drawing an appropriate line of best fit that approximates the data	**1 point**

Example Response



练习 2024 Set 1 Q2 (d)(i)

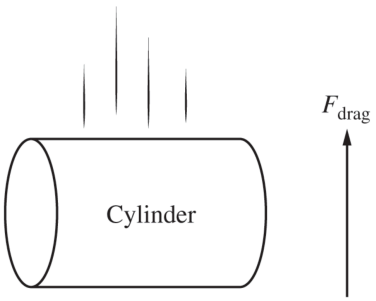
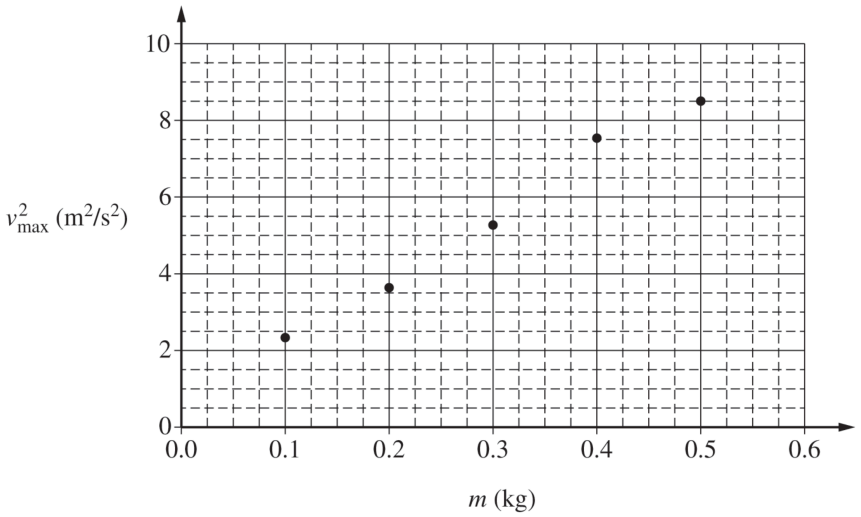


Figure 1

2. A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

(d) The student conducts an experiment to better understand the relationship between maximum speed v_{max} and mass. The student collects data to determine the maximum speed for cylinders dropped from rest, each with the same physical size and shape but a different mass m. The student then graphs v_{max}^2 as a function of mass.

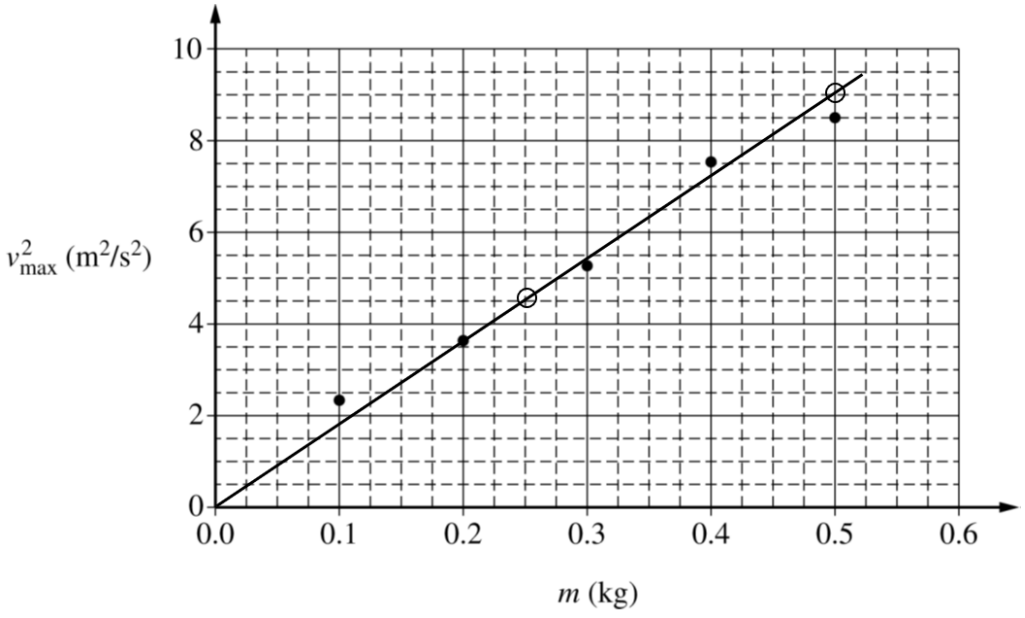


i. **Draw** the best-fit line for the data.

解：

将终速关系改写成纵轴对横轴的直线形式，再拟合图中点群。

$$v_T^2 = \frac{g}{b}m.$$

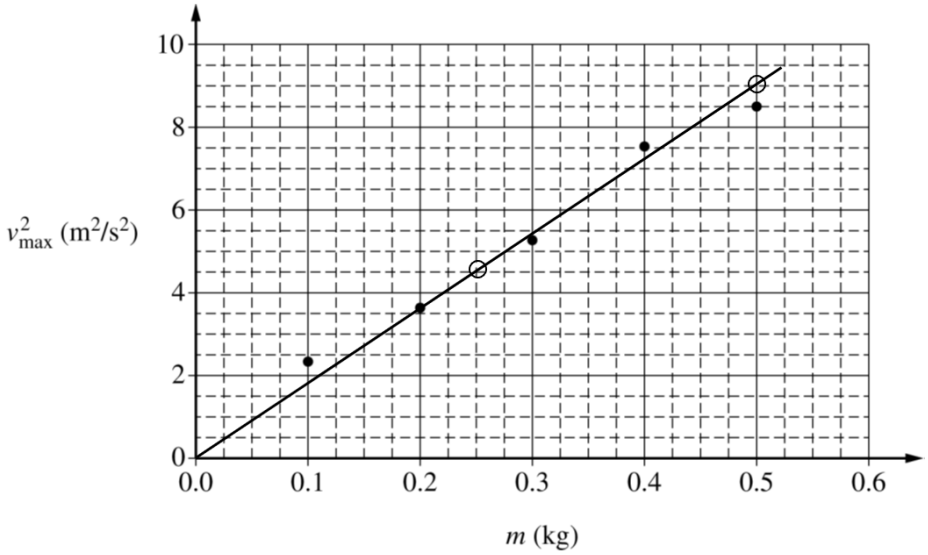


原图数据对应的官方拟合示例。

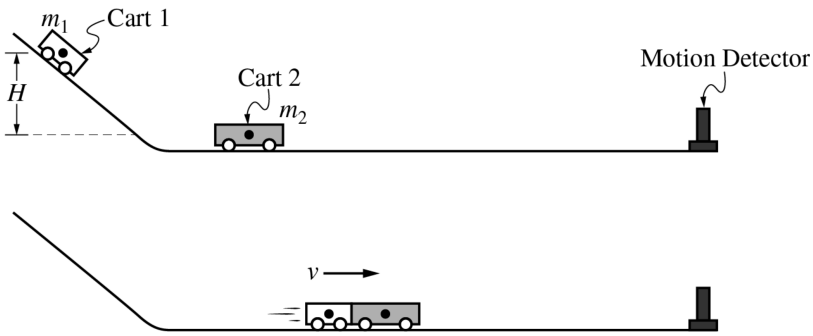
官方评分

(d)(i)	For drawing an appropriate line of best fit that approximates the data	**1 point**

Example Response

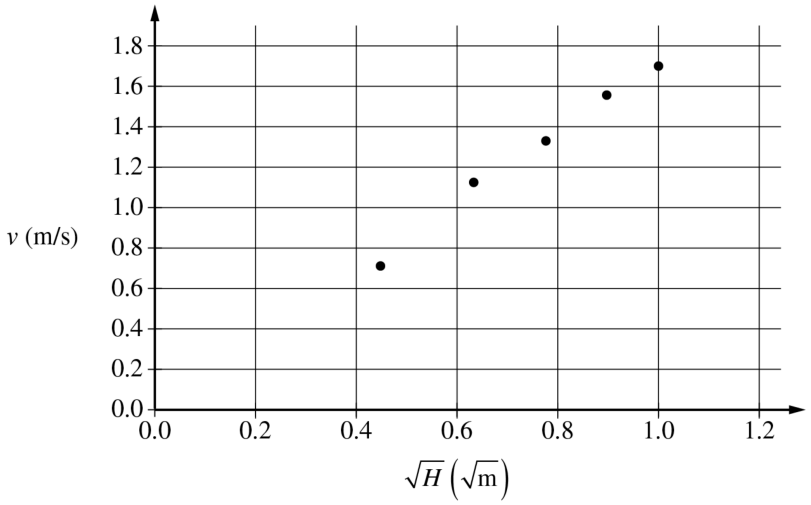


练习 2022 Set 1 Q2 (d)(i)



2. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

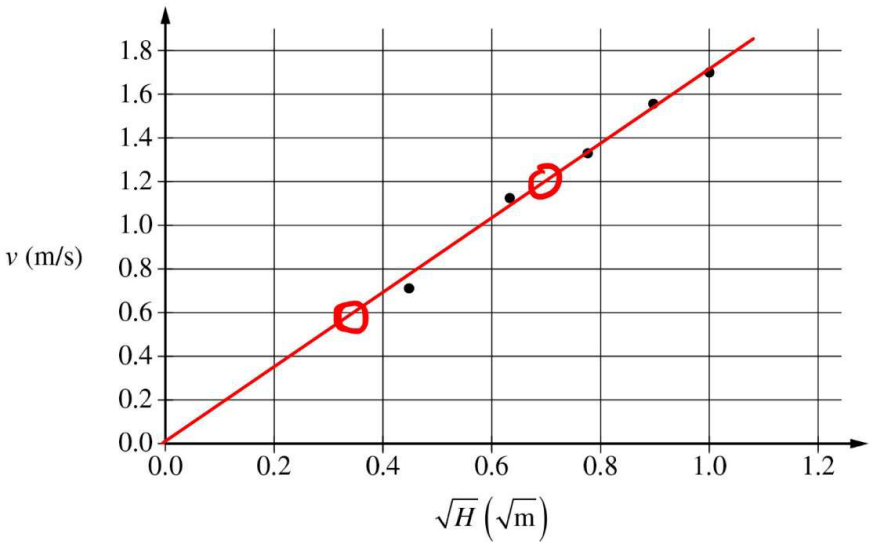
(d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250\,\text{kg}$. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



i. Draw a line that represents the best fit to the data points shown.

解：

在原图上画通过点群中央的最佳拟合线。

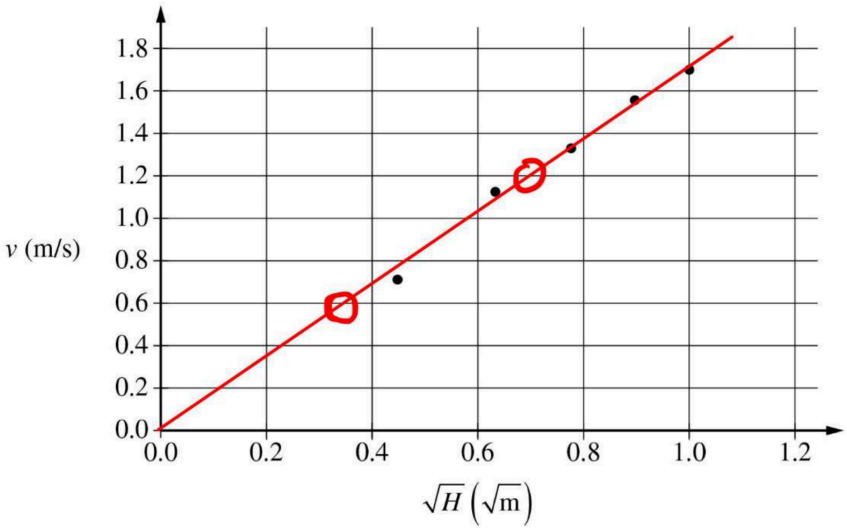


原数据对应的官方拟合示例。

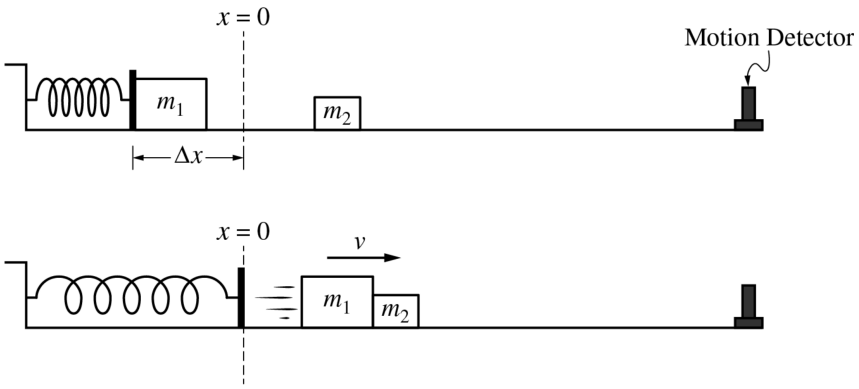
官方评分

(d)(i) For drawing an appropriate best-fit line including approximately the same number of points above and below the line **1 point**

Example Response

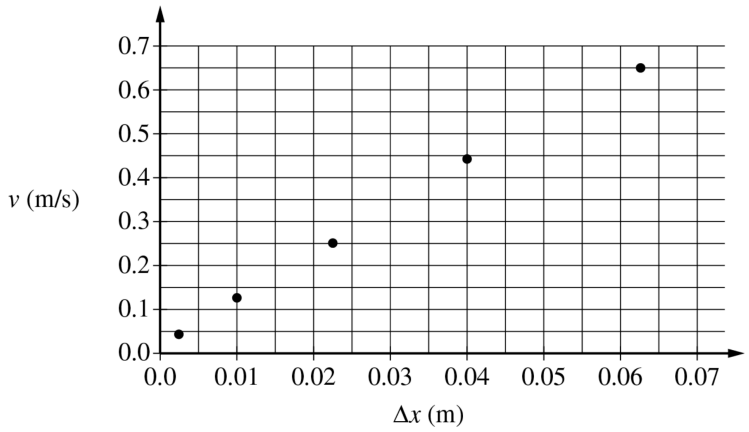


练习 2022 Set 2 Q2 (d)(i)



2. Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

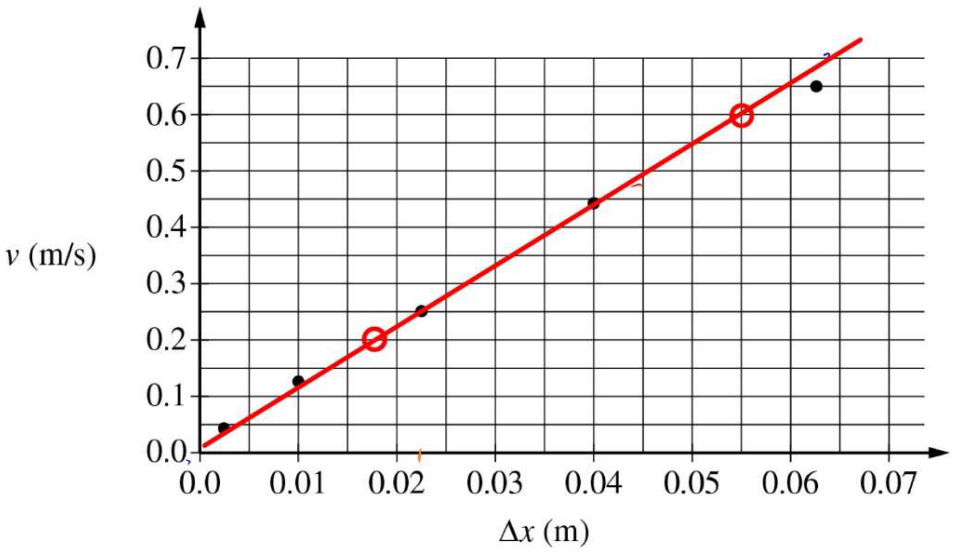
(d) A group of students use the setup to perform an experiment. They measure the mass of Block 1 to be $m_1 = 0.500\,\text{kg}$, and the spring constant k of the spring to be $150\,\text{N/m}$. The mass of Block 2 is unknown. They perform several trials and in each trial the spring is compressed a different distance Δx and the final velocity v of the two-block system is measured. They graph v as a function of Δx, as shown below.



i. Draw a line that represents the best fit to the data points shown.

解：

画经过点群中央的直线。

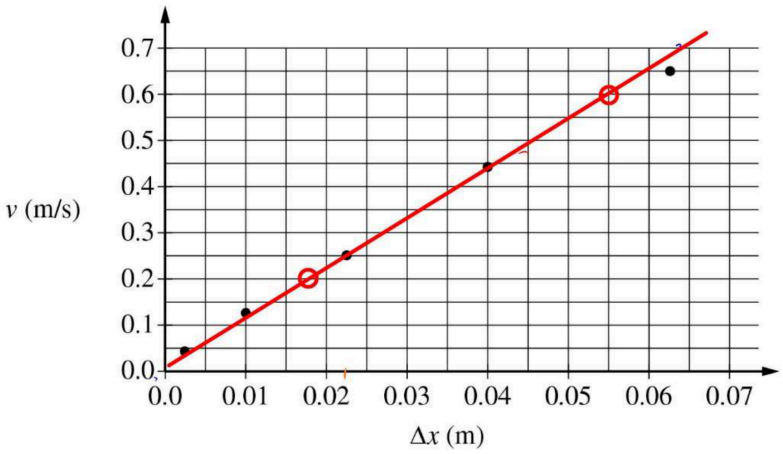


原数据对应的官方拟合示例。

官方评分

(d)(i) For drawing an appropriate best-fit line including approximately the same number of points above and below the line **1 point**

Example Response



练习 2023 Set 2 Q2 (c)(i)

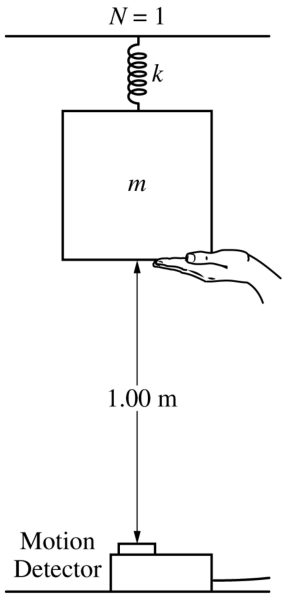


Figure 1

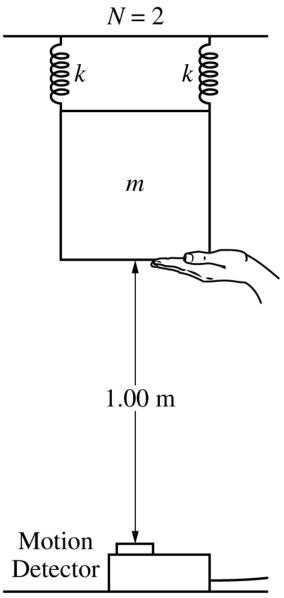
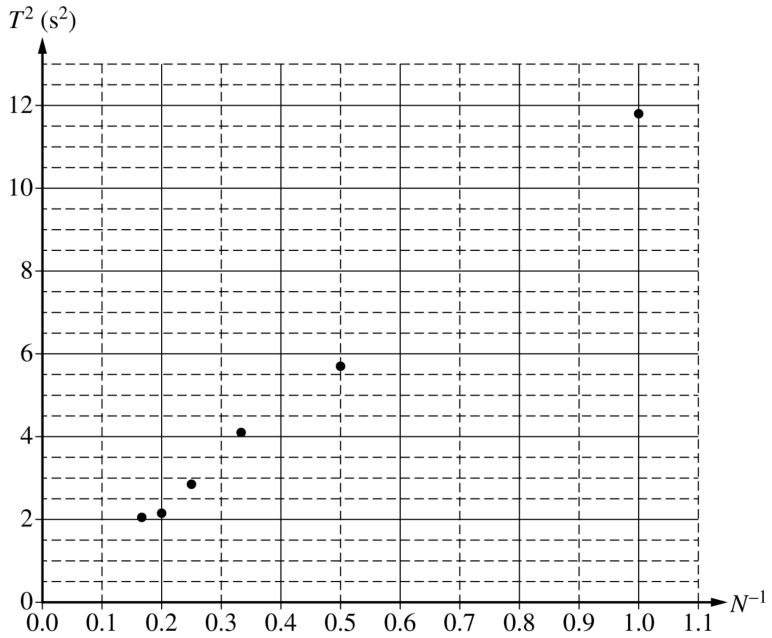


Figure 2

2. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height $1.00\ \text{m}$ above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

(c) The student plots the data for T^2 as a function of N^{-1}, as shown.

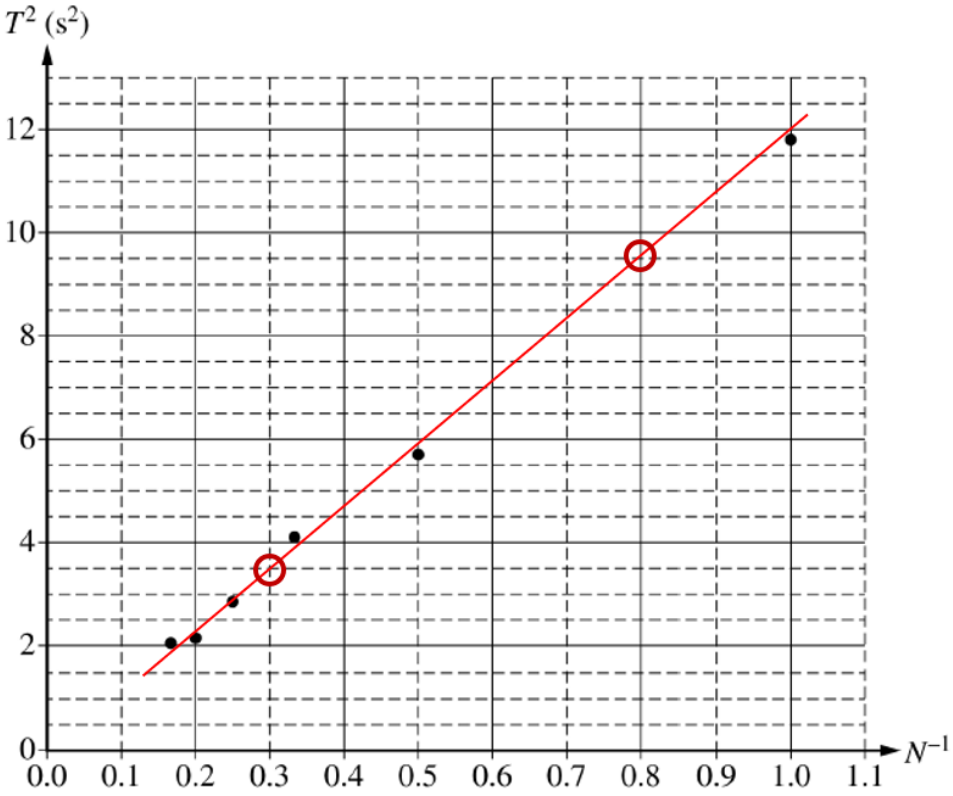


i. Draw the best-fit line for the data.

解：

原图纵轴为周期平方，横轴为根数倒数，理论关系为直线。

$$T^2 = \frac{4\pi^2 m}{k}\frac{1}{N}.$$

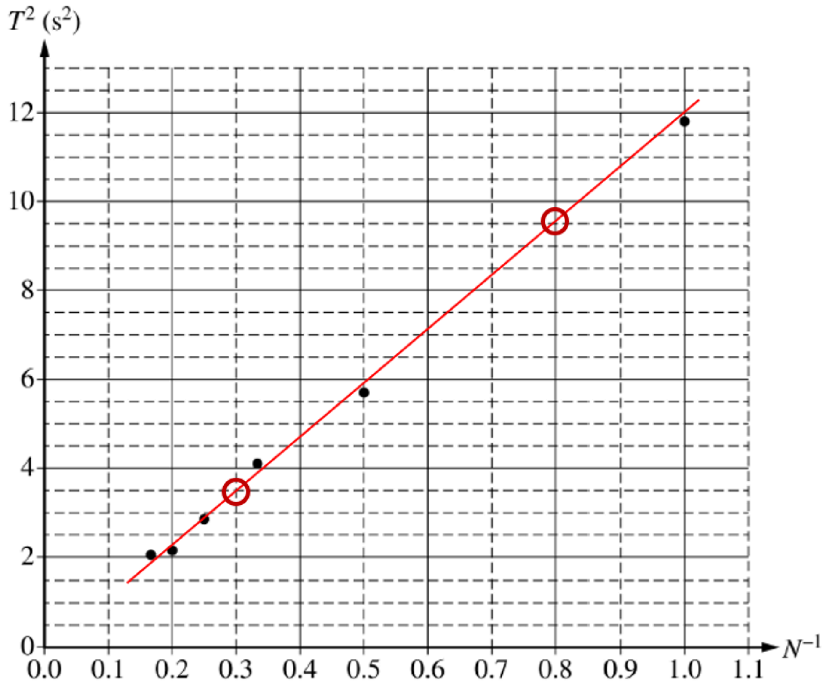


原数据对应的官方直线拟合示例。

官方评分

(c)(i) For drawing an appropriate line of best fit that approximates the data **1 point**

Example Solution



练习 2023 Set 1 Q2 (c)(i)

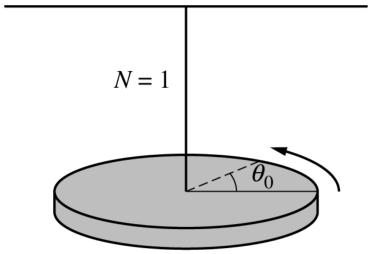


Figure 1

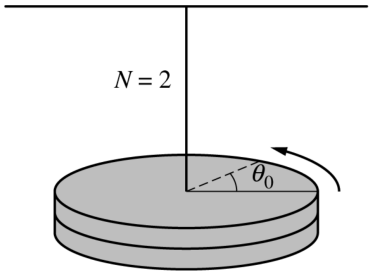
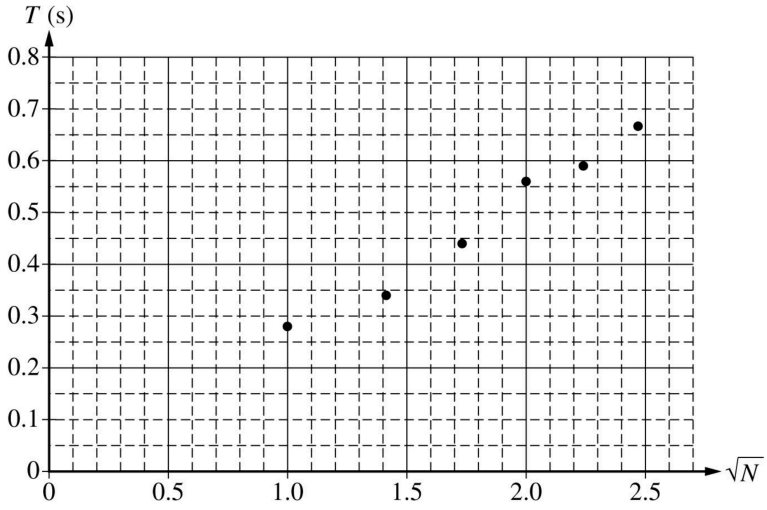


Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

   The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

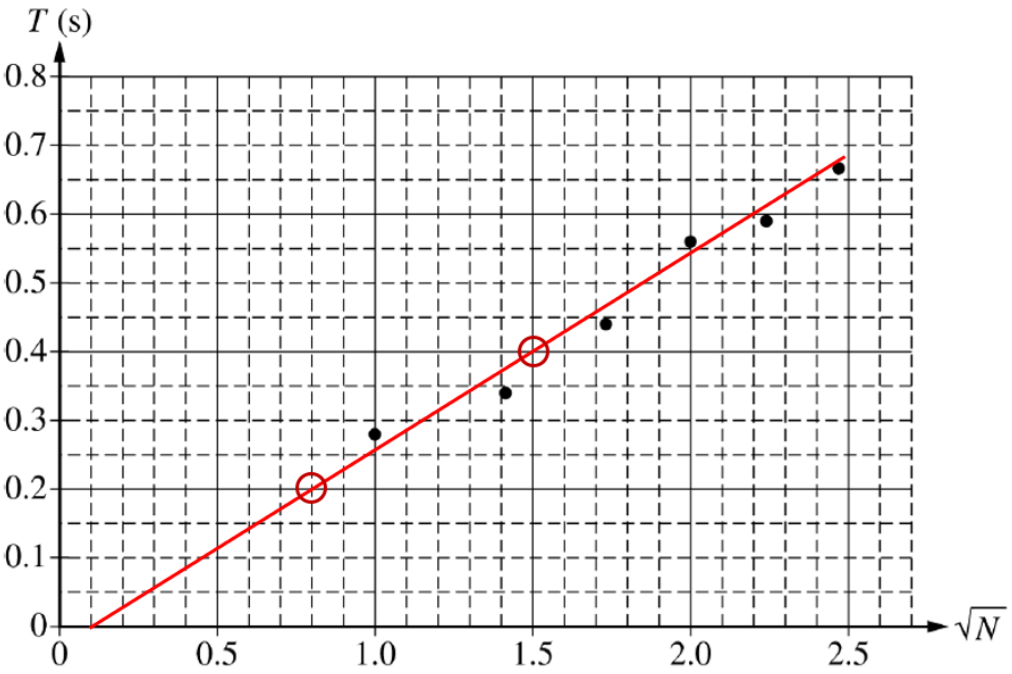
(c) The student plots the data for T as a function of $\sqrt{N}$, as shown.



i. Draw the best-fit line for the data.

解：

周期对盘数平方根的图理论上为直线，画经过点群中央的拟合线。

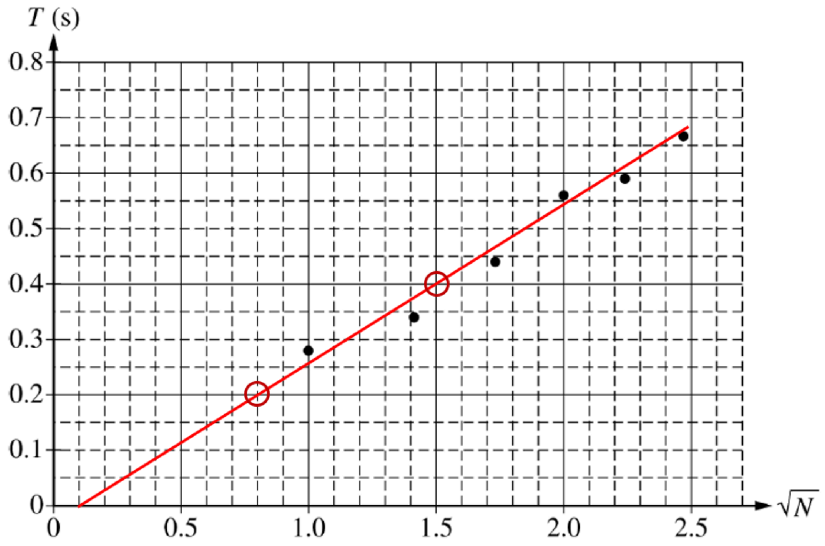


原数据对应的官方拟合示例。

官方评分

(c)(i)	For drawing an appropriate line or curve of best fit that approximates the data	1 point

Example Solution



练习 2025 Q3 (C)(iii)

本问承接 2025 Q3 (C)(ii)。

3. A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance h, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

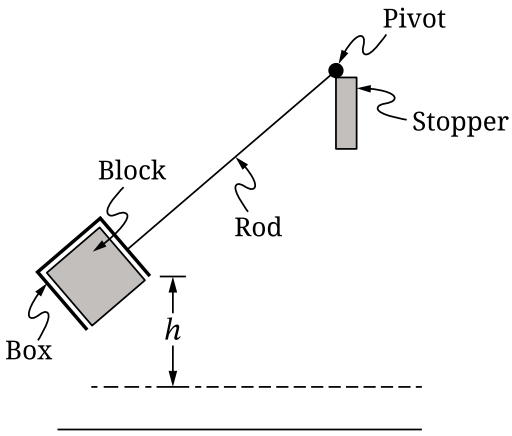


Figure 1

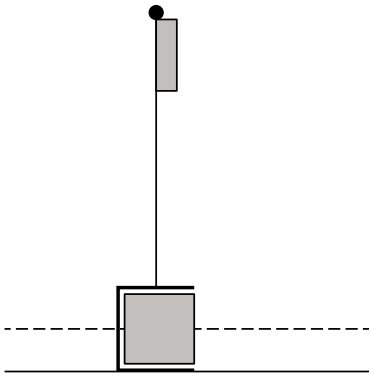


Figure 2

C. The experiment is repeated, but the horizontal surface on which the block slides is replaced with a new rough surface, as shown in Figure 3. The coefficient of kinetic friction between the block and the new surface is μ.

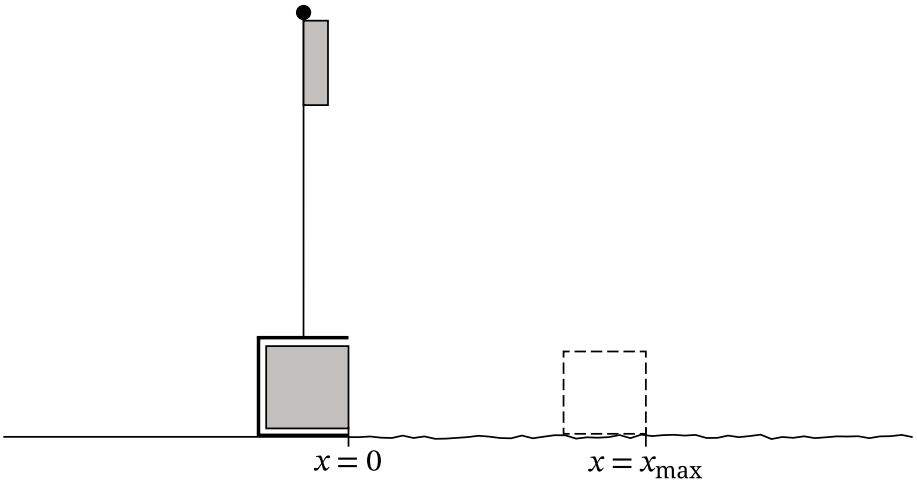


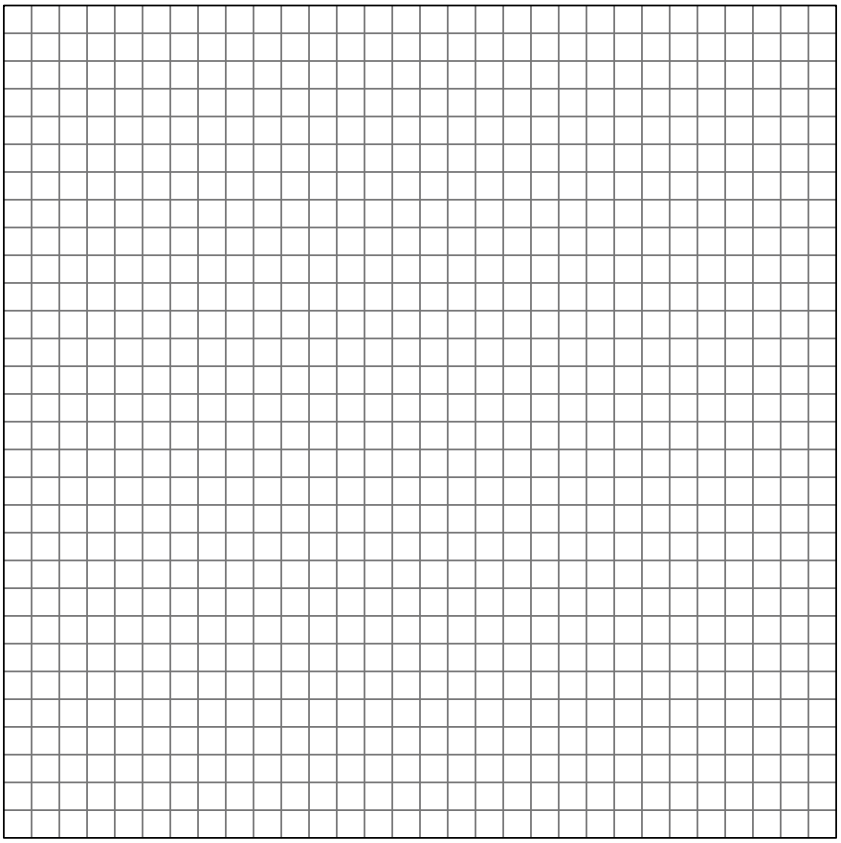
Figure 3

The block-box system is pulled aside so that the center of mass of the block is displaced various vertical distances h and then released from rest. For each vertical distance, students measure the position $x = x_{\max}$ at which the block comes to rest.

The students' measurements of h and $x_{\max}$ are shown in Table 1.

Table 1

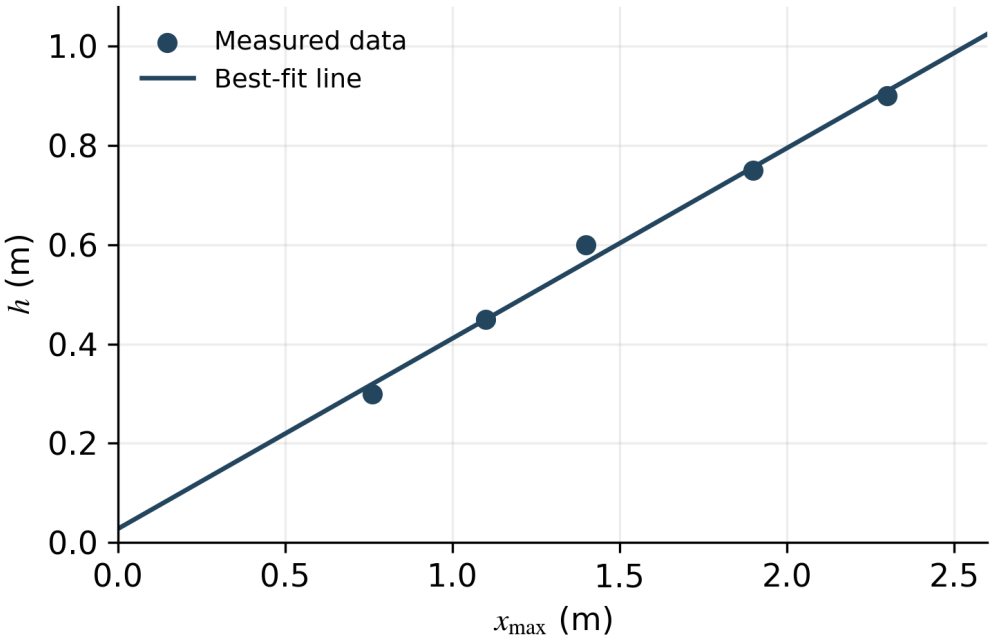
h (m)	$x_{\max}$ (m)
0.30	0.76
0.45	1.10
0.60	1.40
0.75	1.90
0.90	2.30



iii. **Draw** a best-fit line to the data graphed in part C (ii).

解：

拟合线应穿过点群中央，让线上、下方的散点大致均衡，不用折线依次连接测量点。选择线上的相距较远的两点，便于下一问计算斜率。

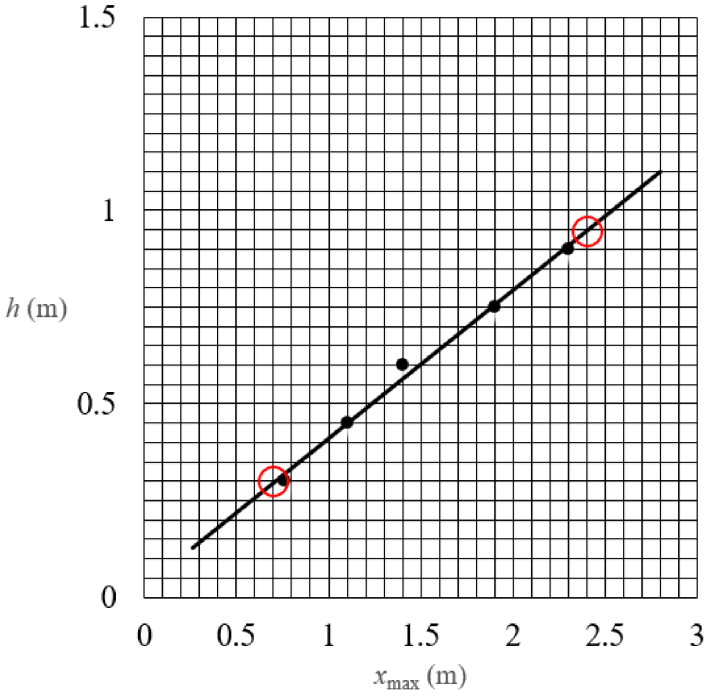


全部五个原数据点与直线拟合；两轴分别为停止位置与释放高度。

官方评分

(iii)	For drawing a line or curve that approximates the trend of the plotted data	**Point C4**

Example Response



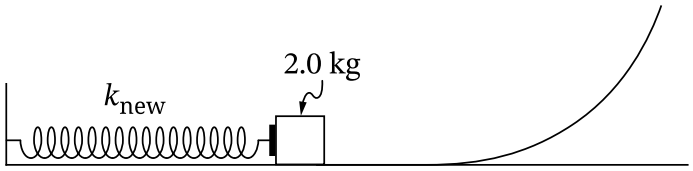
练习 2026 Q3 (C)(iii)

本问承接 2026 Q3 (C)(ii)。

The following information applies to parts C and D.

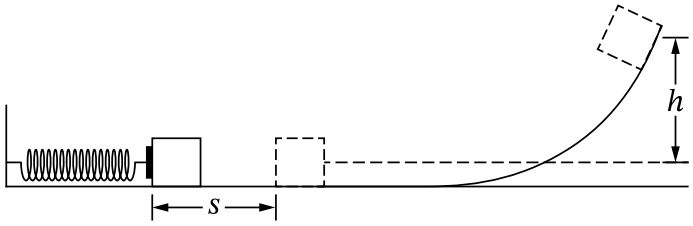
In a different experiment, the student is asked to determine the spring constant k_{new} of a new spring that is attached to a wall. A block of mass $2.0\ \text{kg}$ is placed next to the spring on a surface where frictional forces are negligible. The surface is horizontal near the spring and slopes upward into a ramp to the right of the initial position of the block, as shown in Figure 2.

Figure 2



The student pushes the block, compressing the spring a distance s, as shown in Figure 3. The block is released from rest.

Figure 3



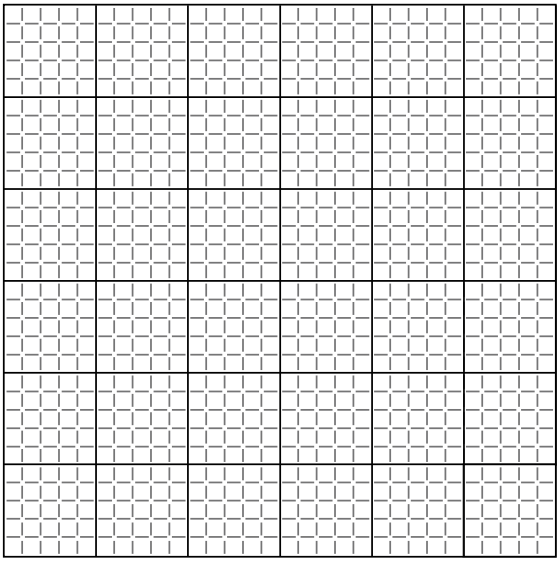
The student uses a meterstick to measure the maximum height h of the block on the ramp. The experiment is repeated several times with different compression distances s. The student's measurements of s and h are shown in Table 1.

Table 1

s (m)	h (m)
0.020	0.02
0.040	0.04
0.060	0.13
0.080	0.18
0.100	0.33

Quantity (units, if appropriate)

_____　　_____



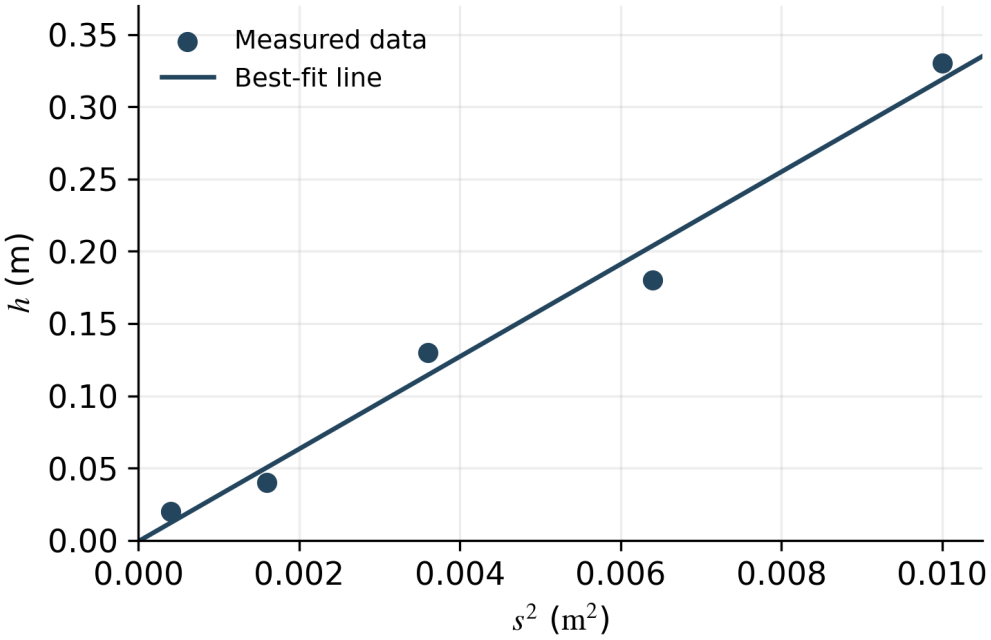
Quantity (units, if appropriate)

_____　　_____

iii. **Draw** a best-fit line for the data graphed in part C (ii).

解：

绘出全部五点，再画经过点群中央的拟合线。



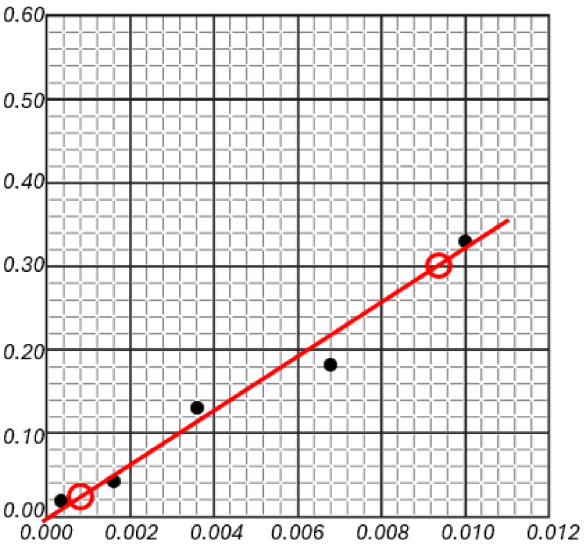
全部五个原数据点；横轴是压缩量平方，不能把压缩量本身标在这里。

官方评分

(iii)	For drawing a line or curve that approximates the trend of the plotted data	**Point C4**

Example Response

Quantity: ***h*** (units, if appropriate): **(m)**



阻力对速率：斜率就是线性阻力系数

纵轴为阻力、横轴为速率，理论关系Fdrag=bv中的b是直线斜率。在线上取相距较远的两点，用纵坐标差除横坐标差；选拟合线上的点，不强迫它们恰是某两个原测量点。

$$b = \frac{\Delta F_{\text{drag}}}{\Delta v}.$$

例题 2024 Set 2 Q2 (d)(ii)

本问承接 2024 Set 2 Q2 (d)(i)。

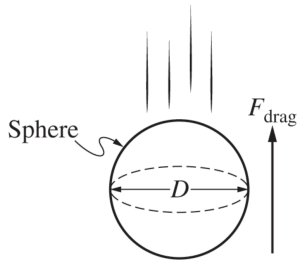
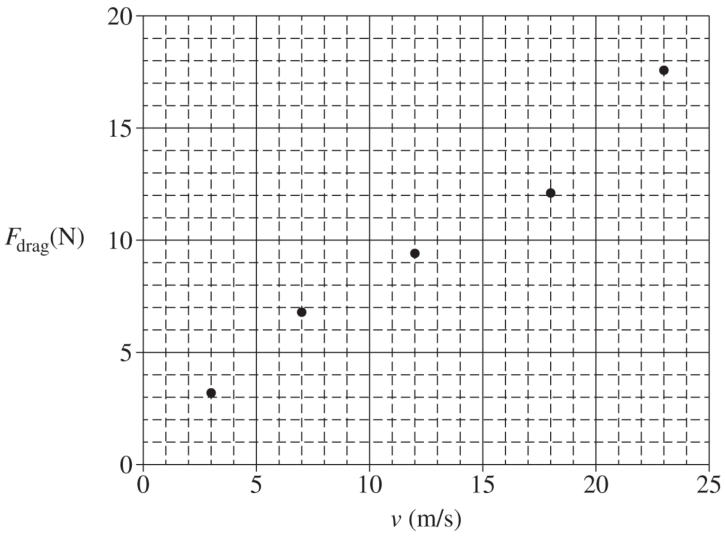


Figure 1

2. A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

(d) The student conducts an experiment to better understand the relationship between F_{drag} and v. The student makes measurements to calculate and graph the magnitude of F_{drag} as a function of v for the falling sphere.



ii. Use the best-fit line to **calculate** an experimental value for b.

解：

选拟合线上的两点，不强求两点恰是测量散点。此图横轴是速率、纵轴是阻力，斜率就是b。

$$b = \frac{\Delta F_{\text{drag}}}{\Delta v} = \frac{15 - 5}{21 - 5} = 0.625\ \text{kg/s}.$$

官方评分

(d)(ii) For calculating a value for the slope of the line using two points on the best-fit line **1 point**

Scoring Note: Using data points that fall on the best-fit line earns this point.

Example Response

$$\text{slope} = \frac{15\ \text{N} - 5\ \text{N}}{21\ \text{m/s} - 5\ \text{m/s}}$$

For using the correct relationship between the slope of the best-fit line and the value of b **1 point**

Example Response

$$\text{slope} = \frac{|F_{\text{drag}}|}{v}$$
$$\text{slope} = b$$

For a calculated value of b that is $0.6\,\text{kg/s} < b < 0.8\,\text{kg/s}$ **1 point**

Example Response

$$b = 0.625\,\text{kg/s}$$

Example Solution

$$|F_{\text{drag}}| = bv$$
$$\frac{|F_{\text{drag}}|}{v} = b$$
$$\text{slope} = b$$
$$b = \frac{15\ \text{N} - 5\ \text{N}}{21\,\text{m/s} - 5\,\text{m/s}}$$
$$b = 0.625\,\text{kg/s}$$

Total for part (d) 4 points

终速平方对质量：斜率是g除以阻力系数

平方阻力的终速关系mg=bvT^2，整理成vT^2=(g/b)m。横轴质量、纵轴终速平方，所以斜率S=g/b，而不是b。先从拟合线求S，再反解b=g/S，核对单位。

$$S = \frac{g}{b}, \qquad b = \frac{g}{S}.$$

例题 2024 Set 1 Q2 (d)(ii)

本问承接 2024 Set 1 Q2 (d)(i)。

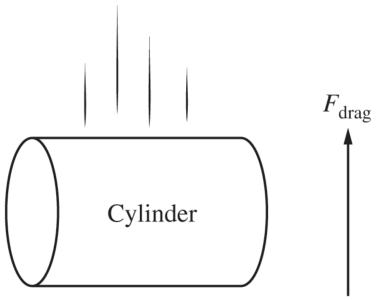
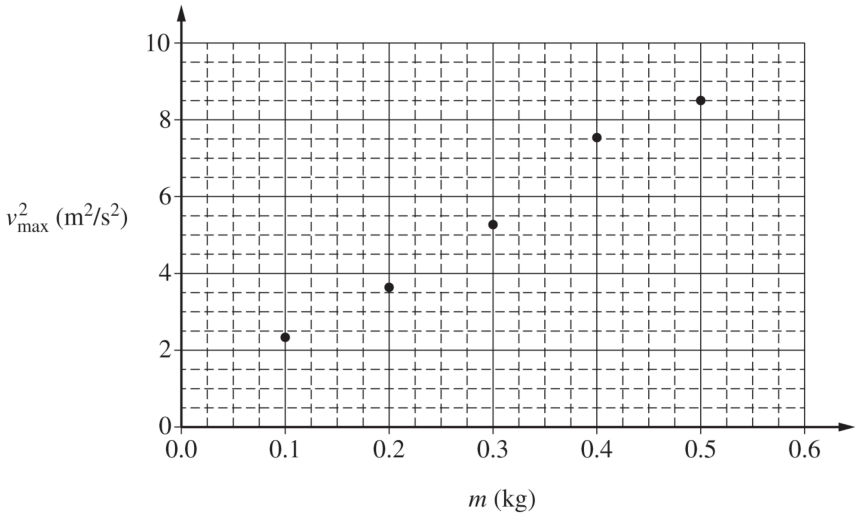


Figure 1

2. A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

(d) The student conducts an experiment to better understand the relationship between maximum speed v_{max} and mass. The student collects data to determine the maximum speed for cylinders dropped from rest, each with the same physical size and shape but a different mass m. The student then graphs v_{max}^2 as a function of mass.



ii. Use the best-fit line to **calculate** an experimental value for b.

解：

斜率是g/b，不是b。由拟合线的两个相距较远的点估计斜率约18，再反求阻力系数。

$$S \approx 18\ \mathrm{m^2/(s^2\,kg)}, \qquad b = \frac{g}{S} \approx \frac{9.8}{18} = 0.544\ \mathrm{kg/m}.$$

官方评分

(d)(ii) For calculating a value for the slope of the line using two points on the best-fit line **1 point**

Scoring Note: Using data points that fall on the best-fit line is acceptable.

Example Response

$$\text{slope} = \frac{9\,\text{m}^2/\text{s}^2 - 4.5\,\text{m}^2/\text{s}^2}{0.5\,\text{kg} - 0.25\,\text{kg}}$$

For using the correct relationship between the slope of the best-fit line and the value of b **1 point**

Example Response

$$\text{slope} = \frac{g}{b}$$

For a calculated value of b that is $0.45\,\text{kg/m} \le b \le 0.75\,\text{kg/m}$ **1 point**

Example Response

$$b = 0.544\,\text{kg/m}$$

Example Solution

$$mg - bv^2 = 0$$
$$bv^2 = mg$$
$$\frac{v^2}{m} = \frac{g}{b}$$
$$\text{slope} = \frac{g}{b}$$
$$b = \frac{g}{\text{slope}}$$
$$b = \frac{9.8\,\text{m/s}^2}{\frac{9\,\text{m}^2/\text{s}^2 - 4.5\,\text{m}^2/\text{s}^2}{0.5\,\text{kg} - 0.25\,\text{kg}}}$$
$$b = 0.544\,\text{kg/m}$$

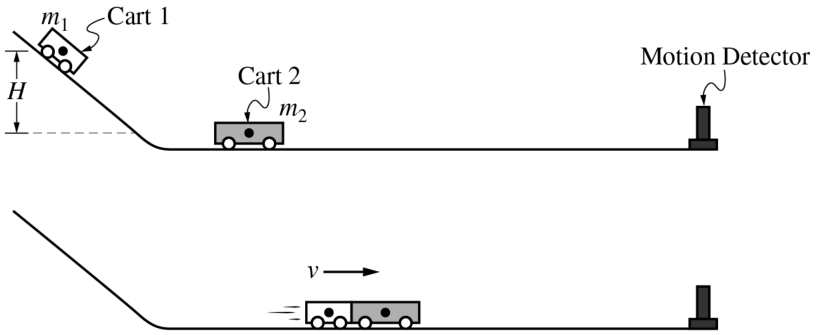
Total for part (d) 4 points

碰后速率图：斜率中含未知质量

先把已经推导出的共同末速度写成横坐标乘一个系数。若横轴是高度平方根，系数含m1/(m1+m2)；若横轴是压缩量，系数含√(km1)/(m1+m2)。由拟合线求斜率后，再把这两个系数分别与斜率对应，反解m2。

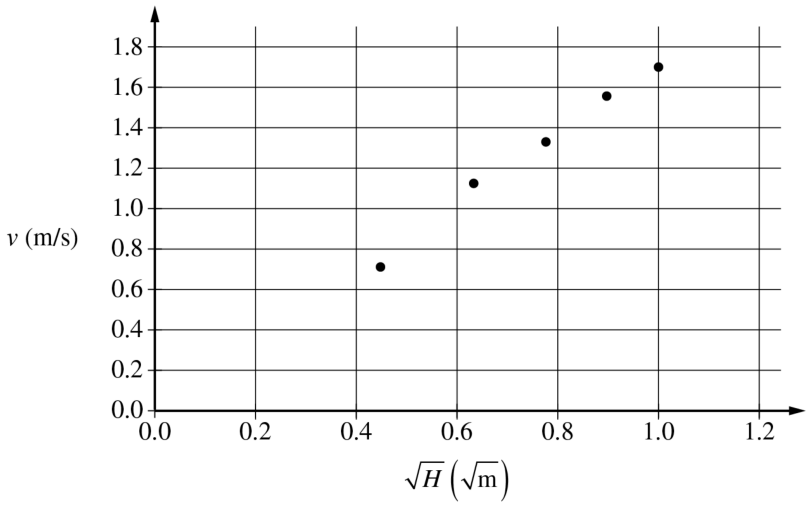
例题 2022 Set 1 Q2 (d)(ii)

本问承接 2022 Set 1 Q2 (c)、2022 Set 1 Q2 (d)(i)。



2. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250\,\text{kg}$. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



ii. Use the best-fit line to calculate the mass of Cart 2.

解：

横轴是高度的平方根。选线上的两点算斜率，再与上式中平方根前的系数对应。

$$S = \frac{1.20 - 0.60}{0.70 - 0.35} \approx 1.72\ \text{m}^{1/2}/\text{s}.$$

$$S = \frac{m_1\sqrt{2g}}{m_1 + m_2}, \qquad m_2 = m_1\left(\frac{\sqrt{2g}}{S} - 1\right) \approx 0.39\ \text{kg}.$$

官方评分

(d)(ii)	For calculating the slope using two points on the best-fit line	**1 point**
	For correctly relating the slope of the best-fit line to the mass of Cart 2	**1 point**
	For a correct mass of Cart 2	**1 point**

Example Response

$$slope = \frac{(1.20 - 0.60)\ \text{m/s}}{(0.70 - 0.35)\sqrt{\text{m}}} = 1.72\ \frac{\sqrt{\text{m}}}{\text{s}}$$

$$slope = \frac{m_1}{m_1 + m_2}\sqrt{2g}$$

$$m_2 = \frac{m_1\sqrt{2g}}{slope} - m_1$$

$$m_2 = \frac{(0.25\ \text{kg})\sqrt{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)}}{1.72\ \frac{\sqrt{\text{m}}}{\text{s}}} - (0.25\ \text{kg})$$

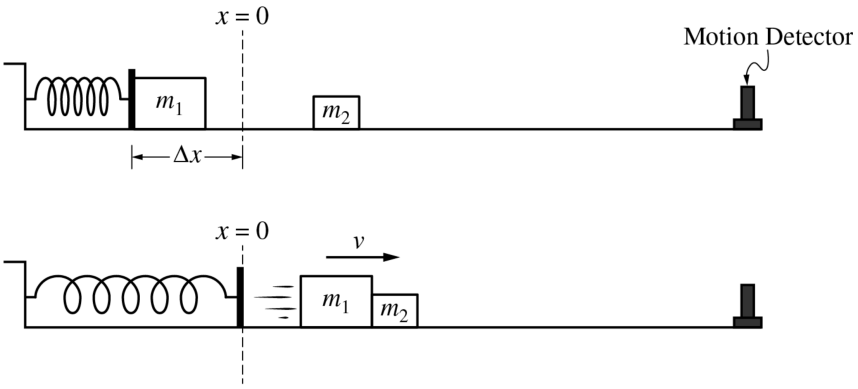
$$\therefore m_2 = 0.39\ \text{kg}$$

Scoring Note: Acceptable responses for mass are 0.30 to 0.60 kg

Total for part (d)　4 points

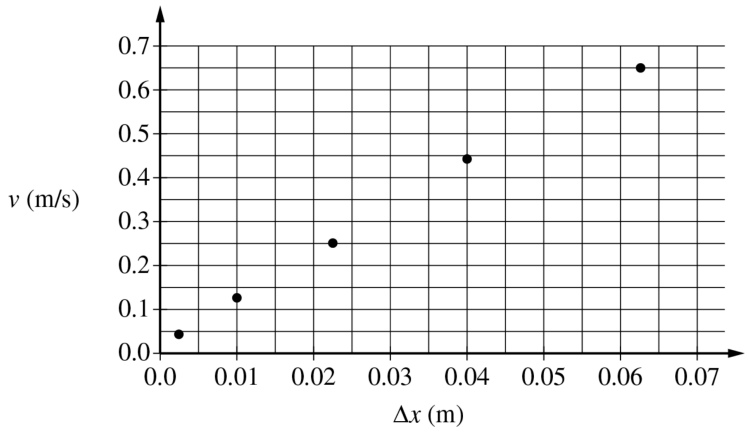
例题 2022 Set 2 Q2 (d)(ii)

本问承接 2022 Set 2 Q2 (c)、2022 Set 2 Q2 (d)(i)。



2. Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

(d) A group of students use the setup to perform an experiment. They measure the mass of Block 1 to be $m_1 = 0.500$ kg, and the spring constant k of the spring to be 150 N/m. The mass of Block 2 is unknown. They perform several trials and in each trial the spring is compressed a different distance Δx and the final velocity v of the two-block system is measured. They graph v as a function of Δx, as shown below.



ii. Use the best-fit line to calculate the mass of Block 2.

解：

横轴就是压缩量，斜率单位为每秒。以图中示例拟合点计算。

$$S = \frac{0.60 - 0.20}{0.055 - 0.0175} \approx 10.67\ \mathrm{s}^{-1}, \qquad m_2 = \frac{\sqrt{km_1}}{S} - m_1 = \frac{\sqrt{150(0.500)}}{10.67} - 0.500 \approx 0.312\ \mathrm{kg}.$$

官方评分

数值核对：原评分示例所取拟合点对应的分子应为0.60−0.20，得到斜率10.67 s^{-1}；原页公式把0.20误印为0.02。质量应按正确斜率反求，约为0.31 kg；评分接受0.27–0.37 kg。

(d)(ii)	For calculating the slope using two points on the line	**1 point**
	For correctly relating the slope of the best-fit line to the mass of Block 2	**1 point**
	For a correct mass of Block 2	**1 point**

Example Response

$$slope = \frac{(0.60 - 0.02)\ \text{m/s}}{(0.055 - 0.0175)\ \text{m}} = 10.67\ \text{s}^{-1}$$

$$slope = \frac{\sqrt{km_1}}{m_1 + m_2}$$

$$m_2 = \frac{\sqrt{km_1}}{slope} - m_1$$

$$m_2 = \frac{\sqrt{\left(150\ \frac{\text{N}}{\text{m}}\right)(0.50\ \text{kg})}}{10.67\ \text{s}^{-1}} - 0.50\ \text{kg}$$

$$m_2 = 0.31\ \text{kg}$$

Scoring Note: Acceptable responses for mass are $0.27\ \text{kg} - 0.37\ \text{kg}$.

Total for part (d) 4 points

停止距离实验：斜率给摩擦系数

原图h对xmax的关系为h=μxmax。在线上取两点求Δh/Δxmax，得到无单位的摩擦系数。散点不必全部落在线上；用同一条拟合线选点，结果可有合理读图差别。

$$\mu = \frac{\Delta h}{\Delta x_{\text{max}}}.$$

例题 2025 Q3 (D)

本问承接 2025 Q3 (C)(iii)。

3. A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance h, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

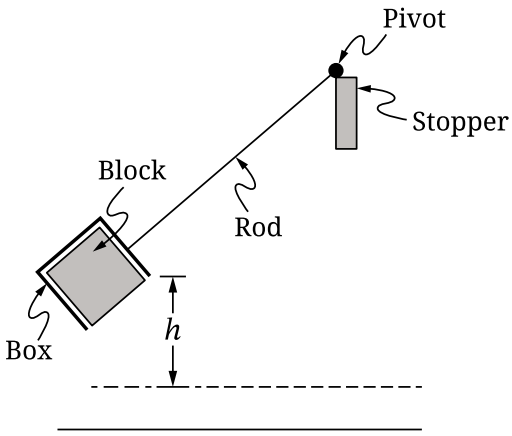


Figure 1

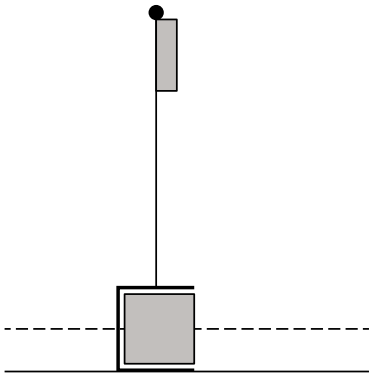


Figure 2

C. The experiment is repeated, but the horizontal surface on which the block slides is replaced with a new rough surface, as shown in Figure 3. The coefficient of kinetic friction between the block and the new surface is μ.

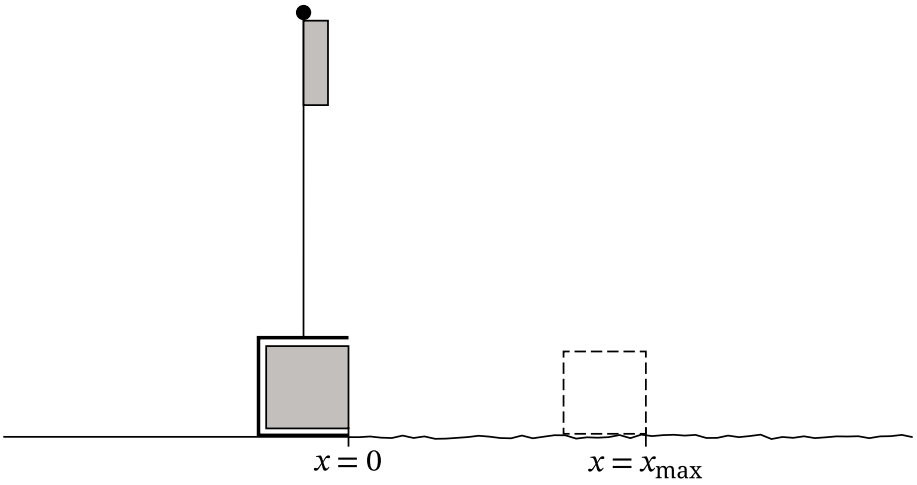


Figure 3

The block-box system is pulled aside so that the center of mass of the block is displaced various vertical distances h and then released from rest. For each vertical distance, students measure the position $x = x_{\max}$ at which the block comes to rest.
The students' measurements of h and $x_{\max}$ are shown in Table 1.

Table 1

h (m)	$x_{\max}$ (m)
0.30	0.76
0.45	1.10
0.60	1.40
0.75	1.90
0.90	2.30

D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for μ.

解：

在线上选两点求斜率，斜率就是摩擦系数。本图用最小二乘线示范，斜率约0.384。读图拟合可以给出相近值。

$$\mu = S \approx 0.384.$$

系数无单位；同一高度下停止距离越短，对应摩擦越强。

官方评分

D For correctly relating the slope of the best-fit line to the value of μ **Point D1**

Examples of acceptable responses includes the following:

Graph	Analysis
h vs. $x_{\max}$	slope = μ

For a value of μ within a range of 0.30 to 0.50 **Point D2**

Example Response

$E_0 + W = E_f$

$mgh - F_f x_{\max} = 0$

$mgh = \mu mg x_{\max}$

$h = \mu x_{\max}$

$y = mx + b$

$h = \mu x_{\max}$

slope = μ

$$\text{slope} = \frac{0.95\ \text{m} - 0.30\ \text{m}}{2.4\ \text{m} - 0.70\ \text{m}}$$

slope = 0.38

μ = slope = 0.38

升高实验：由斜率恢复弹簧常量

原图h对s^2，斜率S=knew/(2mg)。先从拟合线计算S，再乘2mg得到弹簧常量。斜率的单位是每米，弹簧常量的单位是牛顿每米，二者不是同一个量。

$$k_{\text{new}} = 2mgS.$$

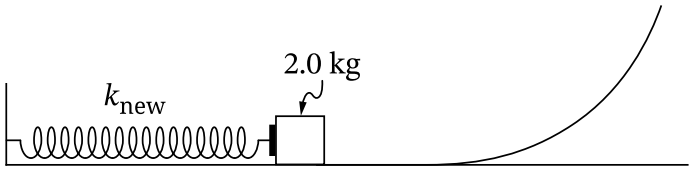
例题 2026 Q3 (D)

本问承接 2026 Q3 (C)(iii)。

The following information applies to parts C and D.

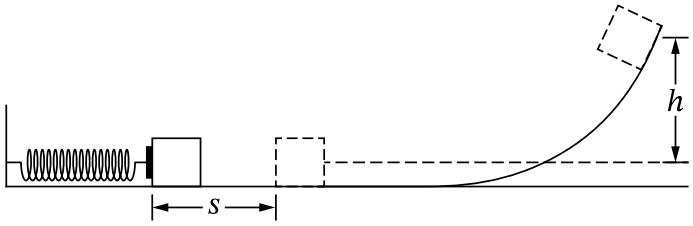
In a different experiment, the student is asked to determine the spring constant k_{new} of a new spring that is attached to a wall. A block of mass $2.0\ \text{kg}$ is placed next to the spring on a surface where frictional forces are negligible. The surface is horizontal near the spring and slopes upward into a ramp to the right of the initial position of the block, as shown in Figure 2.

Figure 2



The student pushes the block, compressing the spring a distance s, as shown in Figure 3. The block is released from rest.

Figure 3



The student uses a meterstick to measure the maximum height h of the block on the ramp. The experiment is repeated several times with different compression distances s. The student's measurements of s and h are shown in Table 1.

Table 1

s (m)	h (m)
0.020	0.02
0.040	0.04
0.060	0.13
0.080	0.18
0.100	0.33

Part D

Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for k_{new}.

解：

原题质量是2.0 kg。图的拟合斜率约31.96每米，代回斜率与弹簧常量的关系。

$$k_{\text{new}} = 2mgS \approx 2(2.0)(9.8)(31.96) = 1.25 \times 10^{3}\ \text{N/m}.$$

这里测量散差允许相近读图值；不能把斜率本身直接当作弹簧常量。

官方评分

D For correctly relating the slope of the best-fit line to the value of k_{new} **Point D1**

Examples of acceptable responses include the following:

Graph	Analysis
h as a function of s^2	slope $= \frac{k_{\text{new}}}{2mg}$
$\sqrt{h}$ as a function of s	slope $= \sqrt{\frac{k_{\text{new}}}{2mg}}$
mgh as a function of $\frac{1}{2}s^2$	slope $= k_{\text{new}}$

For calculating an experimental value of k_{new} within a range of 1150 N/m to 1400 N/m **Point D2**

Example Response

$$mgh = \frac{1}{2}k_{\text{new}}s^2$$

$$\text{slope} = \frac{k_{\text{new}}}{2mg}$$

$$k_{\text{new}} = 2mg(\text{slope})$$

$$\text{slope} = \frac{0.30\text{ m} - 0.024\text{ m}}{0.0094\text{ m}^2 - 0.0008\text{ m}^2} = 32\text{ m}^{-1}$$

$$k_{\text{new}} = 2mg(\text{slope})$$

$$k_{\text{new}} = 2(2.0\text{ kg})\left(9.8\text{ m/s}^2\right)\left(32\text{ m}^{-1}\right)$$

$$k_{\text{new}} = 1254\text{ N/m}$$

平动周期平方图：斜率反求弹簧常量

并联弹簧周期平方满足T^2=($4\pi^2$m/k)N^{-1}。图的纵轴T^2、横轴N^{-1}，斜率是$4\pi^2$m/k。先求线的斜率，再把常量k移到等式另一边；斜率增大时，推算出的k减小。

$$k = \frac{4\pi^2 m}{S}.$$

例题 2023 Set 2 Q2 (c)(ii)

本问承接 2023 Set 2 Q2 (a)、2023 Set 2 Q2 (c)(i)。

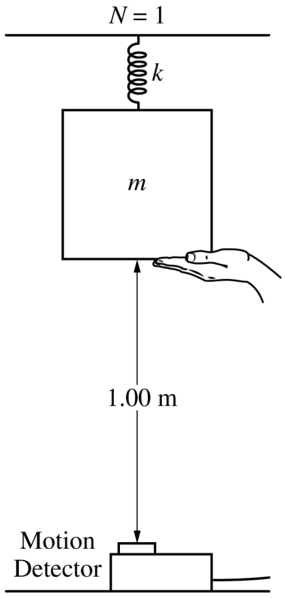


Figure 1

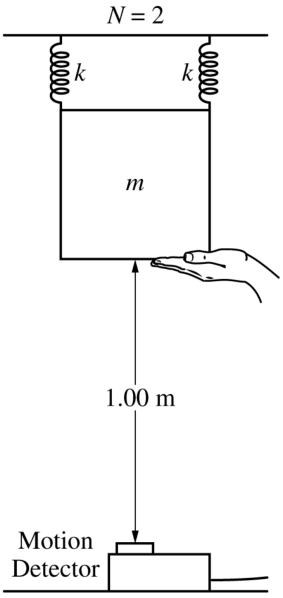
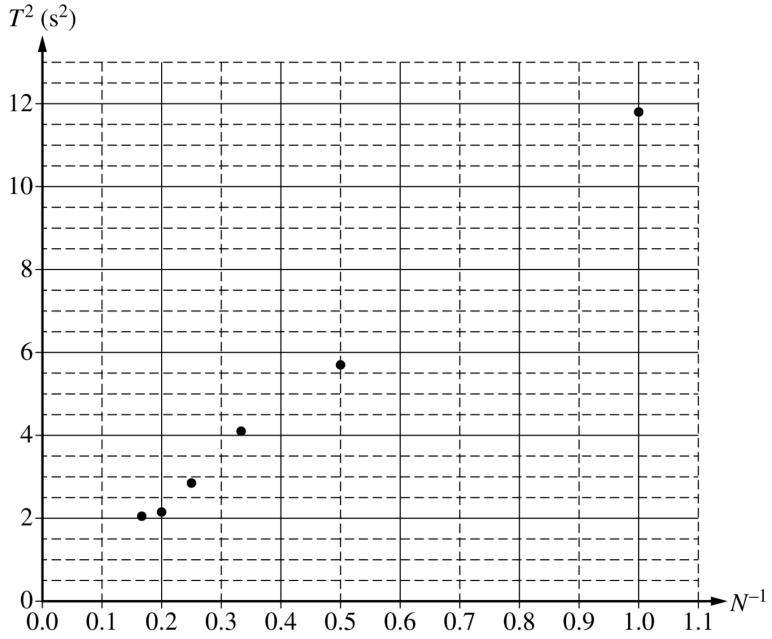


Figure 2

2. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height $1.00\ \text{m}$ above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

(c) The student plots the data for T^2 as a function of N^{-1}, as shown.



ii. The mass of the block is measured to be $m = 1.5$ kg. Using the graph, calculate an experimental value for the spring constant k for a single spring.

解：

由拟合线上两点求斜率，再对应系数。

$$S = \frac{9.5 - 3.5}{0.8 - 0.3} = 12\,\mathrm{s}^2, \qquad k = \frac{4\pi^2 m}{S} = \frac{4\pi^2(1.5)}{12} = 4.93\,\mathrm{N/m}.$$

官方评分

(c)(ii) For using two points from the best-fit line to calculate slope **1 point**

Scoring Note: Using data points that fall on the best-fit line is acceptable.

Example Response

$$\text{slope} = \frac{\Delta\left(T^2\right)}{\Delta\left(N^{-1}\right)}$$

$$\text{slope} = \frac{(9.5\,\text{s}^2 - 3.5\,\text{s}^2)}{(0.8 - 0.3)}$$

$$\therefore \text{slope} = 12\,\text{s}^2$$

For relating the slope of the line of the T^2 vs. N^{-1} graph consistent with the expression derived in part (a) **1 point**

Example Response

$$T^2 = 4\pi^2 \frac{m}{Nk}$$

$$\text{slope} = T^2 N$$

$$\text{slope} = 4\pi^2 \frac{m}{k}$$

For a calculated answer that has units of N/m **1 point**

Example Response

$$k = 4.93\ \text{N/m}$$

Example Solution

$$T^2 = 4\pi^2 \frac{m}{Nk}$$

$$\text{slope} = 4\pi^2 \frac{m}{k}$$

$$k = \frac{4\pi^2 m}{\text{slope}}$$

$$k = \frac{4\pi^2(1.5\ \text{kg})}{\frac{(9.5\,\text{s}^2 - 3.5\,\text{s}^2)}{(0.8 - 0.3)}}$$

$$\rightarrow k = \frac{4\pi^2(1.5\ \text{kg})}{12\,\text{s}^2}$$

$$k = 4.93\ \text{N/m}$$

扭转周期图：斜率平方才给质量

扭转摆的T对√N图是直线。系数中含单盘质量的平方根，所以用斜率求质量时必须把斜率平方，不能一次线性代入。先写周期公式的系数，再解M并核对R^2和扭转常量的位置。

例题 2023 Set 1 Q2 (c)(ii)

本问承接 2023 Set 1 Q2 (a)、2023 Set 1 Q2 (c)(i)。

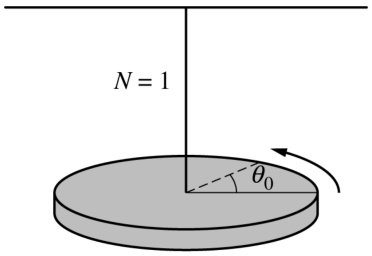


Figure 1

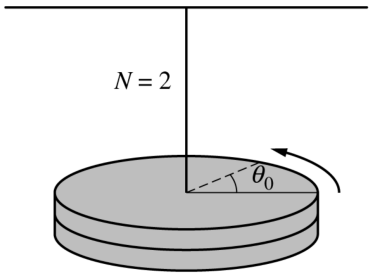
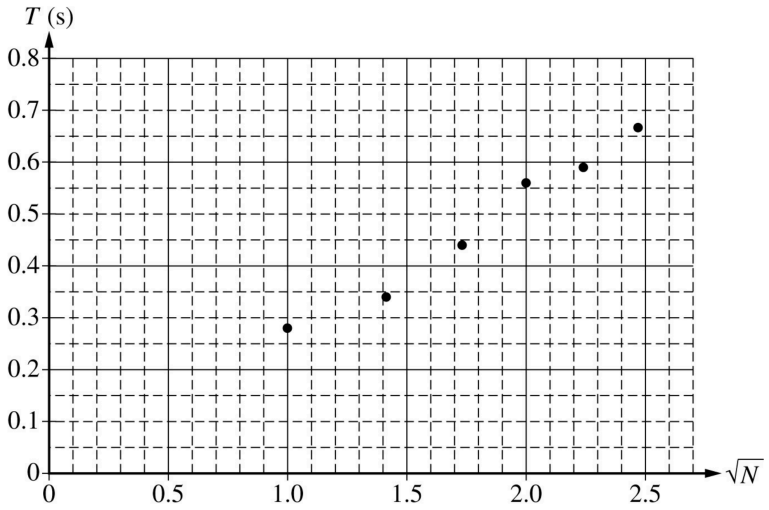


Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

   The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(c) The student plots the data for T as a function of $\sqrt{N}$, as shown.



ii. The student previously determined that the radius of a disk is $R = 0.2\,\text{m}$ and found that $\kappa = 1.6\,\text{N·m}$. Using the graph, calculate the mass M of a single disk.

解：

比较直线式，斜率含单盘质量的平方根；因此解质量时必须把斜率平方。

$$T = \left(2\pi\sqrt{\frac{MR^2}{2\kappa}}\right)\sqrt{N}, \qquad M = \frac{\kappa S^2}{2\pi^2 R^2}.$$

$$S \approx \frac{0.4 - 0.2}{1.5 - 0.8} = 0.286\,\text{s}, \qquad M \approx \frac{1.6(0.286)^2}{2\pi^2(0.20)^2} = 0.166\,\text{kg}.$$

官方评分

(c)(ii) For using two points on the line to calculate the slope **1 point**

Example Response

$$\text{slope} = \frac{T_2 - T_1}{\sqrt{N_2} - \sqrt{N_1}}$$

$$\text{slope} = \frac{0.4\,\text{s} - 0.2\,\text{s}}{1.5 - 0.8} = 0.29\,\text{s}$$

Scoring Note: Slope values may range from .22 s to .33 s.

For correctly relating the slope to the period of the torsional pendulum consistent with part (a) **1 point**

Example Response

$$T = 2\pi\sqrt{\frac{NI}{\kappa}}$$

$$\rightarrow \text{slope} = 2\pi\sqrt{\frac{I}{\kappa}}$$

For substituting the slope into the equation to determine the mass of the disk **1 point**

Example Response

$$\rightarrow M = \frac{\kappa(\text{slope})^2}{2\pi^2 R^2}$$

Example Solution

$$T = 2\pi\sqrt{\frac{NI}{\kappa}}$$

$$\rightarrow \text{slope} = 2\pi\sqrt{\frac{I}{\kappa}}$$

$$(\text{slope})^2 = 4\pi^2\frac{I}{\kappa}$$

$$(\text{slope})^2 = 4\pi^2\frac{\left(\frac{1}{2}MR^2\right)}{\kappa}$$

$$\rightarrow M = \frac{\kappa(\text{slope})^2}{2\pi^2 R^2}$$

$$M = \frac{(1.6\,\text{N}\cdot\text{m})\left(\frac{0.4\,\text{s} - 0.2\,\text{s}}{1.5 - 0.8}\right)^2}{2\pi^2(0.2\,\text{m})^2}$$

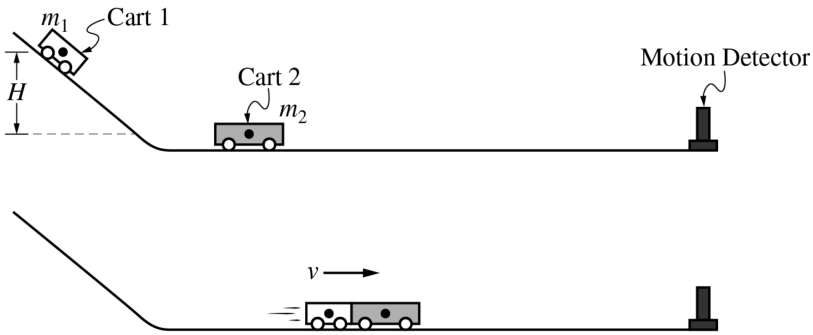
$$\therefore M = 0.17\,\text{kg}$$

同一图线重新校准：固定斜率追踪参数

图线是原来的测量结果，重新测量仪器参数时，把图线斜率保持不变。在同一个斜率等式里追踪未知质量与输入参数：斜面实验m2正比于m1；弹簧实验m2增大需要k增大。说明等式里哪一项必须随之改变，不能只说测量可能有误。

例题 2022 Set 1 Q2 (e)

本问承接 2022 Set 1 Q2 (d)(ii)。



2. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg ?

_____$m_1' > 0.250$ kg　　_____$m_1' < 0.250$ kg　　_____ $m_1' = 0.250$ kg

Justify your answer.

解：

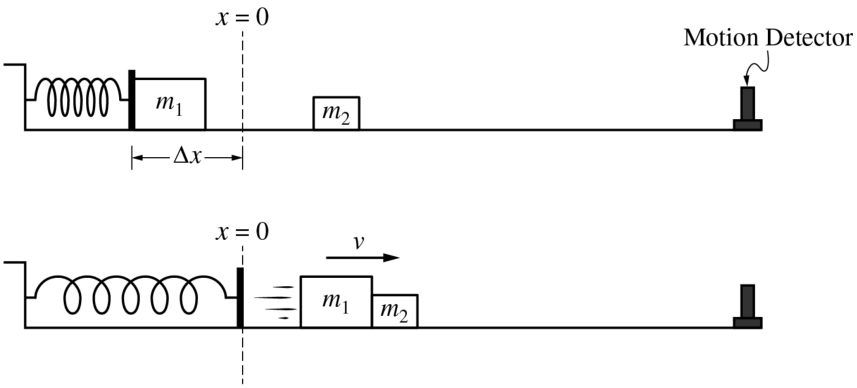
选重新测得的车1质量较小。相同图线斜率使括号中的倍数固定；由刚得到的关系，车2的推算质量正比于车1的输入质量。要使推算的车2质量降低，车1的输入质量也必须降低。

官方评分

(e)	For selecting "$m_1' < 0.250$ kg" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**
	Example Response *A smaller m_2 indicates that the initial energy and momentum was smaller. With identical slope and height H this means that the mass m_1' must be smaller.*	
		Total for part (e) 2 points
		Total for question 2 15 points

例题 2022 Set 2 Q2 (e)

本问承接 2022 Set 2 Q2 (d)(ii)。



2. Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

(e) After the experiment, the students use a balance to measure the mass of Block 2 and find it to be greater than what was determined in part (d). To explain this discrepancy, one of the students proposes that the spring constant was incorrectly measured at the beginning of the experiment. The students measure the spring constant again and record a new value, k'.

Should the students expect that k' be greater than 150 N / m, less than 150 N / m, or equal to 150 N / m ?

_____$k' > 150$ N / m　　_____$k' < 150$ N / m　　_____ $k' = 150$ N / m

Justify your answer.

解：

选新测得的弹簧常量更大。图线斜率和块1质量固定，车2实际质量较大使分母较大；要保持同一斜率，分子中的弹簧常量也必须增大。

$$k = \frac{S^2(m_1 + m_2)^2}{m_1}, \qquad k' > 150\ \mathrm{N/m}.$$

官方评分

评分文字核对：正确选择是新测得的弹簧常量较大。保持图线斜率与块1质量不变，块2质量增大，必须使弹簧常量增大；原页示例最后一句的减小结论与评分选择及前句矛盾，应以这里的斜率关系为依据。

(e)	For selecting "$k' > 150$ N/m" with an attempt at relevant justification	**1 point**
	For a correct justification	**1 point**
	Example Response *A greater* m_2 *indicates the spring constant* k' *should be greater than* 150 N/m. *The slope of the graph is the same so as* m_2 *increases* k *must be smaller.*	
	Total for part (e)	**2 points**
	Total for question 2	**15 points**

周期被系统性测大：误差方向必须与参数相连

提出一个能产生题述偏差的具体误差，并沿公式追踪方向。周期被测大使图线斜率变大；平动图里k与斜率成反比，k偏小；扭转图里M与斜率平方成正比，M偏大。同样的计时偏差在不同反求公式中有不同结果。

例题 2023 Set 2 Q2 (c)(iii)

本问承接 2023 Set 2 Q2 (c)(ii)。

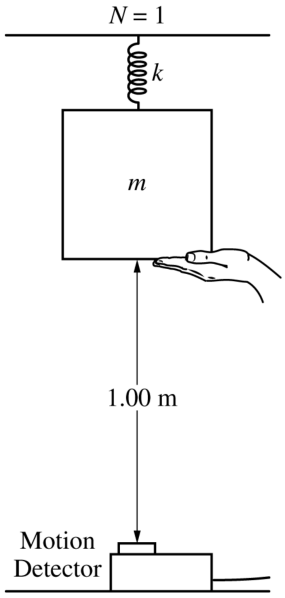


Figure 1

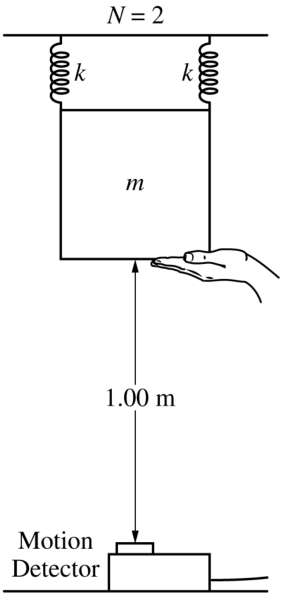
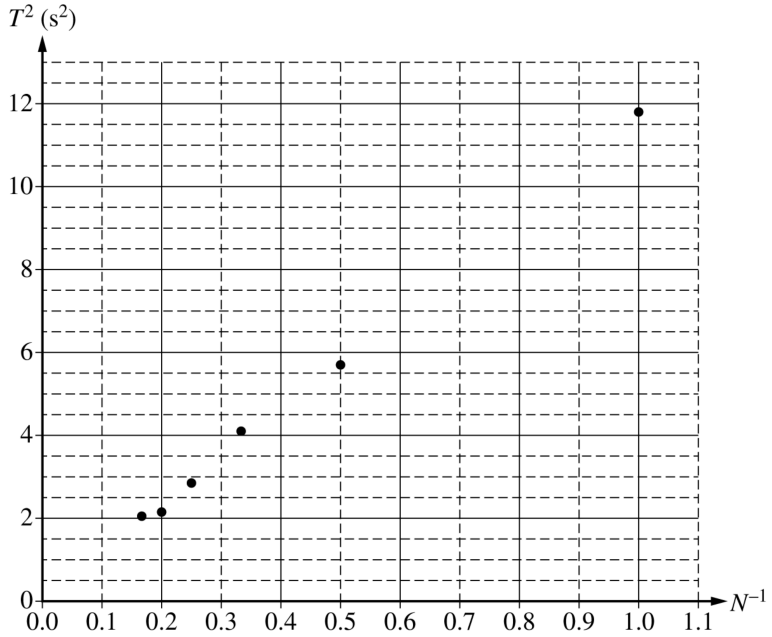


Figure 2

2. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height $1.00\ \text{m}$ above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

(c) The student plots the data for T^2 as a function of N^{-1}, as shown.



iii. The student finds that the value given by the manufacturer for the spring constant is larger than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value for k. Briefly justify your answer.

解：

一个可行误差是探测器校准使周期被系统性测大。周期平方图的斜率偏大，而弹簧常量与斜率成反比，故推算的k偏小，方向与题述一致。

官方评分

(c)(iii) For indicating a source of error that could result in the observed difference with an attempt at a relevant justification **1 point**

Examples include:
- Experimental uncertainties in the mass of the system
- Motion detector miscalibration
- Timing error due to reaction time

For a correct justification that links the source of experimental error to the smaller experimental value of k **1 point**

Examples include:
- Experimental uncertainties in the mass of the system (e.g., mass of block is too small, not accounting for the mass of the spring)
- Calibration of the motion sensor produces a graph from which too large a period is measured
- A larger period due to measurement error will result in a smaller value of k

Total for part (c) 6 points

例题 2023 Set 1 Q2 (c)(iii)

本问承接 2023 Set 1 Q2 (c)(ii)。

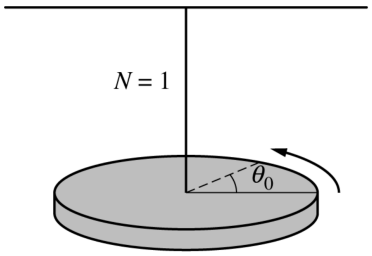


Figure 1

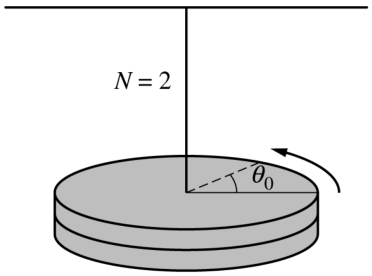
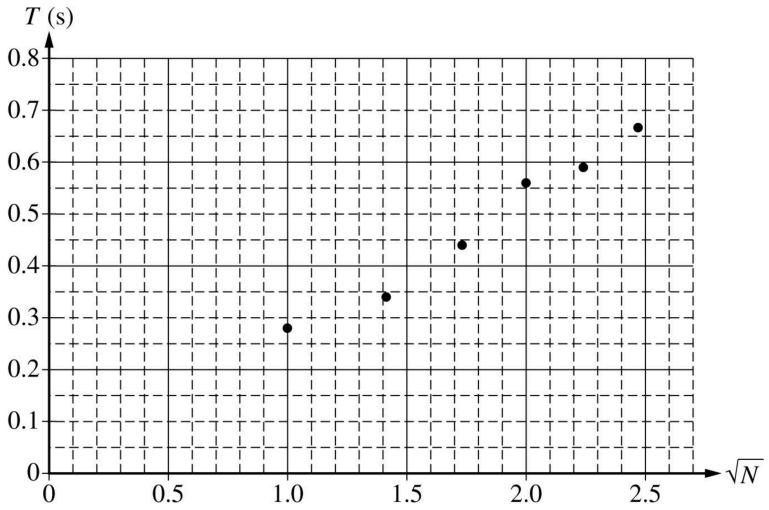


Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

   The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(c) The student plots the data for T as a function of $\sqrt{N}$, as shown.



iii. The student finds that the value given by the manufacturer for the mass of the disk is less than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value of M. Justify your answer.

解：

周期被系统性测大会使图线斜率偏大；质量与斜率平方成正比，所以实验质量偏大，符合题述差异。

官方评分

(c)(iii)	For indicating a source of error that could correctly explain the observed difference with an attempt at a relevant justification	**1 point**
	For a correct justification that links the source of experimental error to the larger experimental value for M . Accept one of the following: • Experimental uncertainties for the period, for example the period is measured to be larger • The mass is more concentrated to the edge, or some other distribution that results in a larger rotational inertia **Scoring Note:** Responses that indicate the given κ is too large or the given radius is too small may earn the second point.	**1 point**
	Example Solution *The experimental value of the mass could be too large because the period measured by the student is too large.* **OR** *The experimental value of the mass could be too large because the mass of the disk is concentrated at the edge of the disk, causing the rotational inertia of the disk to be larger than that used to determine the experimental value of mass.*	
	Total for part (c)	**6 points**